



### Multiplication and division

We divide and multiply the digit terms in the normal way. We then add or subtract the exponential terms. In general for multiplication we write  $(a^n)(a^m) = a^{n+m}$  and for division we write  $\frac{(a^n)}{(a^m)} = a^{n-m}$ .

Examples

$$5 \times 10^3 \times 2 \times 10^4 = (5 \times 2) \times 10^{(3+4)} = 10 \times 10^7 = 1 \times 10^8$$

$$5 \times 10^3 \div 2 \times 10^4 = (5 \div 2) \times 10^{(3-4)} = 2,5 \times 10^{-1} = 0,25$$

### Powers of numbers

We raise the digit term to the indicated power and multiply the exponent by the indicated power. In general we write:  $(a^n)^m = a^{nm}$ .

Example

$$(5 \times 10^3)^2 = 5^2 \times 10^{(3 \times 2)} = 25 \times 10^6 = 2,5 \times 10^7$$

### Roots of exponents

Rewrite the number so that it has an even exponent. Find the square root of the digit term and divide the exponent by 2 to find the answer.

Examples

$$\sqrt{3,6 \times 10^3} = \sqrt{36 \times 10^2} = \sqrt{36} \times 10^{(2 \div 2)} = 6 \times 10 = 60$$

$$\sqrt{5,3 \times 10^7} = \sqrt{53 \times 10^6} = \sqrt{53} \times 10^{(6 \div 2)} = 7,3 \times 10^3$$

## Using your calculator

Not all calculators work in the same manner or with the same sequence of entering keys. Ask your teacher to help you if the described steps do not apply to your calculator.

Very large or very small numbers can be written in scientific notation. This notation reduces mistakes and can be typed in on the calculator. When you multiply 100 000 by 100 000, the answer is displayed as  $1 \cdot \times 10^{10}$  or  $1 \cdot 10$ . This is the scientific notation for 10 000 000 000. To enter a number in scientific notation, use the **Exp** key for the power of 10 numbers and leave out the '× 10' part. The **Exp** key represents 'times 10 to the power ...'.

Examples

1 To enter  $6 \times 10^3$ , press **6** **Exp** **3**. The display shows  $6 \cdot \times 10^{03}$ .

To see what the ordinary form is, press **=**. The display shows 6000.

2 To enter  $3,5 \times 10^{-4}$ , press **3** **.** **5** **Exp** **4** **+/-**.

The display shows  $3.5 \times 10^{-04}$ . The ordinary form is 0,00035.

- 3 To calculate  $6 \times 10^3 \times 3,5 \times 10^{-4}$ :

Press 

The display shows 2.1. The answer is 2,1.

## Conversion of units

When we measure a quantity, we compare it with a specific standard or unit. We say or write the unit with the numerical value of the quantity, for example 45 millimetres. Remember that you must always express a quantity using a number and a unit. Each type of measurement has different units; for example, if we use the metric system we measure length in metres, kilometres, centimetres, and so on.

In science we often need to convert values in various scales of measurement. To do this we need a **conversion factor** that expresses the equivalence of a measurement in two different units (for example, 1 cm = 10 mm). To convert within the decimal system, multiply by the correct factor and use the correct prefix according to the list of metric multiples.

## Conversion symbols

We commonly use the prefixes and symbols listed below to form names and symbols of the decimal multiples of the SI units.

Prefix	Abbreviation	Factor
tera-	T	$10^{12}$
giga-	G	$10^9$
mega-	M	$10^6$
kilo-	k	$10^3 = 1\ 000$
hecto-	h	$10^2 = 100$
deca-	da or D	$10^1 = 10$
deci-	d	$10^{-1} = 0,1$
centi-	c	$10^{-2} = 0,01$
milli-	m	$10^{-3} = 0,001$
micro-	$\mu$	$10^{-6}$
nano-	n	$10^{-9}$
pico-	p	$10^{-12}$
femto-	f	$10^{-15}$

## Dimensional analysis

Dimensional analysis tells us the number by which we must multiply or divide. To convert 60 cm into metres, we know that 100 cm equals 1 m, so common sense tells us to divide by 100. When you are not sure if you have to multiply or divide, follow this method.

