

Cambridge University Press

978-1-107-69274-9 - Theory of Differential Equations: Part IV: Partial
Differential Equations

Andrew Russell Forsyth

Index

[More information](#)

INDEX TO PART IV.

(Part IV occupies volumes v and vi of the present work. The figures refer to the pages in these volumes.

The Table of Contents at the beginning of volume v and of volume vi may also be consulted.)

- Action, equations of, in theoretical dynamics, v, 381.
- Adjoint equations, vi, 111 et seq.; are reciprocally adjoint, vi, 114; effect of Laplace transformations upon, vi, 114; relation between invariants of, vi, 115; construction of, vi, 117; Riemann's use of, vi, 119 et seq.
- Adjoint ordinary equations, used to construct integrals of doubly-finite rank when these are possessed by linear equations of the second order, vi, 89.
- Ampère, v, 206, 282, vi, 1, 8, 16, 17, 21, 200, 201, 266 et seq., 302, 307, 376, 418.
- Ampère's definition of a general integral, vi, 4; compared with the Darboux-Cauchy definition, vi, 6—8.
- Ampère's first class of equations of the second order, as those having integrals without partial quadratures, vi, 16.
- Ampère's method of integration, vi, 266 et seq.; illustrations of, vi, 272; applied to equation of minimal surfaces, vi, 277; and to special equations of the second order, vi, 281; construction of the primitive in, vi, 284; significance of, compared with Monge's and with Boole's, vi, 290; compared with Darboux's method, vi, 303;
extended to equations of the third order in two independent variables, vi, 474, and to equations of the second order in any number of independent variables, vi, 530.
- Ampère's test for equations having integrals free from partial quadratures, vi, 17.
- Ampère's theorem on intermediate integral of an equation of the second order, vi, 248.
- Arbitrary elements, in integrals of equations of order higher than the first, modes of occurrence of, vi, 17; characters of, in general integrals, vi, 21.
- Arbitrary function in integral of equation of second order, the equation for argument of, is invariantive for every compatible equation, vi, 320.
- Arbitrary functions, number of, in the integral in Cauchy's theorem is the same as the order of the equation, v, 47, vi, 22; arguments of, vi, 27 et seq.
- Arguments of arbitrary function in general integrals, number of, vi, 27—29; equation characteristic of, vi, 29, 35, 37; various instances of, for equation of second order, vi, 32.
- Arguments of arbitrary functions in integral made the independent variables in integration by Ampère's method, vi, 271.
- Argument of arbitrary function in integral of equation of the second order, equation for, is invariantive for every compatible equation of that order, vi, 320.
- Asymptotic lines as characteristics, equations which have, v, 245.
- Asymptotic curves on singular integral, v, 251.
- ausgezeichnete Function*, v, 350.
- Bäcklund transformations for equations of the second order, vi, 432 et seq.; kinds of, vi, 433; applied to linear

Cambridge University Press

978-1-107-69274-9 - Theory of Differential Equations: Part IV: Partial

Differential Equations

Andrew Russell Forsyth

Index

[More information](#)

586

INDEX TO PART IV

- equations, vi, 441 et seq.; connected with simultaneous equations of the first order, vi, 450.
- Bäcklund, v, 408, vi, 425, 432.
- Bateman, vi, 16, 580, 581.
- Bertrand, v, 370, 397, 405, vi, 139.
- Bianchi, vi, 328, 377, 454.
- Boehm, vi, 260.
- Boole, vi, 199, 201, 266, 303, 307.
- Boole's method for equations possessing an intermediate integral, vi, 208–212, 215; compared with Monge's method, vi, 209, 212; in practice can be included in Darboux's method, vi, 201; compared with Darboux's method, vi, 303.
- Borel's expression, as a definite integral, for integrals of linear equations, vi, 104.
- Boundary values, Riemann's use of, for adjoint equations, vi, 119 et seq.
- Bour, vi, 201, 367.
- Bourlet, v, 52, 53, vi, 334.
- Bromwich, vi, 581.
- Burgatti, vi, 55, 129.
- Canonical constants, Bertrand's theorem on, deduced from properties of contact transformation, v, 405.
- Canonical form, of complete linear system, v, 81;
 of group of functions, v, 355, how constructed, v, 355–359;
 of linear equation of the second order in two independent variables, vi, 47.
- Canonical forms of equations of dynamics, v, 373: as affected by contact transformations, v, 398: conserved under contact transformations v, 399, and (for general systems) only under contact transformations v, 399, 403: significance of, as contact transformation, v, 405, and as giving Bertrand's theorem on canonical constants, v, 406.
- Cauchy, v, 26, 205, 206, 219, 282, 371, 381.
- Cauchy's construction of the equations of the characteristics, v, 206; and derivation of integrals, v, 210; as modified by Darboux, v, 212.
- Cauchy's integral, of a homogeneous linear equation, v, 58;
 of a non-homogeneous linear equation, v, 73;
 of a system of homogeneous linear equations, v, 88;
 of equations of the second order, vi, 3;
 of linear equation can be represented by a definite integral involving only a single arbitrary function, vi, 104; application of this property to the equation for the conduction of heat, vi, 107, and to the equation for two-dimensional potential, vi, 110.
- Cauchy's problem for equations of the second order and their characteristics, vi, 388.
- Cauchy's theorem, examples when it cannot be applied to linear equations, v, 69, 70.
- Cauchy's theorem, exceptions to, for equations of the first order, v, 32, 36, 158; for equations of the second order, v, 42; for equations of general order, v, 51.
- Cauchy's theorem, based upon Kowalevsky's existence-theorems, v, 11;
 Chapter II;
 for a single equation of the first order, v, 27, 33, 36; similarly for any number of independent variables, v, 33, 35, 36;
 for an equation of the second order, v, 37, 42; vi, 2;
 for equations of any order, v, 43;
 for equations of the second order, as described by Darboux, vi, 305, in particular, as applied to the Monge-Ampère equations, vi, 307; with any number of independent variables, vi, 528.
- Characteristic developable, v, 227.
- Characteristic equation satisfied by arguments of arbitrary functions in integrals of equations of the second order, vi, 29, 320; of any order, vi, 35: and with any number of independent variables, vi, 37.
- Characteristic equations in dynamics, due to Hamilton, v, 371, 376, 381.
- Characteristic invariant, vi, 532; influence of, when resolvable, vi, 550: of Laplace's equation, vi, 571.
- Characteristic number for equations having integrals of doubly finite rank, vi, 70; of self-adjoint equation of finite rank, vi, 133.
- Characteristics, conjugate, v, 251; self-conjugate, v, 251.
- Characteristics in hyper-space, method of, v, 282 et seq.; equations of, v, 284, with use made of their integrals, v, 285–288; kinds of integrals derived from, v, 288–292; equations of, constructed from geometrical properties, v, 293.
- Characteristics, of equations of first order in two independent variables, v, 205 et seq.; equations of, as constructed by Cauchy, v, 206, and are the same as the equations in Charpit's method, v, 208; integral equivalent of these equations, v, 209; equations of, as constructed by Darboux, v, 212;

