

## Essentials of Atmospheric and Oceanic Dynamics

This is a modern, introductory textbook on the dynamics of the atmosphere and ocean, with a healthy dose of geophysical fluid dynamics. It will be invaluable for intermediate to advanced undergraduate and graduate students in meteorology, oceanography, mathematics and physics. It is unique in taking the reader from very basic concepts to the forefront of research. It also forms an excellent refresher for researchers in atmospheric science and oceanography. It differs from other books at this level in both style and content: as well as very basic material, it includes some elementary introductions to more advanced topics. The advanced sections can easily be omitted for a more introductory course, as they are clearly marked in the text. Readers who wish to explore these topics in more detail can refer to this book's parent, *Atmospheric and Oceanic Fluid Dynamics: Fundamentals and Large-Scale Circulation*, now in its second edition.

**Geoffrey K. Vallis** is a professor of applied mathematics at the University of Exeter, UK. Prior to taking up his position there, he taught for many years at Princeton University in the USA. He has carried out research in the atmospheric sciences, oceanography and the planetary sciences, and has published over 100 peer-reviewed journal articles. He is the recipient of various prizes and awards, including the Adrian Gill Prize (Royal Meteorological Society) and the Stanislaw M. Ulam Distinguished Scholar Award (Los Alamos National Laboratory). He is the author of *Atmospheric and Oceanic Fluid Dynamics: Fundamentals and Large-Scale Circulation, Second Edition* (2017, Cambridge University Press).

‘Vallis’ insights into the fundamentals and applications go a long way towards making otherwise complex topics readily grasped by those willing to study. He does not shy away from mathematics where needed, nor does he smother the reader with mathematics where pedagogically unnecessary. Those making it through this book will be ready to tackle a huge suite of research questions related to atmosphere and ocean fluid mechanics. Hence, this book serves an incredibly important role to the academic community. In a nutshell, we need more smart researchers who are adept at atmosphere and ocean dynamics to help understand how those dynamics are increasingly being affected by humanity’s choices.

*Essentials of Atmospheric and Oceanic Dynamics (EAOD)* fills an important niche by offering an articulate and authoritative textbook to be worked through by advanced undergraduates and/or entering graduate students taking courses. The inclusion of exercises in *EAOD* is incredibly valuable for both students and teachers clamouring for more problem sets to test understanding. Whereas Vallis’ previous book *Atmospheric and Oceanic Fluid Dynamics (AOFD)* is the mother reference, *EAOD* offers a pedagogical entrée for those wishing to test the waters, including some deep waters. I will happily keep both books on my shelf and make use of them for personal study and to support the teaching of geophysical fluid dynamics.

Vallis has a clear writing style that brings the reader into the subject in an authoritative and friendly manner. He is a wise guru and gentle tutor. The subject of ocean and atmosphere fluid mechanics has matured greatly through his efforts at writing *AOFD*. *EAOD* furthers that maturation by allowing for a broader readership to tap into his brain. Well done Geoff!’

- Stephen M. Griffies,  
 Geophysical Fluid Dynamics Laboratory,  
 National Oceanic and Atmospheric Administration

‘The “big book” [*AOFD*] by Vallis is a treasure, but I suspect that this new *Essentials* is destined to be used much more widely in classrooms. Vallis does a superb job of communicating the peculiar tensions between deductive reasoning and physical intuition that underlie this science. The new book is more approachable but no less rigorous. I especially appreciate how the various equation sets are derived in succinct but meaningful ways in the first few chapters, and then used as tools to explore the dynamics in the chapters that follow. It’s almost the perfect introductory textbook on this subject, and I plan to use it in my own courses.’

- Brian E. J. Rose,  
 University at Albany

‘He’s done it again. In *Essentials*, Geoff Vallis has produced a text that is useful to the student and the experienced scientist alike. While the content is simplified and shortened compared to its parent text, Vallis now provides even more descriptive explanations to support readers in their quest to navigate the physics of fluid flows. These explanations pair well with the theory, serving as an accessible introduction to students while also supporting the more experienced scientist as they put all of the pieces together. This will certainly be a future favourite for reading groups. Even readers with dog-eared versions of the parent book will want a copy of *Essentials*, for in it Vallis has added an entirely new chapter on planetary atmospheres, allowing the interested reader to venture into outer space to apply their newly honed GFD expertise.’

- Elizabeth A. Barnes,  
 Colorado State University

‘For the past decade, Geoff Vallis’ book *Atmospheric and Oceanic Fluid Dynamics* has been the “go to” encyclopaedic resource, but it is too lengthy and comprehensive to use as a course textbook. With this superb new shorter volume, Geoff Vallis provides us with the definitive graduate-level textbook, with just the right balance of essential topics alongside glimpses of more advanced topics at the cutting edge of research. The extensive use of margin notes, diamonds to indicate advanced topics, and a comprehensive set of problems will ensure that *Essentials of Atmospheric and Oceanic Dynamics* has much to offer students and researchers at all levels. The book opens with the quote: “Seek simplicity, accept complexity. Exploit simplification, avoid complication.” On all counts, this book succeeds magnificently!’

- David Marshall,  
 University of Oxford

‘As its parent book became the bible of the field, but also grew in size and the number of topics it covered in its latest edition, this new book provides a perfect balance and introduction to the essential topics, giving a quick reference without going into all the details. In the Vallis tradition, it is presented clearly, perfectly packaged, and is well organized for both atmospheric and oceanic fluid dynamics. Its simplicity will make it majestically appealing both for people outside the discipline looking for an accessible, yet complete, introduction, and for students within the field at all levels. The inclusion of planetary atmospheres broadens the scope and makes it appealing to a wider and growing audience. Anyone with a background in physics can get the essentials using this book.’