- 1 Write the conversion factor as a fraction:

$$100 \text{ cm} = 1 \text{ m},$$

$$\text{so the conversion factor can be } \frac{100 \text{ cm}}{1 \text{ m}} \text{ or } \frac{1 \text{ m}}{100 \text{ cm}}.$$

- 2 Multiply the factor with the value that must be converted:

$$\frac{60 \text{ cm}}{1} \times \frac{1 \text{ m}}{100 \text{ cm}} \quad \text{or} \quad \frac{60 \text{ cm}}{1} \times \frac{100 \text{ cm}}{1 \text{ m}}.$$

- 3 Treat the units as numbers when you multiply fractions. Cancel the cm at the top with the cm at the bottom in the first option, which leaves the answer in metres (m):  $\frac{60 \text{ cm}}{1} \times \frac{1 \text{ m}}{100 \text{ cm}} = 0,6 \text{ m}.$

$$\text{The second option is incorrect: } \frac{60 \text{ cm}}{1} \times \frac{100 \text{ cm}}{1 \text{ m}} = \frac{6\,000 \text{ cm}^2}{1 \text{ m}}.$$

## Significant figures

The number of reliable known digits in a number is called the number of significant figures. When you multiply or divide, the answer should have no more digits than the number with the least number of significant figures used in the calculation.

Example:

$$2 \times 3,55 = 7$$

$$\text{But also, } 2,00 \times 3,55 = 7,10$$

In the example, 2 has only one significant figure. It can be a rounded-off number that can range from 1,5 to 2,4. 2,00 has three significant figures.

## The SI system

When we work with laws and equations in science, it is very important to use a consistent set of units. We use the *Système International d'Unités*, abbreviated to SI, which is the international system of units for measurement. There are seven basic units from which all other units are derived. The basic units are:

- length (distance) in metres (m)
- time in seconds (s)
- mass in kilograms (kg)
- electric current in ampere (A)
- temperature in kelvin (K)
- light intensity in candela (cd)
- amount of matter in mole (mol).

### Symbols and units used in this book:

Quantity	Symbol	SI unit
Position	$x, y$	metres (m)
Displacement	$\Delta x, \Delta y$ or $s$	metres (m)
Acceleration	$a$	metres per second squared ( $\text{m}\cdot\text{s}^{-2}$ )
Initial velocity	$v_i$ or $u$	metres per second ( $\text{m}\cdot\text{s}^{-1}$ )
Final velocity	$v_f$ or $v$	metres per second ( $\text{m}\cdot\text{s}^{-1}$ )
Average velocity	$\bar{v}$	metres per second ( $\text{m}\cdot\text{s}^{-1}$ )
Time	$t$	seconds (s)
Time interval	$\Delta t$	seconds (s)
Mass	$m$	kilogram (kg)
Force	$F$	newton (N)
Weight	$W$	newton (N)
Gravitational acceleration	$g$	metres per second squared ( $\text{m}\cdot\text{s}^{-2}$ )
Friction	$f$	newton (N)
Coefficient of friction	$\mu_s, \mu_k$	
Normal force	$N$	newton (N)
Tension (force)	$T$	newton (N)
Wave speed	$v$	metres per second ( $\text{m}\cdot\text{s}^{-1}$ )
Wavelength	$\lambda$	metres (m)
Refractive index	$n$	
Voltage or potential difference	$V$	volts (V)
Electric charge	$Q, q$	coulomb (C)
Electric current	$I$	ampere (A)
Resistance	$R$	ohm ( $\Omega$ )
Work done	$W$	joule (J)
Magnetic field	$B$	tesla (T)
Magnetic flux	$\Phi$	weber (Wb)
Emf	$\varepsilon$	volts (V)
Power	$P$	watts (W)
Amount of substance	$n$	mole (mol)
Pressure	$P$	pascal (Pa)
Temperature	$t$	degrees celsius ( $^{\circ}\text{C}$ )
Temperature	$T$	kelvin (K)
Molar mass	$M$	grams per mole ( $\text{g}\cdot\text{mol}^{-1}$ )
Concentration	$c$	mole per cubic decimetre ( $\text{mol}\cdot\text{dm}^{-3}$ )