Cambridge University Press

978-1-107-69274-9 - Theory of Differential Equations: Part IV: Partial
Differential Equations

Andrew Russell Forsyth

Index

[More information](#)

INDEX TO PART IV

587

- properties of, v, 224 et seq.; their relation to integral surfaces, v, 224; equations of, deduced from geometrical properties, v, 228, and connected with the general integral, v, 230; equations of, in terms of surface parameters, v, 233; envelope of, is edge of regression, v, 337; used by Lie as basis of classification of equations, v, 244; equations having asymptotic lines as, v, 245; equations having lines of curvature as, v, 246; equations having geodesics as, v, 248.
- Characteristics of equations of the second order, vi, 388 et seq.; primitive as locus of, vi, 393; when the equation is of the Monge-Ampère type, vi, 394; geometrical interpretation of, vi, 397; used to classify the equations, vi, 400.
- Characteristics, singularities of, in ordinary space, v, 261 et seq.
- Charpit, v, 157, vi, 303.
- Charpit's equations for integration of intermediate integral of a system, vi, 217, 248, 285.
- Charpit's method for equations in two independent variables, v, 156 et seq.; used to integrate equations not subject to Cauchy's theorem, v, 158 et seq.; the equations in, are the equations of characteristics, v, 208.
- Chrystal, v, 70, 190.
- Clairin, vi, 425, 433, 441.
- Classes of integrals, of a complete system, v, 193 et seq.; how far the customary classes are comprehensive, v, 198; derived by use of theory of contact transformations, v, 326, 331, 334, 338: by use of groups of functions, v, 369; of simultaneous equations of the first order, v, 419, 424; that are intermediate for equation of the second order, vi, 8, but are not entirely comprehensive, vi, 261.
- Classification (Lie's) of equations of the first order, according to the nature of the characteristics, v, 244; and of equations of the second order similarly, vi, 400.
- Coexistence of equations, Jacobian relations for (see *Jacobian conditions*).
- Coexistence of equations of the second order, vi, 339.
- Combinants of two functions, properties of, v, 112 et seq.; commonly called the Poisson-Jacobi, v, 113; in connection with the canonical equations of dynamics, v, 392; cannot be generalised for equations in several dependent variables, v, 474.
- Compatible equations, of the second order, how constructed by Darboux's method, vi, 314, and by Hamburger's method, vi, 336; of order higher than second, vi, 353, 339.
- Complete integral, v, 171; relations between, general integral and singular integral, with limitations, v, 172, 251, 255; can be particular cases of distinct general integrals, v, 176, 182; tests for, v, 178; of system of equations, v, 194; as related to the equations of the characteristic, v, 211, 212, 215; in hyperspace, derived from characteristics, v, 290; contact of two, along a characteristic, v, 299; contact of, with singular integrals, v, 312; derived through contact transformations, v, 326, 331, 334, 338: derived through groups of functions, v, 369; of simultaneous equations in several dependent variables, v, 419.
- Complete integrals of equation of second order, vi, 8; subjected to variation of parameters, vi, 361.
- Complete intermediate integrals of equations of second order generalised, vi, 377.
- Complete linear system of equations, v, 79 (see also *complete systems*).
- Complete systems of linear equations, that are homogeneous, v, 76 et seq.; remain complete, when replaced by an algebraic equivalent, v, 79, or when the independent variables are changed, v, 80; canonical form of, v, 81; number of independent integrals of, v, 83; Mayer's method of integrating, v, 89; Jacobi's method of integrating, v, 91; that are not homogeneous, v, 97; conditions of coexistence of, v, 98; integration of, v, 99.
- Complete systems of equations and groups of functions, how related, v, 347, 349.
- Complete systems of non-linear equations, v, 109 et seq.; when in involution, v, 120; number of independent integrals of, in Mayer's method, v, 122; classes of integrals of, v, 193.
- Completely integrable equations, König's, v, 411; conditions for, v, 416; various cases, v, 416; integration of, v, 419; different kinds of integrals of, v, 419, with general result, v, 424.
- Cones associated with the geometrical interpretation of an equation of the

Cambridge University Press

978-1-107-69274-9 - Theory of Differential Equations: Part IV: Partial

Differential Equations

Andrew Russell Forsyth

Index

[More information](#)

