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Optimal Transportation

Theory and Applications

Edited by

YANN OLLIVIER
Université Paris-Sud, France

HERVÉ PAJOT
Université de Grenoble, France

CÉDRIC VILLANI
Université de Lyon, France

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Contributors

Filippo Santambrogio, Laboratoire de Mathématiques, Université Paris-Sud, Orsay Cedex 91405, France.

Ivan Gentil, Institut Camille Jordan, Université Lyon I, Villeurbanne Cedex 69622, France.

Luigi Ambrosio, Faculty of Sciences, Scuola Normale Superiore di Pisa, Pisa 56126, Italy.

Alessio Figalli, Department of Mathematics, University of Texas, Austin, TX 78712, USA.

Peter Topping, Mathematics Institute, University of Warwick, Coventry CV4 7AL, England.

Sara Danieri, S.I.S.S.A., Trieste 34151, Italy.

Guiseppe Savare, Departimento di Matematica, Università di Pavia, Pavia 27100, Italy.

Shin-ichi Ohta, Department of Mathematics, Kyoto University, Kyoto 606-8502, Japan.

Olivier Besson, Institut de Mathématiques, Université de Neuchatel, Neuchatel 2009, Switzerland.

Martine Picq, Université de Lyon CNRS, INSA-Lyon ICJ UMR, Villeurbanne Cedex F-69100, France.

Jérome Pousin, Université de Lyon CNRS, INSA-Lyon ICJ UMR, Villeurbanne Cedex F-69100, France.

Mathias Beiglbock, Faculty of Mathematics, University of Vienna, Vienna 1090, Austria.

Christian Léonard, Laboratoire de Mathématiques, Université Paris Ouest, Nanterre Cedex 92001, France.

Walter Schachermayer, Faculty of Mathematics, University of Vienna, Vienna 1090, Austria.

François Bolley, Université de Paris IX (Paris-Dauphine), Paris Cedex 16 F-75775, France.

Patrick Cattiaux, Institut de Mathématiques de Toulouse, Université Paul Sabatier, Toulouse Cedex 09, France.

Arnaud Guillin, Laboratoire de Mathématiques, Université Blaise Pascal, Aubière 63170, France.

Quentin Mérigot, Laboratoire Jean Kuntzmann, Université Grenoble Alpes and CNRS, Grenoble, France.

Preface

This book contains the proceedings of the summer school "Optimal transportation: Theory and Applications" held at the Fourier Institute (University of Grenoble I, France). The first 2 weeks were devoted to courses that described the main properties of optimal transportation and discussed its applications to analysis, differential geometry, dynamical systems, partial differential equations and probability theory. Courses were addressed both to students and researchers. A workshop took place during the last week. The aim of this conference was to present very recent developments of optimal transportation and also its applications in biology, mathematical physics, game theory and financial mathematics.

The first part of the book contains (expanded) versions of the courses. There are two sets of notes by F. Santambrogio. The first one gives a short introduction to optimal transport theory. In particular, the Kantorovich duality, the structure of Wasserstein spaces and the Monge–Ampère equations related to optimal transport are presented to the readers. These notes could be seen as an introduction for the other papers of the book. The second one describes applications to economics, game theory and urban planning.

The notes of I. Gentil, P. Topping and S.-I. Ohta describe (with different flavours) the connections between optimal transport and the notion of Ricci curvature, which is a very important tool in classical Riemannian geometry. A notion of curvature-dimension condition was defined by D. Bakry and M. Émery to study geometric properties of diffusions and to get functional inequalities. I. Gentil's notes study the Bakry–Émery condition in the case of the Ornstein–Uhlenbeck semigroup. A quite different approach using optimal transport theory to obtain logarithmic Sobolev-type inequalities is also discussed. A definition of metric measure spaces with lower Ricci curvature bound (which coincides with the classical definition in the case of Riemannian manifolds) was proposed very recently by K.-T. Sturm and J. Lott–C.

Villani independently. S.-I. Ohta's long paper discusses in detail the geometry of such spaces. For instance, versions of the Brunn–Minkowski inequality, of the Lichnerowicz inequality, of Bishop–Gromov volume comparison, of the Bonnet–Myers diameter bound and also the stability under Hausdorff–Gromov convergence are proved in this general setting. The theory of Ricci flow as developed by R. Hamilton and others since 1982 is an essential element in the proofs by G. Perelman of the Poincaré conjecture and Thurston's geometrisation conjecture. The objective of P. Topping's lectures is to explain this theory from the point of view of optimal transport.

The fundamental work of Y. Brenier related to the Euler equation played an important role in the renewal of optimal transport in the 1980s. The notes of L. Ambrosio and A. Figalli describe some recent results on Brenier's variational models for the incompressible Euler equations.

The paper by S. Daneri and G. Savaré gives an overview of the theory of gradient flows in Euclidean spaces and then in metric spaces. Applications to evolution equations in the Wasserstein spaces of probability measures are also discussed.

Apart from these mini-courses, this book also contains five research/survey papers. O. Besson, M. Picq and J. Pousin present an algorithm for a computing mass transport problem inspired from optimal transport and whose origin lies in hearts' images tracking. M. Beigblöck, C. Léonard and W. Schachermayer discuss the duality theory for the Monge–Kantorovich transport problem. In particular, they give a version of Fenchel's perturbation method. The paper of F. Bolley reviews recent quantitative results on the approximation of mean field diffusion equations by large systems of interacting particles, obtained by optimal coupling methods. P. Cattiaux and A. Guillin describe some recent results on Poincaré-type inequalities, transportation-information inequalities or logarithmic Sobolev inequality obtained via Lyapounov conditions. Q. Mérigot proves the stability of the Federer curvature measures with respect to the Wasserstein distance. This was motivated by problems of reconstruction of curves and surfaces from point cloud approximation that come from image analysis for instance. These five contributions illustrate the variety of possible applications of optimal transport to pure and applied mathematics, and also to computer science.