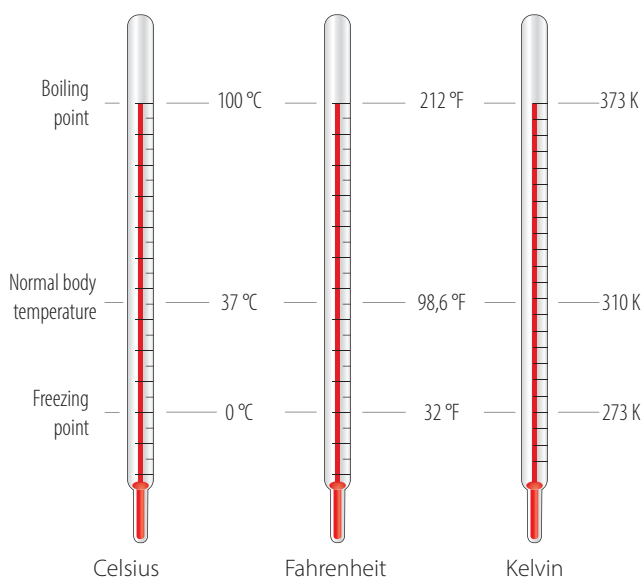
Volume	$V$	cubic metre ( $\text{m}^3$ )
Enthalpy change	$\Delta H$	kilojoule per mole ( $\text{kJ}\cdot\text{mol}^{-1}$ )

## Indicating symbols

We distinguish between vector quantities, that have magnitude and direction, and scalar quantities, with only magnitude. Vectors are indicated with a bold symbol, or with an arrow on top of the symbol. Force is a vector and is indicated as  $\vec{F}$ . When we refer to the magnitude of the force, we use  $F$ .

## Indicating units

We measure speed in metres per second. This means that you divide the distance in metres by the time in seconds. We can abbreviate the unit for speed to  $\text{m/s}$ ,  $\text{m s}^{-1}$ ,  $\text{m}\cdot\text{s}^{-1}$  or  $\text{m}\cdot\text{s}^{-1}$ .



## Temperature

In everyday life, temperature is a measure of how hot or cold an object is. We measure the temperature according to temperature scales and three temperature scales are in current use. These are Celsius, measured in degrees Celsius ( $^{\circ}\text{C}$ ), Fahrenheit, measured in degrees Fahrenheit ( $^{\circ}\text{F}$ ) and Kelvin, measured in kelvin (K).

In South Africa we use the Celsius scale for temperature readings in everyday life. The freezing point of water is  $0^{\circ}\text{C}$  and the boiling point of water at sea level is  $100^{\circ}\text{C}$ . Some countries, such as the United States of America, use the Fahrenheit scale. In this scale the freezing point of water is  $32^{\circ}\text{F}$  and the boiling point is  $212^{\circ}\text{F}$ . Scientists use the Kelvin scale for scientific temperature measurements. To convert from degrees Celsius ( $^{\circ}\text{C}$ ) to kelvin (K), we use the conversion  $T = t + 273$  where  $T$  stands for temperature in K and  $t$  stands for temperature in  $^{\circ}\text{C}$ . To convert from kelvin to degrees Celsius we use  $t = T - 273$ .

## Length

The SI unit for length is **metres**. We use the metric prefixes to show various lengths in this order: kilometre, hectometre, decametre, metre, decimetre, centimetre, millimetre. We multiply the numerical values by a factor of ten to convert between units to give the equivalent amount in a smaller decimal. We divide the numerical values by a factor of ten to convert between units to give the equivalent amount in a larger decimal.

$$\begin{array}{cccccc}
 & \times 10 & & \times 10 & & \times 10 & & \times 10 & & \times 10 & & \times 10 \\
 \text{km} & & \text{hm} & & \text{dam} & & \text{m} & & \text{dm} & & \text{cm} & & \text{mm} \\
 & \div 10 & & \div 10 & & \div 10 & & \div 10 & & \div 10 & & \div 10
 \end{array}$$

$$1 \text{ km} = \overset{\rightarrow}{1} \times \overset{\rightarrow}{10} \times \overset{\rightarrow}{10} \times \overset{\rightarrow}{10} \text{ metres} = 1\,000 \text{ m, and}$$

$$100 \text{ mm} = \overset{\leftarrow}{100} \div \overset{\leftarrow}{10} \div \overset{\leftarrow}{10} \div \overset{\leftarrow}{10} = 0,1 \text{ m}$$

### Mass

Mass is a measure of the amount of matter in an object. We can determine the mass of an object with a balance or scale. The standard unit for mass is the **kilogram**. We use the decimal conversions to change the mass values to grams:

$$1 \text{ kg} = 1 \times 10 \times 10 \times 10 = 1\,000 \text{ g or } 1 \times 10^3 \text{ g.}$$

### Pressure

**Pressure** is defined as the force per unit area applied to a surface. The SI unit for pressure is newtons per square metre ( $\text{N}\cdot\text{m}^{-2}$ ). This unit is also called the **pascal** (Pa) and  $1 \text{ Pa} = 1 \text{ N}\cdot\text{m}^{-2}$ . To convert from pascal (Pa) to kilopascal (kPa), we divide the number by a 1 000, so  $1\,000 \text{ Pa} = 1 \text{ kPa}$ .

