

## ARGUMENT OF DRAMA.

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### ACT I.

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*Preliminaries to examination of  
 Modern Rivals.*

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#### SCENE I.

[MINOS *and* RHADAMANTHUS.]

|  | PAGE |
|--|------|
| Consequences of allowing the use of various Manuals<br>of Geometry : that we must accept |      |
| (1) 'Circular' arguments . . . . .   | 1    |
| (2) Illogical do. . . . .  | 2    |
| Example from Cooley . . . . .  | „    |
| „ „ Wilson . . . . .   | 4    |

## SCENE II.

[MINOS and EUCLID.]

§ 1. *A priori reasons for retaining Euclid's Manual.*

|   | PAGE |
|---|------|
| We require, in a Manual, a selection rather than a complete repertory of Geometrical truths . . . . . | 6    |
| Discussion limited to subject-matter of Euc. I, II. . . . .   | 8    |
| One fixed logical sequence essential . . . . .  | „    |
| One system of numbering desirable . . . . .   | 10   |
| <i>A priori</i> claims of Euclid's sequence and numeration to be retained . . . . .                   | 11   |
| New theorems might be interpolated without change of numeration . . . . .                             | „    |

§ 2. *Method of procedure in examining Modern Rivals.*

|   |    |
|---|----|
| Proposed changes which, even if proved to be essential, <i>would not</i> necessitate the abandonment of Euclid's Manual:— . . . . . | 13 |
| (1) Propositions to be omitted ;  |    |
| (2) „ to be replaced by new proofs ;  |    |
| (3) New propositions to be added.   |    |
| Proposed changes which, if proved to be essential, <i>would</i> necessitate such abandonment:— . . . . .                            | 15 |
| (1) Separation of Problems and Theorems ;   |    |
| (2) Different treatment of Parallels.   |    |

---

*ARGUMENT OF DRAMA.*

xv

|  | PAGE |
|--|------|
| Other subjects of enquiry :— . . . . .   | 15   |
| (3) Superposition ;  |      |
| (4) Use of diagonals in Euc. II ;  |      |
| (5) Treatment of Lines ;   |      |
| (6)       "       of Angles ;  |      |
| (7) Euclid's propositions omitted ;  |      |
| (8)       "       "       newly treated ;  |      |
| (9) New propositions ;   |      |
| (10) Style, &c.  |      |
| List of authors to be examined, viz. :— . . . . .  | 16   |
| Legendre, Cuthbertson, Wilson, Pierce, Willock,<br>Chauvenet, Loomis, Morell, Reynolds, Wright,<br>Syllabus of Association for Improvement of Geo-<br>metrical Teaching, Wilson's 'Syllabus'-Manual. |      |

---

§ 3. *The combination, or separation, of Problems  
and Theorems.*

---

|   |    |
|---|----|
| Reasons assigned for separation . . . . .               | 18 |
| Reasons for combination :— . . . . .                    | 19 |
| (1) Problems are also Theorems ;                        |    |
| (2) Separation would necessitate a new numer-<br>ation, |    |
| (3) and hypothetical constructions.                     |    |

---

§ 4. *Syllabus of propositions relating to Pairs of Lines.*

---

|  |    |
|--|----|
| Three classes of Pairs of Lines :— . . . . . | 20 |
| (1) Having two common points ;               |    |