- first order, v, 221: how related to integral surfaces, v, 223, and to the characteristics, v, 224.
- Conjugate characteristics, v, 251.
- Contact, of integral surfaces, v, 225; of edge of regression of general integral with complete integral, v, 240; of selected edge of regression with integral surface, v, 241; of singular integral with other integrals, v, 255–261; of integrals in general, v, 306–310.
- Contact of integrals in hyperspace, v, 297–306, 310.
- Contact transformations, v, 131, 315; definition and specific equations of, v, 315–317: which are infinitesimal, v, 317: which do not involve the dependent variable, v, 318, and also are infinitesimal, v, 322: which are homogeneous, v, 323;
 applied to the integration of an equation or equations, v, 324 et seq.; classes of integrals thus derived, v, 326, 331, 334, 338; general relation of, to the integration of equations, v, 343; applicable only when certain relations are satisfied unconditionally and must be replaced by theory of groups of functions when the relations are satisfied only conditionally, v, 344;
 effect of, upon group of functions, v, 346: and upon reciprocal groups, v, 349: invariants of group of functions under, v, 364;
 and canonical equations in dynamics, v, 370: relations between, v, 398–404: can be translated into each other, v, 405;
 effect of, on equations of the second order possessing two intermediate integrals, vi, 295;
 arising from Imschenetsky's variation of parameters applied to Laplace's linear equation, vi, 382.
- Cosserat, vi, 159, 161.
- Cosserat's proof of Moutard's theorem on equations of the second order having integrals in explicit finite form without partial quadratures, vi, 161 et seq.; summary of results in, vi, 195.
- Coulon, vi, 490.
- Critical relation for transformation of equations of the second order, vi, 428; significance of, vi, 429, 430, 436–441.
- Curvature, lines of, as characteristics, v, 246.
- Curves associated with the geometrical interpretation of an equation of the first order, v, 222: how related to integral surfaces, v, 223: integral, v, 238.
- Darboux, v, 205, 212, 226, 227, 243, 251 et seq., 282, 408, vi, 5, 39, 47, 55, 60, 70, 78, 82 et seq., 111, 120, 127, 128, 131, 139, 157, 158, 159, 161, 200, 201, 295, 302 et seq., 377, 432, 454.
- Darboux-Cauchy definition of a general integral, vi, 5; compared with Ampère's definition, vi, 6–8.
- Darboux's forms of linear equations of the second order, having integrals of finite rank, vi, 82 et seq.
- Darboux's modification of Cauchy's method of characteristics, v, 212 et seq.
- Darboux's method for integrating equations of the second order in two independent variables, vi, 302 et seq.; central aim of, vi, 302; compared with methods of Monge, Ampère, Boole, vi, 303, 313; property of subsidiary system in, vi, 309, and integrals of that system, vi, 313; includes Vályi's process for integration of simultaneous equations of second order, vi, 328; applied to equations $f(r, s, t) = 0$, vi, 344: applied to obtain compatible equations of order higher than the second, vi, 353;
 extended to equations of the third order in two independent variables, vi, 478;
 extended to equations of the second order in any number of independent variables, vi, 539, 562; applied to Laplace's equation, vi, 571.
- De Boer, vi, 343, 344, 351, 352.
- Deformable surfaces, equation of, referred to minimal lines as parametric curves, vi, 344.
- Delassus, v, 53, vi, 104.
- De Morgan, vi, 201.
- Developable touching an integral surface along characteristic, properties of, v, 227.
- distinguée, fonction*, v, 350.
- Dixon, vi, 260.
- Dominant functions and equations used, v, 13, 14.
- Donkin, v, 370.
- Doubly finite rank, linear equations of the second order having integrals of, vi, 69; how affected by Laplace transformations, vi, 70; characteristic number of, vi, 70; construction of the equations, vi, 72–78, with Darboux's modified forms for, vi, 82.
- Dynamics, equations of theoretical, v,

Cambridge University Press

978-1-107-69274-9 - Theory of Differential Equations: Part IV: Partial
Differential Equations

Andrew Russell Forsyth

Index

[More information](#)

INDEX TO PART IV

589

- 370 et seq.; canonical form of, v, 373, conserved by contact transformation, v, 399—404: represent an infinitesimal contact transformation, v, 405.
- Dziobek, v, 370.
- Edge of regression of integral surface, v, 237; is envelope of characteristics, v, 237; is general form of an integral curve, v, 239; of general integral has contact of second order with complete integral, v, 240; selected curves from the infinitude, v, 240, with properties, v, 241; equations of, deduced from the differential equation, v, 243.
- Element of integral in hyperspace, v, 299.
- Elliptic case of linear equations of the second order, vi, 44.
- Energy, when it gives an integral of dynamical equations, v, 375: is the source of the infinitesimal contact transformation represented by canonical equations, v, 405.
- Envelope of characteristics, is edge of regression of surface, v, 237; on a hypersurface, equations of, v, 300.
- Equal invariants, linear equations of the second order having, vi, 131 et seq. (see also *self-adjoint equations*).
- Essential parameters, number of, in an integral of an equation, v, 192.
- Euler, vi, 127, 159, 521.
- Exceptional integrals, v, 185 (see also *special integrals*); geometry of, v, 188.
- Exceptions to Cauchy's theorem for equations of the first order in two independent variables, v, 158 et seq.
- Existence-theorems for integrals of system of equations, of the first order and linear, v, 11; of the first order and not linear, v, 21; of the first order and any degree in any number of independent variables, v, 35; of the second order, v, 37; of any order, v, 43;
 - for integrals of a complete system of homogeneous linear equations, v, 83;
 - for single equation, exceptional case omitted from, v, 110; can lead to a singular integral, v, 111.
- Falk, vi, 166, 456, 469.
- Finite form of general integral, characteristic property of, vi, 14; equations of second order determined by, vi, 159 et seq.
- Finite rank, linear equations of the second order having integrals of, vi, 64; in both variables. vi, 69; how affected by Laplace transformations, vi, 70; Goursat's theorem on, vi, 90; of an equation and its adjoint, vi, 116; of self-adjoint equations, vi, 133 et seq., and as affected by Moutard's theorem, vi, 141.
- First class of equations of the second order, after Ampère, vi, 16.
- First method, Jacobi's, v, 371, 380; is a generalisation of Hamilton's results in theoretical dynamics, v, 382; statement of general process, v, 386; how modified by assignment of initial conditions, v, 387, 390; when the dependent variable occurs, v, 391.
- First order, any system of partial equations can be changed so as to contain only equations of the, v, 8; Cauchy's theorem for a single irreducible equation of the, v, 27, 33, 35, 36.
- First order, characteristics of, possessed by equations of the second order, vi, 394; are included in those of second order, vi, 395; geometrical interpretation of, vi, 397; connected with intermediate integrals, vi, 401.
- Fourier, vi, 109.
- Fredholm, vi, 582.
- Frobenius, vi, 73.
- Functions, group of (see *Group of functions*).
- Fundamental system of integrals of a complete system of homogeneous linear equations, v, 86; can be used to express any integral, v, 87.
- General integral, of homogeneous linear equation is completely comprehensive, v, 57; of non-homogeneous linear equation is not completely comprehensive, v, 65, 68; range of, in the case when a non-homogeneous equation has been made homogeneous, v, 71;
 - classes of, v, 168, 171: the most comprehensive class of, v, 169, 171; deduced from complete integral by variation of parameters, v, 164 sqq.; of system of equations, v, 195;
 - as related to the equations of the characteristic, v, 211, 212, 215; how related to Cauchy's integral, v, 218;
 - and singular integral, relations between, v, 254, 255;
 - in hyperspace derived from characteristics, v, 289; contact of, with singular integral, v, 310;
 - derived through contact transformations, v, 327, 331, 334, 339.