The pressure of the air varies slightly according to the weather. At sea level the average atmospheric pressure is  $1,013 \times 10^5 \text{ N}\cdot\text{m}^{-2}$  or  $1,013 \times 10^2 \text{ kPa}$ . We use this value to define the atmosphere (atm), and  $1 \text{ atm} = 101,3 \text{ kPa}$ . The value of 101,3 kPa is known as standard pressure.

### Greek symbols

We sometimes use the letters from the Greek alphabet as symbols in science. In Grade 11, you will use these symbols:

Greek letter	Greek name	Meaning in science	Example
$\delta$	delta	partially	$\delta^-$ = partially negative
$\lambda$	lambda	wavelength	$\lambda = 5 \text{ m}$
$\Delta$	Delta (capital)	change in...	$\Delta t$ = change in time
$\Phi$	phi	magnetic flux	$\Phi = B \times A$
$\Omega$	omega	ohm (unit of resistance)	$R = 5 \Omega$
$\mu$	mu	coefficient of friction	$\mu_k$ = coefficient of kinetic friction $\mu_s$ = coefficient of static friction

### Changing the subject of the formula

There are many formulae and equations in science that show the relationship between different quantities in nature. We use these formulae and equations to calculate unknown quantities.

## Choosing the correct formula

When you need to calculate an unknown quantity, the first step is to decide what scientific principle or formula relates the unknown quantity with the given, known quantities. Identify and list all the known quantities in the question. Then look for a formula in which all the quantities are known except for one, which is the quantity that you have to calculate.

## Variables and substitution

We call the letters in an equation or formula variables because we can replace them with different number values. When we are given particular values for the variables in an equation, we substitute these values in the equation to work out the numerical answer. This is called **substitution**.

An algebraic equation is similar to a balance or scale. The numerical values of the expression on both sides of the equal sign must be the same. We solve the equation by working out the value of the variable that will make the sides of the equation equal.

## Changing the subject of an equation

Consider the equation as a balance. When we do something to the one side, we must always do the same to the other side.

- Step 1** Move all the terms with the unknown symbol to the left side.  
 Remove the unknown terms on the right side by adding the additive inverses to both sides.
- Step 2** Move all the known terms and numbers to the right side.  
 Remove the known terms and numbers that are on the left by adding the additive inverse to both sides.
- Step 3** Simplify both sides.
- Step 4** Divide both sides by the coefficient of the variable.

### Example

The equation that relates initial velocity ( $v_i$ ), final velocity ( $v_f$ ), acceleration ( $a$ ) and time ( $\Delta t$ ) is given by the equation  $v_f = v_i + a\Delta t$ . Follow Steps 1 to 4 to change the subject of the equation to calculate the acceleration.

**Step 1**  $v_f - a\Delta t = v_i + a\Delta t - a\Delta t$

**Step 2**  $v_f - v_f - a\Delta t = v_i - v_f + a\Delta t - a\Delta t$

**Step 3**  $-a\Delta t = v_i - v_f$

To obtain a positive subject of the formula,  $a$ , multiply throughout by  $-1$ :

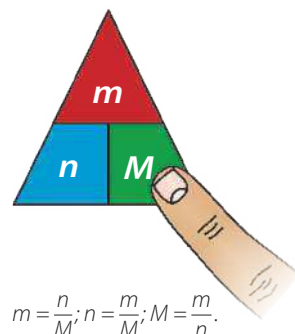
$$a\Delta t = -v_i + v_f = v_f - v_i$$

**Step 4**  $\frac{a\Delta t}{\Delta t} = \frac{v_f - v_i}{\Delta t}$   
 $\therefore a = \frac{v_f - v_i}{\Delta t}$



When there are three quantities involved in an equation, it is often easier to use the equation triangle. In the triangle the horizontal line represents division and the vertical line represents multiplication.