|   | PAGE |
|---|------|
| (2) Having a common point and a separate point ;  |      |
| (3) „ no common point.  |      |
| Four kinds of ‘ properties ’ ; as to . . . . .  | 21   |
| (1) common or separate points ;   |      |
| (2) equality, or otherwise, of angles made with transversals ;  |      |
| (3) equidistance, or otherwise, of points on the one from the other ;   |      |
| (4) direction.  |      |
| Symbols to be employed . . . . .  | 22   |
| Method of deducing contranominal propositions . . . . .   | 25   |
| Propositions divisible into two classes :— . . . . .  | 27   |
| (1) Deducible from undisputed axioms ;  |      |
| (2) „ disputable „  |      |
| Three classes of Pairs of Lines :— . . . . .  | „    |
| (1) Coincidental ;  |      |
| (2) Intersectional ;  |      |
| (3) Separational.   |      |
| Subjects and predicates of propositions concerning these three classes :—   |      |
| Coincidental . . . . .  | „    |
| Intersectional . . . . .  | 29   |
| Separational . . . . .  | 30   |
| TABLE I. <i>Containing undisputed Axioms, and Theorems admitted to be real, valid, and deducible from undisputed Axioms</i> . . . . .   | 32   |
| Subjects and predicates of other propositions concerning Separational Lines . . . . .   | 34   |
| TABLE II. <i>Consisting of Theorems admitted to be real and valid, but not deducible from undisputed Axioms. They require, for proof, at least one Axiom, viz. either one of themselves, or else the proposition ‘ sD is real ’</i> . . . . . | 35   |

*ARGUMENT OF DRAMA.* xvii

|  | PAGE |
|--|------|
| Theorems of Table II, which have been proposed as              |      |
| Axioms:— . . . . .   | 37   |
| (1) Euclid's Axiom ;   |      |
| (2) T. Simpson's Axiom ;                                       |      |
| (3) Clavius'           ,,                                      |      |
| (4) Playfair's         ,,                                      |      |
| (5) The 'Ahmedabad' Axiom.                                     |      |
| Method by which it will be shown (in App. V) that              |      |
| <i>any</i> Theorem of Table II is sufficient logical basis     |      |
| for all the rest . . . . .                                     | 38   |
| 'Real,' and 'valid,' propositions defined . . . . .            | ,,   |
| TABLE III. <i>Consisting of valid but ideal propositions ;</i> |      |
| <i>some being undisputed Axioms, the rest Theorems</i>         |      |
| <i>deducible from undisputed Axioms: but all depending</i>     |      |
| <i>for their reality on that of 'sD' . . . . .</i>             | 40   |
| TABLE IV. <i>Consisting of real Theorems, not admitted to</i>  |      |
| <i>be valid: their validity depending on the reality of</i>    |      |
| <i>'sD' . . . . .</i>  | 41   |

§ 5. *Playfair's Axiom.*

|   |    |
|---|----|
| Is Euclid's Ax. 12 axiomatic? . . . . .                 | 42 |
| Need of test for meeting of finite Lines . . . . .      | 43 |
| Euclid's and Playfair's Axioms deducible, each from     |    |
| the other . . . . .                                     | ,, |
| Reasons for preferring Euclid's Axiom:—                 |    |
| (1) Playfair's does not show <i>which way</i> the Lines |    |
| will meet; . . . . .                                    | 44 |
| (2) Playfair's is less comprehensible; . . . . .        | 45 |
| (3) and asserts more than Euclid's, the additional      |    |
| matter being superfluous . . . . .                      | ,, |

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 Charles L. Dodgson  
 Table of Contents  
[More information](#)

|   |                           |      |
|---|---------------------------|------|
| xviii   | <i>ARGUMENT OF DRAMA.</i> |      |
|   |                           | PAGE |
| Objection to Euclid's Axiom (that it is the converse<br>of I. 17) untenable . . . . . |                           | 46   |
| <hr/> <i>§ 6. Principle of Superposition.</i> <hr/>                                   |                           |      |
| Used by Moderns in Euc. I. 5 . . . . .  |                           | 46   |
| „ „ I. 24 . . . . .   |                           | 48   |
| <hr/> <i>§ 7. Omission of Diagonals in Euc. II.</i> <hr/>                             |                           |      |
| Proposal tested by comparing Euc. II. 4, with Mr.<br>Wilson's version of it . . . . . |                           | 49   |

## ACT II.

[MINOS *and* NIEMAND.]

*Manuals which reject Euclid's treatment  
of Parallels.*

### SCENE I.

|                        |    |
|------------------------|----|
| Introductory . . . . . | 35 |
|------------------------|----|

---

*ARGUMENT OF DRAMA.*

xix

---

SCENE II.