Cambridge University Press

978-1-107-69274-9 - Theory of Differential Equations: Part IV: Partial
Differential Equations

Andrew Russell Forsyth

Index

[More information](#)

- General integral of equations of the second order, as defined by Ampère, vi, 4, 8; as defined by Darboux, after Cauchy, vi, 5; comparison of two definitions of, vi, 6–8; character of, vi, 13;
characteristic properties of the arbitrary elements in, vi, 21; number of arbitrary functions in, vi, 22.
- General method for constructing intermediate integrals (if any) of an equation of the second order, vi, 220; applied to the Monge-Ampère equations, vi, 226; subsidiary equations in, coincide with Boole's, vi, 227; when based upon Darboux's method, vi, 314.
- General order, Cauchy's theorem for integrals of systems of equations of, v, 43; limitation upon the form of equations of, and its importance, v, 48; equations of, in two independent variables, vi, 487.
- Generalisation of integrals of equations of the second order, vi, 361 et seq.; of intermediate integrals, vi, 377; in case of Laplace's linear equation, vi, 379.
- Generalised form of Cauchy's theorem for equations of the first order, v, 33, 36; for equations of the second order, v, 42.
- Geodesics as characteristics, equations having, v, 248.
- Geometry of space and relation between different kinds of integrals, v, 186; illustrated by means of the characteristics, v, 205 et seq.; of the various integrals of an equation of the first order, v, 224 et seq.
- Goursat, v, 26, 55, 72, 100, 164, 180, 205, 223, 243, 248, 314; vi, 7, 27, 39, 91, 94, 129, 159, 198, 261, 301, 303, 328, 333, 334, 344, 388, 397, 418, 424, 425, 434, 441, 454, 455.
- Goursat's theorem on primitive of equation of second order to be deduced from intermediate integral, vi, 406.
- Graindorge, v, 370, 397.
- Group of functions, v, 314; definition of, as applied to partial equations, v, 345; order, sub-group, involution, defined, v, 345; limit to order when group is in involution, v, 346, 349; how affected by contact transformation, v, 346; connected with complete Jacobian system of equations, v, 347, 349; group reciprocal to, or polar of, v, 349; properties of indicial functions of, v, 350; relation between order of, and number of indicial functions, v, 355, 359, 360, 366; canonical form of, v, 355; when in canonical form, can be amplified into another group, v, 361; two invariants of, under contact transformation, v, 364; highest order, and construction of, a sub-group in involution, v, 364; applied to integrate a system of equations, v, 367.
- Guichard, vi, 130.
- Hamburger, v, 407, 408, 428, 455, 474; vi, 303, 336, 456.
- Hamburger's method of constructing equations compatible with an equation of the second order, vi, 336 et seq.; subsidiary system of equations in, compared with those in Darboux's method, vi, 338; applied to equations of the third order in two independent variables, vi, 482.
- Hamburger's systems of simultaneous equations, when linear, v, 428; the method limited to the case of two independent variables, v, 430; applied to the special case of two dependent variables, v, 435; with examples, v, 439; applied to the case with any number of dependent variables, v, 442 et seq.; integrable also by means of partial equations, v, 449;
when non-linear, v, 458; transformed so as to be linear equations in an increased number of variables, v, 456; general result, v, 459; special method for, v, 467; generalisation of Jacobi's process not generally effective for, v, 474.
- Hamilton, v, 370.
- Hamilton's characteristic equations in dynamics, v, 371, 376, 381.
- Hamilton's theorem on integrals of a dynamical system, v, 379; is the basis of Jacobi's first method, v, 380, 382.
- Harmonic equations and their integrals, vi, 157.
- Hilbert, v, 230.
- Hill, M. J. M., v, 249.
- Homogeneous contact transformations, v, 323.
- Homogeneous linear equations, v, 56 et seq.; number of independent integrals of, v, 57; most general integral of, v, 57; Cauchy's integral of, v, 58; systems of, v, 76 et seq.
- Hyperbolic case of linear equations of the second order, vi, 44; see also *linear equations*.
- Imschenetsky, v, 100, 164, 370; vi, 1, 8, 10, 21, 46, 68, 199, 237, 266, 361, 376.
- Imschenetsky's generalisation of sub-complete integrals of Monge-Ampère equations, vi, 366 et seq.; applied to