- Draw the triangle.
- Write the quantities in the correct positions.
- Use your finger to cover the quantity that you must calculate; this will tell you if you must multiply or divide the other two quantities.



## Equations of motion

Equations of motion that were explained in Grade 10 are still in use:

$$\begin{array}{ll} v_f = v_i + a\Delta t & \text{or} \quad v = u + a\Delta t \\ v_f = v_i^2 + 2a\Delta x & \text{or} \quad v^2 = u^2 + 2as \\ \Delta x = v_i\Delta t + \frac{1}{2}a\Delta t^2 & \text{or} \quad s = u\Delta t + \frac{1}{2}a\Delta t^2 \\ \Delta x = \left(\frac{v_f + v_i}{2}\right)\Delta t & \text{or} \quad s = \left(\frac{u + v}{2}\right)\Delta t \end{array}$$

## What is rate?

The rate at which a change happens is the amount of change that happens over a period of time. Rate is change per second whether it is change in mass, velocity, concentration or energy.

Look at these examples of rate:

Power is the amount of energy delivered per unit time,

$$\text{so power (in watt)} = \frac{\text{energy (in joules)}}{\text{time (in seconds)}}$$

Acceleration is the rate of change in velocity,

$$\text{so acceleration (in m}\cdot\text{s}^{-2}\text{)} = \frac{\text{change in velocity (in m}\cdot\text{s}^{-1}\text{)}}{\text{time (in s)}}$$

Reaction rate is the change in concentration of a reagent per unit time.

## Direct and inverse proportions

Proportion or variation describes certain relationships between two variables. We often want to determine the degree to which a quantity changes (dependent quantity) when the magnitude of another quantity is varied in predetermined fixed amounts (independent quantity). This degree of change enables us to determine the relationship between different physical quantities. When we compare two quantities in an experiment we must keep all other factors constant. If we change more than one factor, this could affect the results.

## Direct proportionality

When an increase in the independent quantity ( $x$ ) leads to a constant increase in the dependent quantity ( $y$ ), the two quantities are directly proportional.

## Inverse proportionality

When an increase in the independent quantity ( $x$ ) results in a decrease in the dependent quantity ( $y$ ), the quantities are inversely proportional.

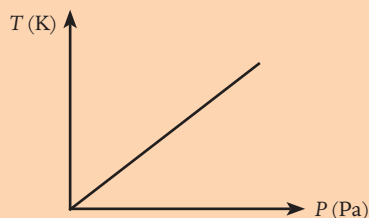
## Proportionality graphs

We can show proportionality on a graph. The variables are dependent on each other, and when one variable changes, the other will also change. Graphs use a set of axes, and we describe position by referring to these horizontal and vertical lines. We find the  $x$ -value by looking along the  $x$ -axis, and the  $y$ -value by looking up or down on the  $y$ -axis.

The graphical representation of a direct proportionality is a straight line through the origin. The gradient of the line is equal to the constant  $k$ .

### Example 1

The temperature of an enclosed mass of gas is directly proportional to its pressure, so  $T \propto P$ .

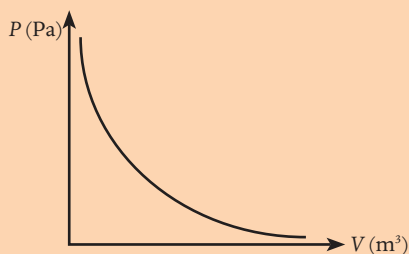


The graph is a straight line through the origin.

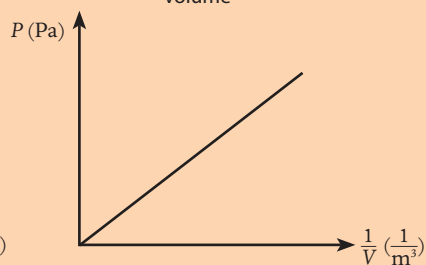
The graphical representation of an inverse proportionality is a hyperbola. We can obtain a straight line by plotting the values of the one variable against the reciprocal of the values of the other variable.

### Example 2

The pressure of an enclosed mass of gas is inversely proportional to its volume, and the pressure is directly proportional to  $\frac{1}{\text{volume}}$ .



This graph is a hyperbola.



This graph is a straight line.