---

*Treatment of Parallels by methods involving  
 infinite series.*

---

LEGENDE.

|  | PAGE |
|--|------|
| Treatment of Line . . . . .                            | 54   |
| „ Angle . . . . .                                      | 56   |
| „ Parallels . . . . .                                  | „    |
| Test for meeting of finite Lines . . . . .             | 57   |
| One instance of ‘ <i>Petitio Principii</i> ’ . . . . . | 58   |
| Manual unsuited for beginners . . . . .                | „    |

SCENE III.

---

*Treatment of Parallels by angles made with transversals.*

---

COOLEY.

|                                    |    |
|------------------------------------|----|
| Style of Preface . . . . .         | 59 |
| Treatment of Parallels . . . . .   | 61 |
| Utter collapse of Manual . . . . . | 62 |

b 2

## SCENE IV.

*Treatment of Parallels by equidistances.*

CUTHBERTSON.

|   | PAGE |
|---|------|
| Treatment of Line . . . . .   | 63   |
| Attempted proof of Euclid's (tacitly assumed) Axiom,<br>that two Lines cannot have a common segment . . . . . | 64   |
| Treatment of Angle . . . . .  | 65   |
| „ Parallels . . . . .   | „    |
| Assumption of the Ahmedabad Axiom . . . . .   | 66   |
| Euclid's 12th Axiom replaced by a Definition, two<br>Axioms, and five Theorems . . . . .                      | 67   |
| Test for meeting of two finite Lines . . . . .  | 68   |
| Manual a modified Euclid . . . . .  | 69   |

## SCENE V.

*Treatment of Parallels by direction.*

§ I. WILSON.

|  |    |
|--|----|
| Introductory . . . . .   | 70 |
| Treatment of Line . . . . .  | 71 |
| „ Angle . . . . .  | 72 |
| Extension of limit of 'angle' to sum of four right<br>angles . . . . . | „  |



## ARGUMENT OF DRAMA.

xxi

|  | PAGE |
|--|------|
| 'Straight' angles . . . . .  | 74   |
| Meaning of 'direction' . . . . .   | 76   |
| 'Opposite' directions . . . . .  | 78   |
| 'Same' and 'different' directions . . . . .  | 79   |
| Axiom ' $sD$ is real' discussed . . . . .  | 81   |
| Property ' $sD$ ' can neither be defined, nor constructed,<br>nor tested . . . . .   | 82   |
| 'Separational directions' <i>not</i> identical with 'coincidental directions' . . . . .  | 83   |
| Virtual assumption of ' $sS$ is real' (which Euclid proves<br>in I. 27), as Axiom . . . . .  | 86   |
| Axiom ' $sD$ is real' discussed . . . . .  | "    |
| Axiom ' $sD$ is real' rejected, and Axiom ' $sD$ is real'<br>granted with limitations . . . . .  | 88   |
| Axiom 'different Lines which meet one another have<br>different directions' granted . . . . .  | "    |
| Axiom 'Lines with different directions would meet'<br>discussed . . . . .  | "    |
| and rejected . . . . .   | 90   |
| Diagram of 'same' and 'different' directions con-<br>demned . . . . .  | "    |
| ' $sD$ ' accepted as (ideal) definition . . . . .  | 91   |
| 'Parallel,' as used by Wilson, to be replaced by phrase<br>' $sD$ -related' . . . . .  | "    |
| Definition discussed . . . . .   | "    |
| Theorem ' $sD$ -related Lines do not meet' accepted . . . . .  | 93   |
| Theorem 'Lines, $sD$ -related to a third, are so to each<br>other,' discussed, and condemned as a ' <i>Petitio Prin-</i><br><i>cipiū</i> ' . . . . . | "    |
| Axiom 'Angle may be transferred, preserving direc-<br>tions of sides' discussed . . . . .  | 95   |
| If angle be variable, it involves fallacy ' <i>A dicto secun-</i><br><i>dum Quid ad dictum Simpliciter</i> ' . . . . .                               | "    |