Cambridge University Press

978-1-107-69274-9 - Theory of Differential Equations: Part IV: Partial
Differential Equations

Andrew Russell Forsyth

Index

[More information](#)

INDEX TO PART IV

591

- Laplace's linear equations, vi, 379, and is a contact-transformation, vi, 382.
- Independence of linear equations in a homogeneous complete system, v, 77.
- Independent integrals, of homogeneous linear equation, v, 57;
of system of homogeneous linear equations, v, 83; of system in involution, number of, v, 122.
- Indicial functions of a group, v, 350; their number is invariant under contact transformations, v, 351; other properties of, v, 352, 354; relation between number of, and the order of the group, v, 355, 366.
- Infinitesimal contact transformations, v, 317; form of, v, 318; which do not involve the dependent variable, v, 322; determination of all, is equivalent to integrating an equation, v, 324;
determined by energy of a dynamical system, v, 405; significance of, leads to Bertrand's theorem on canonical constants, v, 405.
- Infinitesimal transformation, invariant for, v, 72; invariant equation for, v, 73.
- Integrability, conditions of, of a single differential expression, v, 101; of a system of simultaneous equations, v, 103.
- Integrable equations, König's completely, v, 411; conditions for, v, 416, with various systems, v, 416; construction of integral equivalent of, v, 418; kinds of integrals of, v, 419, with general result, v, 424, and examples, v, 425.
- Integral curves, v, 238; can always be obtained as an edge of regression, v, 239; equations of, v, 239.
- Integrals of an equation of first order, different kinds of, and relations between, v, 164 sqq.; particular kinds, v, 171; complete, general, singular, special, exceptional (see under these titles respectively); of a complete system, classes of, v, 193;
deduced by method of characteristics, v, 210, 214, 288–292; relations of different, to one another, v, 297 et seq.
- Intermediate integrals of equations of order higher than the first, vi, 8; general, and complete, vi, 10; not necessarily possessed, vi, 10.
- Intermediate integrals, equations of the second order and the Monge-Ampère type which possess, vi, 200; Monge's method of obtaining, vi, 201; assumption of particular type of, necessary for Monge's argument, vi, 203, and for Boole's argument, vi, 208; Boole's method of obtaining, vi, 210; simultaneous, can exist, vi, 205, 227; general method for (see *general method*); Ampère's theorem on integration of, vi, 248;
equations of the second order possessing two, are reducible by contact transformations to $s=0$, vi, 295; construction of, after Darboux's method, vi, 314; generalised by variation of parameters, vi, 377.
- Intermediate integrals of equations of second order and the characteristics, vi, 401; general theory of, vi, 403 et seq.; can lead to primitive, vi, 406; various cases and examples, vi, 409 et seq.
- Intermediate integrals, of equations of the third order in two independent variables, vi, 457 et seq.; general theory of, vi, 470 et seq.
- Intermediate integrals of equations of the second order in any number of independent variables, vi, 490 et seq.
- Invariant, and invariant equation, for infinitesimal transformation, v, 72, 73.
- Invariant, characteristic, vi, 532 (see *characteristic invariant*).
- Invariants of an equation of the second order, significance of, when equal to one another, vi, 131 et seq.
- Invariants of linear equation of the second order, vi, 44; when they vanish, the equation can be integrated by quadratures, vi, 46; used to construct canonical forms of the equation, vi, 47; of equations arising through Laplace-transformations, vi, 52; when they vanish for a transformed equation, the original equation can be integrated, vi, 56; Darboux's expressions for successive, vi, 83.
- Invariants of parabolic linear equations of the second order, vi, 98; effect of their vanishing upon the form of the equations, vi, 100–102.
- Involution, equations of the second order in, vi, 330.
- Involution, systems in, v, 82, 120; number of independent integrals of, v, 122.
- Involution, system of functions in, v, 345; function in, with a group, v, 345; limit to order of a system in, v, 346, 349; highest order of a sub-group in, v, 365.
- Irreducibility, significance of, for equations of first order, v, 33.
- Irreducible differential expressions, vi, 60, 73.

Cambridge University Press

978-1-107-69274-9 - Theory of Differential Equations: Part IV: Partial
Differential Equations

Andrew Russell Forsyth

Index

[More information](#)