- Yohai Kaspi,  
 Weizmann Institute of Science

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GEOFFREY K. VALLIS

University of Exeter





Shaftesbury Road, Cambridge CB2 8EA, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India

103 Penang Road, #05–06/07, Visioncrest Commercial, Singapore 238467

Cambridge University Press is part of Cambridge University Press & Assessment,  
a department of the University of Cambridge.

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education, learning and research at the highest international levels of excellence.

[www.cambridge.org](http://www.cambridge.org)

Information on this title: [www.cambridge.org/9781107692794](http://www.cambridge.org/9781107692794)

DOI: 10.1017/9781107588431

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First published 2019 (version 2, August 2022)

*A catalogue record for this publication is available from the British Library*

ISBN 978-1-107-69279-4 Paperback

Additional resources for this publication at [www.cambridge.org/vallisessentials](http://www.cambridge.org/vallisessentials).

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Note: In the text itself more advanced sections are marked with a diamond, ♦, and may be omitted on a first reading. If a section is so marked then the marking applies to all the subsections within it.



## Preface

*Seek simplicity, accept complexity.  
Exploit simplification, avoid complication.*

This is an introductory book on the dynamics of atmospheres and oceans, with a healthy dose of geophysical fluid dynamics. It is written roughly at the level of advanced or upper-division undergraduates and beginning graduate students, but parts of it will be accessible to first- or second-year undergraduates and I hope that practising scientists will also find it useful. The book is designed for students and scientists who want an introduction to the subject but who may not want all the detail, at least not yet, and its prerequisites are just familiarity with some vector calculus and basic classical physics. Thus, it is meant to be accessible to non-specialists and students who will not necessarily go on to become professional dynamists. However, as well as very basic material the book does include some elementary introductions to a few ‘advanced’ topics, such as the residual circulation and turbulence theory, as well as material on the general circulation of the atmosphere and ocean. The more advanced parts could easily be omitted for a first course and, like difficult ski slopes, are marked with a diamond, ♦. Readers may explore these topics more in the references provided, or in this book’s parent, *Atmospheric and Oceanic Fluid Dynamics*. Nearly all the topics in this book, except those in the chapter on planetary atmospheres, are dealt with in greater detail there.

### *What is in the book*

The book is divided into three Parts. The first, and longest, provides the foundation for the study of the dynamics of the atmosphere and ocean. It does not assume any prior knowledge of fluid dynamics or thermodynamics, although readers who have such knowledge may be able to skim Chapter 1. The rest of Part I provides an introduction to ‘geophysical fluid dynamics’, the subject that remains at the heart of atmospheric and oceanic dynamics and without which the subject would be largely qualitative and/or computational. Here we discuss the effects of rota-

tion and stratification, leading into shallow water theory and the quasi-geostrophic and planetary-geostrophic equations. Rossby waves, gravity waves, baroclinic instability and elementary treatments of wave–mean-flow interaction and turbulence round out Part I.

Parts II and III focus on the large scale dynamics and circulation of the atmosphere and ocean, respectively. Our main focus in both Parts is what is sometimes called ‘the general circulation’, meaning the large-scale quasi-steady and/or time-averaged circulation, but this circulation depends on the effects of time-dependent eddies — the atmosphere’s Ferrel Cell may be considered to be ‘driven’ by the effects of baroclinic instability and Rossby waves. And the El Niño phenomenon, described in the final chapter, is explicitly time dependent. One feature of this book that is not in the parent book is a chapter discussing some of the general principles of planetary atmospheres, a topic of increasing interest because of the new, sometimes quite spectacular, observations of the planets in our Solar System and beyond.

### *How to use the book*

The contents of the book are about enough for a two-term course in atmosphere–ocean dynamics. A term-long, first course in geophysical fluid dynamics could, for example, be based on Part I, omitting some of the earlier or later chapters depending on the students’ backgrounds and interests. A term-long course in atmospheric and/or oceanic circulation could be based on Part II and/or Part III, supplementing the material with review articles or research papers as needed, perhaps using data sets to look at the real world (and other planets, if Chapter 13 is to be studied). Alternatively, one could combine aspects of Parts I and II, or Parts I and III, to construct an ‘Atmospheric Dynamics’ or ‘Oceanic Dynamics’ course.

If the book is to be used for self-study it could simply be read from beginning to end, although many other pathways are possible and may be preferable. Parts II and III depend on the material in Part I, but the material is reasonably self-contained, and readers who already have some knowledge of geophysical fluid dynamics should feel free to start at a later chapter, or with Part II or Part III. A few problems are collected at the end of some chapters; these are designed to test understanding as well as to fill in gaps and extend the material in the book itself. Many other problems at varying levels of difficulty can be found on the web site of this book, which can easily be found with a search engine. The reader will also see a number of margin notes throughout the book, rather like the ones to the left. The book itself was typeset using  $\text{\LaTeX}$  with Crimson fonts for text, Cronos Pro for sans serif and Minion Math for equations.

I would like to thank Matt Lloyd, Zoë Pruce and Richard Smith at Cambridge University Press for their expert guidance through the writing and production process, as well as many colleagues and students — too many to list, but they know who they are — for their many comments, corrections and criticisms. If you, the reader, have other comments, major or minor, do please contact me.

Margin notes that are set in a roman (i.e., upright) font emphasize or expand on something that is in the main text.

*Margin notes set in italics are asides or historical anecdotes.*