|   | PAGE |
|---|------|
| If it be constant, the resulting Theorem (virtually identical with the Axiom) involves fallacy ' <i>Petitio Principii</i> ' . . . . . | 96   |
| If angle be constant, the Axiom involves two assumptions: viz. that   |      |
| (1) there can be a Pair of different Lines that make equal angles with <i>any</i> transversal . . . . .                               | 100  |
| (2) Lines, which make equal angles with a certain transversal, do so with <i>any</i> transversal . . . . .                            | 101  |
| Axiom rejected . . . . .  | 102  |
| Ideas of 'direction' discussed . . . . .  | ”    |
| Theory of 'direction' unsuited for teaching . . . . .   | 104  |
| Test for meeting of finite Lines discussed:— . . . . .  | ”    |
| it virtually involves Euclid's Axiom . . . . .  | 105  |
| or if not, it causes hiatus in proofs . . . . .   | 106  |
| List of Euclid's propositions which are omitted . . . . .   | 107  |
| General survey of book:— . . . . .  | 108  |
| An ' <i>Illicit process of the Minor</i> ' . . . . .  | ”    |
| A false Corollary . . . . .   | 109  |
| A plethora of negatives . . . . .   | ”    |
| A superfluous <i>datum</i> . . . . .  | 110  |
| Cumbersome proof of Euc. I. 24 . . . . .  | 111  |
| An unintelligible Corollary . . . . .   | ”    |
| A unique 'theorem of equality' . . . . .  | 112  |
| A bold assumption . . . . .   | 113  |
| Two cases of ' <i>Petitio Principii</i> ' . . . . .   | ”    |
| A problem $3\frac{1}{2}$ pages long . . . . .   | ”    |
| A fifth case of ' <i>Petitio Principii</i> ' . . . . .  | 114  |
| A sixth . . . . .   | ”    |
| Summing-up, and rejection of Manual . . . . .   | 115  |

---

*ARGUMENT OF DRAMA.*

xxiii

§ 2. PIERCE.

|   | PAGE |
|---|------|
| Treatment of Line . . . . .                           | 118  |
| Introduction of Infinitesimals . . . . .              | 119  |
| Treatment of Parallels . . . . .                      | ”    |
| Angle viewed as ‘difference of direction’ . . . . .   | 120  |
| Assumption of Axiom ‘ $sD$ is real’ . . . . .         | 121  |
| List of Euclid’s Theorems which are omitted . . . . . | 122  |
| Manual <i>not</i> adapted for beginners . . . . .     | ”    |

§ 3. WILLOCK.

|   |     |
|---|-----|
| Treatment of Parallels . . . . .  | 124 |
| Virtual assumption of Axiom ‘ $sD$ is real’ . . . . .                               | ”   |
| Assumption of Axiom ‘separational Lines have the<br>same direction’ . . . . .       | 126 |
| General survey of book :— . . . . .   | 127 |
| Difficulties introduced too soon . . . . .  | ”   |
| Omission of ‘coincidental’ Lines . . . . .  | 129 |
| ‘Principle of double conversion’ discussed, and<br>condemned as illogical . . . . . | ”   |
| Mysterious passage about ‘incommensurables’ . . . . .                               | 131 |
| Manual rejected . . . . .   | ”   |

---

 ACT III.
 

---

*Manuals which adopt Euclid's treatment  
 of Parallels.*

---

## SCENE I.

## § 1.

|                        | PAGE |
|------------------------|------|
| Introductory . . . . . | 132  |

## § 2. CHAUVENET.

|                          |     |
|--------------------------|-----|
| General survey . . . . . | 134 |
|--------------------------|-----|

## § 3. LOOMIS.

|                          |     |
|--------------------------|-----|
| General survey . . . . . | 136 |
|--------------------------|-----|

## § 4. MORELL.

|   |     |
|---|-----|
| Treatment of Line . . . . .                               | 137 |
| "    Angle . . . . .                                      | 139 |
| "    Parallels . . . . .                                  | "   |
| General survey :—   |     |
| 'Direct,' 'reciprocal,' and 'contrary' Theorems . . . . . | 140 |
| Sentient points . . . . .                                 | 141 |
| A false assertion . . . . .                               | 143 |
| A speaking radius . . . . .                               | 144 |
| Ratios and common measures . . . . .                      | "   |