- Jacobi, v, 100, 113, 137, 157, 370, 380, 382, 397, 407, 417, 432, vi, 302.
- Jacobian conditions of integrability, of a single differential expression, v, 101; of a system of simultaneous equations, v, 103; sufficient as well as necessary, v, 104 et seq.
- Jacobi-Hamiltonian method, v, 371; constructed by Jacobi on Hamilton's theorem on dynamical equations, v, 380, 382; general result stated, v, 386; modification of, when the dependent variable occurs, v, 391.
- Jacobi's method of integrating complete linear systems, v, 91; is a method of successive reduction, v, 92.
- Jacobi's methods of integrating equations of the first order (see *first method*, *second method*).
- Jacobian process of combination of equations in one dependent variable not effective for equations in several dependent variables, v, 474; form of, v, 476.
- Jacobi's second method, as developed by Mayer, v, 117 et seq.; applied to a single equation, v, 137 et seq.
- Jacobian system of equations of the first order (see *complete systems*).
- Jacobian system of linear equations, v, 82 (see also *complete linear system*, *complete systems*).
- Jordan, v, 26, 164.
- Kapteyn, vi, 261.
- Kinds of integrals of an equation of the first order, v, 164 sqq.; tests for, v, 178; geometrical illustration of, v, 186 (see also *classes of integrals* of a complete system); as connected with the characteristics, v, 210, 214.
- König, v, 408, 411, vi, 303, 335.
- König's systems of completely integrable equations, v, 411: conditions to be satisfied by, v, 416; integration of, v, 418; different kinds of integrals of, v, 419, with general result, v, 424.
- Königsberger, v, 407, 419, 425, 428, 439.
- Kowalevsky, v, 11, 26, 48.
- Lacroix, v, 157.
- Lagrange, v, 131, 164, 370, vi, 9, 111, 159, 361.
- Laplace, vi, 39.
- Laplace's equation for potential, vi, 571; integral of, provided by extension of Darboux's method, vi, 573; Whitaker's integral of, vi, 576, and its relation to the Cauchy integral, vi, 578.
- Laplace's linear equation, vi, 160, 297; integral of, generalised by Imschenetsky, through variation of parameters, vi, 379; and by R. Liouville, vi, 384.
- Laplace's method for linear equations of the second order, vi, 39 et seq.
- Laplace transformations of linear equations, vi, 49; the two are inverses of each other, vi, 50; successive applications of, vi, 51; as affecting integrals of finite rank, vi, 57, 70; Goursat's theorem on, vi, 91; how affecting Lévy transformations, vi, 96; applied to adjoint equations, vi, 114.
- Legendre's equations for minimal surfaces, vi, 280.
- Legendrian transformation of the dependent variable so as to construct a primitive, v, 127, 131, 217, 292.
- Lévy, vi, 94, 96.
- Lévy's transformation, vi, 94; how related to Laplace's transformations, vi, 96.
- Lie, v, 137, 157, 205, 244, 248, 314 et seq., 370, vi, 295, 324, 332, 424.
- Lie's classification of equations of the first order according to the characteristics, v, 244.
- Lie's theorem that equations of the second order possessing two intermediate integrals can be changed into $s=0$ by contact transformations, vi, 295.
- Linearly distinct integrals of a linear equation of the second order, Goursat's theorem on, v, 90.
- Linear equations, in several dependent variables, Hamburger's system of, v, 428; subsidiary equations for, with the critical algebraic equation, v, 430 et seq.: in two dependent variables, v, 435; in any number of dependent variables, v, 442: can be integrated (when integral exists) by simultaneous systems of partial equations, v, 449.
- Linear equations in the parabolic case (see *parabolic*).
- Linear equations of the second order in three independent variables, vi, 520.
- Linear equations of the second order, Laplace's method for, vi, 39 et seq.; reduced to one of two alternative forms, vi, 42; three cases, when variables are real, vi, 43; its invariants, vi, 44; canonical forms of, vi, 47; transformations of, in succession, vi, 49; can be integrated when any invariant of any transformed equation vanishes, vi, 56; having integrals of finite rank (see *finite rank*, *doubly finite rank*).
- Linear equations of the second order subjected to Bäcklund transformations, vi, 441 et seq.
- Linear equations, that are homogeneous, v, 56 et seq.; that are not homogeneous, v, 60 et seq.; complete systems of homogeneous, v, 76 et seq.

Cambridge University Press

978-1-107-69274-9 - Theory of Differential Equations: Part IV: Partial
Differential Equations

Andrew Russell Forsyth

Index

[More information](#)

INDEX TO PART IV

593

- Lines of curvature as characteristics, equations having, v, 246.
- Liouville (J.), v, 382, vi, 143, 160, 194, 197.
- Liouville, R., vi, 69, 111, 119, 384.
- Liouville's equation of the second order, vi, 143, 160, 177, 194, 197.
- Lovett, v, 371.
- Mansion, v, 100, 164, 205, 220, 371.
- Mayer, v, 55, 89 et seq., 100, 115, 117, 127, 137, 157, 316, 388, 417.
- Mayer's development of Jacobi's second method, v, 117 et seq.; with use of Legendre's transformation, v, 127 et seq.
- Mayer's form of Lie's general theorem on contact transformations, v, 316.
- Mayer's method of integrating complete linear systems, v, 89.
- Méray, v, 53.
- Minimal surfaces, equation of, integrated by Ampère's method, vi, 277; integrals of, due to Legendre, Monge, Weierstrass, vi, 280.
- Monge, v, 205, 237, 248; vi, 199 et seq., 266, 280, 301, 302, 307.
- Monge-Ampère equation generalised, when there are more than two independent variables, vi, 511.
- Monge-Ampère equation of the second order, vi, 200, 202, 208, 213, 226 et seq., 281, 307, 367, 433; construction of classes of, vi, 236, 246, 252; characteristics of, vi, 393—395.
- Monge's method for equations possessing an intermediate integral, vi, 201—208, 215; compared with Boole's method, vi, 209, 212; in practice is included in Ampère's method, vi, 201; compared with Darboux's method, vi, 303.
- Moutard, vi, 111, 159, 160.
- Moutard's theorem on self-adjoint equations, vi, 139, and their construction in successively increasing rank, vi, 141; also the integrals of such equations, vi, 147.
- Moutard's theorem on equations of the second order having integrals of explicit finite form, vi, 160; Cosserat's proof of, vi, 161 et seq., with a summary of results, vi, 195.
- Natani, v, 407, vi, 456.
- Non-homogeneous linear equation, v, 60 et seq.;
 a general integral of, v, 62;
 special integrals of, v, 65;
 general theorem as to integral of, v, 67;
 can be made homogeneous, v, 71.
- Non-linear equations of the first order, Chapter iv, *passim*, v, 100 et seq.
- Number of arbitrary functions in Cauchy's theorem is same as order of the equation, v, 47.
- Number of equations in a system must, in general, be the same as the number of dependent variables, v, 6.
- Number of independent integrals, of homogeneous linear equation, v, 57; of system of homogeneous linear equations, v, 83.
- Number of independent variables, equations involving any general, vi, 527.
- Number of quadratures in Mayer's method of integrating complete linear systems compared with the number in Jacobi's method, v, 94, 95.
- One integral common to subsidiary system for Monge-Ampère equations, vi, 253; conditions for, vi, 256, 257; causes the intermediate integral to lead to two equations of the second order, vi, 258.
- Order of contact of integral surfaces (see *contact*).
- Order of group of functions, v, 345; how limited, when the group is in involution, v, 346, 349; unaffected by contact transformation, v, 346; exceeds number of indicial functions by even integer, v, 355.
- Parabolic linear equations of the second order, vi, 44; invariants of, vi, 98; form of, when the invariants vanish, vi, 100—102; cannot have a general integral in finite terms free from partial quadratures, vi, 103.
- Parameters, variation of, v, 165; number of essential, in an integral of an equation, v, 192; for systems of equations, v, 419, 464; for equations of the second order, vi, 361 et seq.
- Partial quadratures, integrals with or without, as determining classes of equations of the second order, vi, 16; as affecting character of integrals, vi, 17; Ampère's test for equations having integrals free from, vi, 17;
 must occur in general integral of parabolic linear equations of the second order, if in finite terms, vi, 103; in Borel's expression for integrals of linear equations, vi, 106;
 equations of the second order having integrals in finite form free from, vi, 159.
- Particular integrals of equations of the second order, vi, 8.
- Pfaff's problem, v, 55.
- Picard, v, 24.
- Poisson, v, 113, 370, 397.

