

Games and Mathematics

The appeal of games and puzzles is timeless and universal. In this unique book, David Wells explores the fascinating connections between games and mathematics, proving that mathematics is not just about calculation but also about imagination, insight and intuition.

The first part of the book introduces games, puzzles and mathematical recreations, including the Tower of Hanoi, knight tours on a chessboard, Nine Men's Morris and more. The second part explains how thinking about playing games can mirror the thinking of a mathematician, using scientific investigation, tactics and strategy, and sharp observation. Finally, the author considers game-like features found in a wide range of human behaviours, illuminating the role of mathematics and helping to explain why it exists at all.

This thought-provoking book is perfect for anyone with a thirst for mathematics and its hidden beauty; a good high-school grounding in mathematics is all the background that's required, and the puzzles and games will suit pupils from 14 years.

DAVID WELLS is the author of more than a dozen books on popular mathematics, puzzles and recreations. He has written many articles on mathematics teaching, and a secondary mathematics course based on problem solving. A former British under-21 chess champion and amateur 3-dan at Go, he has also worked as a game inventor and puzzle editor.

Games and Mathematics
Subtle Connections

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The figure of the Al Mani knight tour (page 15) can be found at www.mayhematics.com/t/history/1a.htm and elsewhere.