---

*ARGUMENT OF DRAMA.*

XXV

|                                      | PAGE |
|--------------------------------------|------|
| Derivation of 'homologous' . . . . . | 145  |
| Mensuration of areas . . . . .       | 146  |
| A logical <i>fiasco</i> . . . . .    | 147  |
| Manual rejected . . . . .            | 148  |

§ 5. REYNOLDS.

|   |     |
|---|-----|
| General survey . . . . .                    | 149 |
| List of Euclid's Theorems omitted . . . . . | 150 |

§ 6. WRIGHT.

|   |     |
|---|-----|
| Quotations from preface . . . . .       | 151 |
| General survey :— . . . . .             | 152 |
| Specimen of verbose obscurity . . . . . | 153 |

SCENE II.

---

§ 1. *Syllabus of the Association for the Improvement of Geometrical Teaching.*

---

|  |     |
|--|-----|
| Introduction of <i>Nostradamus</i> , a member of the Association . . . . . | 155 |
| Treatment of Line . . . . .  | 157 |
| „ Angle . . . . .  | 158 |
| „ Parallels . . . . .  | 161 |
| Test for meeting of finite Lines . . . . .                                 | „   |
| Re-arrangement of Euclid's Theorems . . . . .                              | 162 |

|   |                           |      |
|---|---------------------------|------|
| xxvi                                      | <i>ARGUMENT OF DRAMA.</i> |      |
|   |                           | PAGE |
| General survey :—                         |                           | 163  |
| A 'Theorem' is a 'statement of a Theorem' |                           | "    |
| Rule of Conversion                        |                           | "    |
| Miscellaneous inaccuracies                |                           | 164  |
| Summing-up                                |                           | 167  |

§ 2. *Wilson's 'Syllabus'-Manual.*

|   |     |
|---|-----|
| Introductory                                    | 169 |
| A Theorem is a 'statement of a Theorem'         | 170 |
| Rule of Conversion                              | "   |
| Every Theorem a 'means of measuring'            | "   |
| 'Straight angles'                               | "   |
| Miscellaneous inaccuracies                      | "   |
| The Manual's one great merit                    | 171 |
| No test for meeting of finite Lines             | 172 |
| Propositions discussed in detail :—             |     |
| An important omission                           | 173 |
| An illogical conversion                         | "   |
| 'Un enfant terrible'                            | 175 |
| Summary of results :—                           | 180 |
| Of 73 propositions of Euclid, this Manual has   |     |
| 14 omitted ;                                    |     |
| 43 done as in Euclid ;                          |     |
| 10 done by new but objectionable methods, viz.— |     |
| 1 illogical ;                                   |     |
| 1 'hypothetical construction' ;                 |     |
| 2 needlessly using 'superposition' ;            |     |
| 2 algebraical ;                                 |     |
| 4 omitting the diagonals of Euc. II. ;          |     |
| 6 done by new and admissible methods.           |     |

*ARGUMENT OF DRAMA.* xxvii

|  | PAGE |
|--|------|
| No reason for abandoning Euclid's sequence and numeration . . . . .      | 181  |
| Nor for regarding this Manual as anything but a revised Euclid . . . . . | ”    |
| Summing-up . . . . .   | ”    |

ACT IV.

[MINOS *and* EUCLID.]

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*Manual of Euclid.*

---

§ 1. *Treatment of Pairs of Lines.*

---

|  |     |
|--|-----|
| Modern treatment of Parallels . . . . .    | 183 |
| Playfair's Axiom . . . . .                 | 184 |
| Test for meeting of finite Lines . . . . . | ”   |

---

§ 2. *Euclid's constructions.*

---

|   |     |
|---|-----|
| ‘Arbitrary restrictions’ . . . . .                  | 186 |
| ‘Exclusion of hypothetical constructions’ . . . . . | 187 |

§ 3. *Euclid's demonstrations.*

|  | PAGE |
|--|------|
| ‘Invariably syllogistic form’ . . . . .        | 188  |
| ‘Too great length of demonstration’ . . . . .  | ”    |
| ‘Too great brevity of demonstration’ . . . . . | 189  |
| ‘Constant reference to axioms’ . . . . .       | ”    |

