

Contents

<i>Acknowledgements</i>	<i>page x</i>
PART I MATHEMATICAL RECREATIONS AND ABSTRACT GAMES	1
Introduction	1
Everyday puzzles	5
1 Recreations from Euler to Lucas	9
Euler and the Bridges of Königsberg	9
Euler and knight tours	12
Lucas and mathematical recreations	17
Lucas's game of solitaire calculation	20
2 Four abstract games	23
From Dudeney's puzzle to Golomb's Game	24
Nine Men's Morris	25
Hex	26
Chess	29
Go	35
3 Mathematics and games: mysterious connections	40
Games and mathematics can be analysed in the head . . .	40
Can you 'look ahead'?	41
A novel kind of object	42
They are abstract	45
They are difficult	45
Rules	47
Hidden structures forced by the rules	47
Argument and proof	48

	Certainty, error and truth	49
	Players make mistakes	50
	Reasoning, imagination and intuition	51
	The power of analogy	52
	Simplicity, elegance and beauty	52
	Science and games: let's go exploring	53
4	Why chess is not mathematics	54
	Competition	54
	Asking questions <i>about</i>	55
	Metamathematics and game-like mathematics	57
	Changing conceptions of problem solving	57
	Creating new concepts and new objects	57
	Increasing abstraction	58
	Finding common structures	59
	The interaction between mathematics and sciences	60
5	Proving versus checking	61
	The limitations of mathematical recreations	61
	Abstract games and checking solutions	62
	How do you 'prove' that 11 is prime?	63
	Is '5 is prime' a coincidence?	63
	Proof versus checking	64
	Structure, pattern and representation	65
	Arbitrariness and un-manageability	65
	Near the boundary	67
	PART II MATHEMATICS: GAME-LIKE, SCIENTIFIC AND PERCEPTUAL	73
	Introduction	73
6	Game-like mathematics	75
	Introduction	75
	Tactics and strategy	76
	Sums of cubes and a hidden connection	79
	A masterpiece by Euler	81
7	Euclid and the rules of his geometrical game	85
	Ceva's theorem	88
	Simson's line	90
	The parabola and its geometrical properties	90
	Dandelin's spheres	93

Contents

vii

8	New concepts and new objects	95
	Creating new objects	96
	Does it exist?	98
	The force of circumstance	98
	Infinity and infinite series	99
	Calculus and the idea of a tangent	102
	What is the shape of a parabola?	105
9	Convergent and divergent series	108
	The pioneers	108
	The harmonic series diverges	110
	Weird objects and mysterious situations	111
	A practical use for divergent series	113
10	Mathematics becomes game-like	115
	Euler’s relation for polyhedra	115
	The invention-discovery of groups	118
	Atiyah and MacLane disagree	120
	Mathematics and geography	121
11	Mathematics as science	122
	Introduction	122
	Triangle geometry: the Euler line of a triangle	123
	Modern geometry of the triangle	126
	<i>The Seven-Circle Theorem, and other New Theorems</i>	129
12	Numbers and sequences	131
	The sums of squares	131
	Easy questions, easy answers	133
	The prime numbers	133
	Prime pairs	134
	The limits of conjecture	135
	A Polya conjecture and refutation	136
	The limitations of experiment	136
	Proof versus intuition	140
13	Computers and mathematics	142
	Hofstadter on good problems	143
	Computers and mathematical proof	144
	Computers and ‘proof’	146
	Finally: formulae and yet more formulae	147

14	Mathematics and the sciences	148
	Scientists abstract	148
	Mathematics anticipates science and technology	148
	The success of mathematics in science	150
	How do scientists use mathematics?	151
	Methods and technique in pure and applied mathematics	152
	Quadrature: finding the areas under curves	153
	The cycloid	156
	Science inspires mathematics	160
15	Minimum paths: elegant simplicity	163
	A familiar puzzle	163
	Developing Heron's theorem	166
	Extremal problems	168
	Pappus and the honeycomb	169
16	The foundations: perception, imagination, insight	170
	Archimedes' lemma and proof by looking	171
	Chinese proofs by dissection	172
	Napoleon's theorem	173
	The polygonal numbers	176
	Problems with partitions	180
	Invented or discovered? (Again)	182
17	Structure	184
	Pythagoras' theorem	185
	Euclidean coordinate geometry	190
	The average of two points	192
	The skew quadrilateral	193
18	Hidden structure, common structure	197
	The primes and the lucky numbers	197
	Objects hidden behind a veil	198
	Proving consistency	201
	Transforming structure, transforming perception	202
19	Mathematics and beauty	207
	Hardy on mathematics and chess	208
	Experience and expectations	209
	Beauty and Brilliances in chess and mathematics	210
	Beauty, analogy and structure	210

<i>Contents</i>		ix
	Beauty and individual differences in perception	212
	The general versus the specific and contingent	214
	Beauty, form and understanding	215
20	Origins: formality in the everyday world	217
	The psychology of play	219
	The rise and fall of formality	221
	Religious ritual, games and mathematics	222
	Formality and mathematics	223
	Hidden mathematics	224
	Style and culture, style in mathematics	225
	The spirit of system versus problem solving	227
	Visual versus verbal: geometry versus algebra	228
	Women, games and mathematics	229
	Mathematics and abstract games: an intimate connection	231
	<i>References</i>	234
	<i>Index</i>	243