Cambridge University Press

978-1-107-69274-9 - Theory of Differential Equations: Part IV: Partial Differential Equations

Andrew Russell Forsyth

Index

[More information](#)

- Polar groups of functions, v, 349.
- Potential, equation satisfied by, in free space, vi, 15, 37, 576.
- Primitive (see *complete integral*, *general integral*); of systems of equations, v, 410; kinds of, v, 419; of equations of the second order, vi, 8; deduced from intermediate integral, vi, 406; of Monge-Ampère equations, when the subsidiary system possesses three common integrals, vi, 232–235; how limited in use of Ampère's method, vi, 267; how constructed, vi, 284.
- Quadratures, presence or absence of partial, as determining classes of integrals of equations of the second order, vi, 16; character of integral affected by, vi, 17 (see also *partial quadratures*).
- Raabe, vi, 10.
- Rank (*rang*) of a differential expression or an integral, vi, 60; Goursat's theorem on, vi, 90.
- Rank of a self-adjoint equation as affected by Moutard's substitution, vi, 141, 145.
- Real variables and reduced forms of linear equations of the second order, vi, 43.
- Reciprocal groups of functions, v, 349; effect of contact transformations upon, v, 349; relations between, v, 353.
- Reducibility of differential expressions, conditions for, vi, 61.
- Regression, edge of, of integral surface, v, 237 (see *edge*).
- Regular integrals of systems of equations of the first order, conditions for, v, 11, 21; uniqueness of, v, 11, 18, 20, 22; of single irreducible equation of first order, v, 27; of single irreducible equation of the second order, v, 37, 42; for equations of any order, v, 43; of systems of linear equations, v, 88.
- Resolubility of characteristic invariant, effect of, vi, 550, 553.
- Riemann, vi, 111, 120, 527.
- Riemann's use of adjoint equations, vi, 119 et seq.; examples of, vi, 124.
- Riquier, v, 23, 53.
- Routh, v, 370.
- Schwartz, A., vi, 42.
- Schwarz, H. A., vi, 360.
- Second method of Jacobi, as developed by Mayer, v, 117 et seq.; applied to the integration of a single equation, v, 137.
- Second order, equations of Cauchy's theorem for integrals of, in two independent variables, v, 37, 42, vi, 2; having intermediate integrals, vi, 199 et seq.; in any number of independent variables, vi, 490 et seq.
- Second order, characteristics of, possessed by equations of the second order, vi, 395; include those of the first order, vi, 395; geometrical interpretation of, vi, 397.
- Selected edges of regression, v, 240; contact of, with integral surface, v, 241.
- Self-adjoint equations, vi, 131 et seq.; construction of, when of finite rank, vi, 133; integrals of, when of finite rank, vi, 136, 147 et seq.; Moutard's theorem on, vi, 139 et seq.; examples of, vi, 152 et seq.
- Self-conjugate characteristics, v, 251.
- Sersawy, vi, 82, 156, 303, 490.
- Several dependent variables, simultaneous equations in, v, 407.
- Simultaneous equations of the first order in several variables, v, 407 et seq.; as derived from primitives, v, 409; different kinds of integrals of, v, 419, with general result, v, 424; Hamburger's linear systems of, v, 428; Hamburger's non-linear systems of, v, 456; in general lead to equations of higher order in one dependent variable, v, 477; as affected by Bäcklund transformations, vi, 450, 452.
- Simultaneous equations of the second order, vi, 325; integration of, vi, 327; Vályi's process for, included in Darboux's method, vi, 328; systems of, in involution, vi, 330; of order higher than second, vi, 353.
- Simultaneous integrals of different resolutions of a subsidiary system are compatible, in Monge's method, vi, 205, 227; in Darboux's method, vi, 315, 339; and in Hamburger's method, vi, 339.
- Singularities of characteristics in ordinary space, v, 261–280.
- Singular integrals, v, 65 (see also *special integrals*).
- Singular integral of an equation of the first order, conditions for, v, 111, 171; exceptional case of, v, 112; how deduced from complete integrals, v, 166, 170; can be affected by changing the form of the equation, v, 178; can be deduced from the differential equation, v, 182; additional limitations and conditions for existence of, v, 184; of system of equations, v, 196, with appropriate tests, v, 201; when deducible from the system, v, 203;

Cambridge University Press

978-1-107-69274-9 - Theory of Differential Equations: Part IV: Partial
Differential Equations

Andrew Russell Forsyth

Index

[More information](#)