§ 4. *Euclid's style.*

|   |     |
|---|-----|
| Artificiality, unsuggestiveness, and want of simplicity . . . . . | 191 |
|---|-----|

§ 5. *Euclid's treatment of Lines and Angles.*

|   |     |
|---|-----|
| Treatment of Line . . . . .                         | 191 |
| „ Angle :—  |     |
| ‘declination from’ accepted . . . . .               | 192 |
| must be less than sum of two right angles . . . . . | ”   |
| ‘multiple angles’ in VI. 33 . . . . .               | ”   |
| proof for Ax. 10 accepted . . . . .                 | 193 |

§ 6. *Omissions, alterations, and additions, suggested by Modern Rivals.*

|                                      |     |
|--------------------------------------|-----|
| Omission of I. 7 suggested . . . . . | 194 |
|--------------------------------------|-----|



---

*ARGUMENT OF DRAMA.*

xxix

|   | PAGE |
|---|------|
| Reasons for retaining it :—   |      |
| needed to prove I. 8 . . . . .  | 194  |
| not included in new I. 8 . . . . .  | ”    |
| proves <i>rigidity</i> of triangle . . . . .                                | ”    |
| I. 7, 8 analogous to III. 23, 24 . . . . .                                  | 195  |
| bears on practical science . . . . .  | ”    |
| Omission of II. 8 suggested . . . . .                                       | ”    |
| Reason for retaining it, its use in Geometrical<br>Conic Sections . . . . . | ”    |
| Alterations suggested :—  |      |
| New proofs for I. 5 :—  |      |
| ‘hypothetical construction’ . . . . .                                       | 196  |
| superposition . . . . .   | ”    |
| treating sides as ‘obliques’ . . . . .                                      | ”    |
| ” as radii of a circle . . . . .  | 197  |
| Inversion of order of I. 8, 24; rejected . . . . .                          | ”    |
| ” I. 18, 19, 20; do. . . . .  | ”    |
| Fuller proof of I. 24; accepted . . . . .                                   | ”    |
| Algebraical proofs of II.; rejected . . . . .                               | ”    |
| Additions suggested :—  |      |
| New Axiom; accepted . . . . .   | 198  |
| Two new Theorems; do. . . . .   | ”    |

---

§ 7. *The summing-up.*

---

|                                    |     |
|------------------------------------|-----|
| Euclid’s farewell speech . . . . . | 199 |
|------------------------------------|-----|

## APPENDICES.

---

### I.

|  | PAGE |
|--|------|
| <i>Extract from Mr. Todhunter's essay on 'Elementary Geometry,' included in 'The Conflict of Studies, &amp;c.'</i> . . . . . | 201  |

### II.

|  |     |
|--|-----|
| <i>Extract from Mr. De Morgan's review of Mr. Wilson's Geometry, in the 'Athenæum' for July 18, 1868</i> . . . . . | 221 |
|--|-----|

### III.

|   |     |
|---|-----|
| <i>The enunciations of Tables I-IV stated in full: with references to the various writers who have assumed or proved them</i> . . . . . | 227 |
|---|-----|

## APPENDICES.

xxx1

## IV.

|   | PAGE |
|---|------|
| <i>The various methods of treating Parallels,<br/>adopted by Euclid and his Modern Rivals</i> | 240  |

## V.

|   |     |
|---|-----|
| <i>Proof that, if any one proposition of Table II<br/>be granted as an Axiom, the rest can be<br/>deduced from it . . . . .</i> | 275 |
|---|-----|

## VI.

*List of propositions of Euc. I, II, with refer-  
ences to their occurrence in the manuals  
of his Modern Rivals:—*

|  |     |
|--|-----|
| § 1. <i>References to Legendre, Cuthbertson, Wilson, Pierce,<br/>Willock, and Wilson's 'Syllabus'-Manual . . . . .</i> | 286 |
| § 2. <i>References to the other Modern Rivals . . . . .</i>  | 294 |