INDEX TO PART IV

595

- is the exception for the equations of the characteristic, v, 212, 217;
 geometrical properties of, v, 249 et seq.; represent envelope (if any) of complete integrals, v, 251; equations of, deduced from geometrical properties, though with exceptions, v, 253; how related to the general integral, v, 254; order of contact of, with the other integrals, v, 255–261;
 in hyperspace and the equations of the characteristics, v, 291; its contact with other integrals, v, 302, 310–313;
 derived through contact transformations, v, 328, 336: but does not occur usually when the dependent variable is explicitly absent from the equation, v, 331, 338.
- Singular integral for equations of the second order, vi, 3, 9.
- Sommerfeld, vi, 527.
- Sonin, vi, 303.
- Special integrals, v, 65, 67, 97, 172 et seq.; illustrations of, v, 69, 177, 188, 200; (see also *exceptional integrals*); geometry of, v, 188; general process for construction of equations which possess, v, 190; of system of equations, v, 200;
 are connected with some of the non-ordinary positions on the characteristic, v, 212, 217;
 in hyperspace and the equations of the characteristics, v, 292.
 do not arise under the merely formal theory of contact transformations, v, 343.
- Special integrals of equations of order higher than the first, vi, 9.
- Special integrals, which may arise in the use of Ampère's method, one kind of, vi, 276.
- Special intermediate integrals of equations of second order, not included in other classes, vi, 261; with examples, vi, 264.
- Speckman, v, 408, vi, 303.
- Sub-group of a group of functions, v, 345; highest order of, when in involution, v, 365, with mode of construction of the most extensive sub-group, v, 366.
- Subsidiary equations in Ampère's method, how obtained, vi, 282; agree with Monge's, in application to equations, vi, 283; their significance, vi, 289.
- Subsidiary system in Darboux's method, property of, vi, 309; utility of integrable combinations of the equations in, vi, 313; simultaneous integrals of distinct resolutions of, vi, 315, 339; construction of, vi, 316, and simplified expression of, vi, 318; compared with subsidiary system in Hamburger's method, vi, 338.
- Subsidiary system of equations, in Monge's method involving differential elements, vi, 203; in Boole's method involving differential coefficients, vi, 211; integrals of, how used, vi, 207, 215; relation between sets of, in Monge's method and in Boole's method respectively, vi, 212, 215; in the general method, coincides with Boole's, vi, 227; when it possesses three common integrals, vi, 230, or two, vi, 242, or one, vi, 253, or none, vi, 259.
- Successive reduction, Jacobi's method of, for the integration of complete linear systems, v, 92.
- Surfaces representing integrals, v, 221: their characteristics, v, 224; properties of, v, 225; edge of regression of, v, 237 (see also *edge*).
- System of partial equations, in general, must contain the same number of equations as of dependent variables, v, 6; can be transformed so as to contain equations of first order only, v, 8; can be made to depend upon an equation or equations of higher order in one dependent variable only, v, 10.
- Systems of linear equations, complete, that are homogeneous, v, 76 et seq.
- Systems of non-linear equations that are complete, v, 109 et seq.
- Tangential transformation (see *contact transformations*).
- Tanner, vi, 159, 197, 456, 460, 462, 464, 469, 490, 509.
- Teixeira, vi, 453.
- Theoretical dynamics, equations of, v, 370 et seq.
- Third order, equations of the, and of higher orders, vi, 456 et seq.; classes of, which possess intermediate integrals, vi, 457 et seq.; Ampère's method extended to, vi, 474; Darboux's method extended to, vi, 478; Hamburger's method extended to, vi, 482.
- Three integrals common to subsidiary system for Monge-Ampère equations, vi, 230; conditions for, vi, 231; lead to a primitive, vi, 232–235; construction of such equations, vi, 236.
- Transformation (Legendrian) of dependent variable so as to construct a primitive, v, 127 et seq.

Cambridge University Press

978-1-107-69274-9 - Theory of Differential Equations: Part IV: Partial
Differential Equations

Andrew Russell Forsyth

Index

[More information](#)

596

INDEX TO PART IV

- Transformation of equations of the second order, general theory of, vi, 425 et seq.; critical relation for, vi, 428; forms of, after Bäcklund, vi, 432; applied to linear equations, vi, 441 et seq.
- Transformations of contact (see *contact transformations*).
- Transformed equations, series of, by application of Laplace-transformations to linear equation of second order, vi, 51; invariants of, vi, 52; possessing any vanishing invariant lead to integration of original equation, vi, 56; Darboux's expression of, vi, 85; by application of Lévy's transformations, vi, 94.
- Tresse, v, 53.
- Two integrals common to subsidiary system, for Monge-Ampère equations, vi, 242; conditions for, vi, 243, 244; use made of, vi, 245, 249; construction of such equations, vi, 246, 252.
- Uniqueness of regular integrals, of systems of equations of the first order, v, 11, 18, 20, 22; is a part of Cauchy's theorem, v, 28, 35; for equations of second order, v, 37, 42; for equations of any order, v, 45; for systems of linear equations, v, 88.
- Vályi, vi, 328, 333.
- Vanishing invariants of linear equations of the second order, effect of, vi, 46, 56, 100.
- Variation of parameters, v, 165; for systems of equations in several variables, v, 419 et seq.; with special examples, v, 425; in general, v, 464.
- Variation of parameters applied, to integrals of equations of the second order, vi, 361 et seq.; to Laplace's linear equation, vi, 379, being a contact transformation, vi, 382.
- Vivanti, vi, 490, 509, 522, 525.
- von Weber, v, 53, 407; vi, 328, 490.
- Weierstrass, v, 432; vi, 280.
- Whittaker, v, 371; vi, 15, 16, 576, 580, 581.
- Winckler, vi, 81, 156, 303, 360.
- Zajačowski, v, 407.