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LOGIC

PART I

BY

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ΠΑΝΤΑ ΡΕΙ ΕΙ ΜΗ ΤΟ ΔΛΗΘΕΪΣ

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‘MAN IS A RATIONAL ANIMAL’

Definition

PREFACE

THE present work is intended to cover the whole field of Logic as ordinarily understood. It includes an outline of elementary Formal Logic, which should be read in close connection with Dr Keynes's classical work, in which the last word has been said on most of the fundamental problems of the subject. As regards Material Logic, I have taken Mill's *System of Logic* as the first basis of discussion, which however is subjected to important criticisms mostly on the lines of the so-called conceptualist logicians.

I have to express my great obligations to my former pupil, Miss Naomi Bentwich, without whose encouragement and valuable assistance in the composition and arrangement of the work, it would not have been produced in its present form.

W. E. J.

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[More information](#)

CONTENTS

INTRODUCTION

| | PAGE |
|---|--------|
| § 1. Definition of Logic. Grounds for including the theory of induction | xiii |
| § 2. Thinking includes perceptual judgments | xvii |
| § 3. All ulterior motives of the pursuit of truth irrelevant to the analysis and criticism of thought | xvii |
| § 4. Relation of Logic to the Art of Thinking | xix |
| § 5. Logic, Aesthetics and Ethics as the three normative studies | xx |
| § 6. Relation of Logic to Universal Grammar | xxi |
| § 7. Logic and Mathematics. Methodology. Pure and Applied Mathematics | xxii |
| § 8. Historical sketch of the problems connecting Logic with Philosophy and with Psychology. Realism, Conceptualism and Nominalism. Materialistic and Empirical Logic | xxvii |
| § 9. Special features of the author's treatment of Logic | xxxiii |

CHAPTER I

THE PROPOSITION

| | |
|--|----|
| § 1. Sentence, assertion and proposition | 1 |
| § 2. Assertion as conscious belief | 4 |
| § 3. Necessity for recognising the mental attitude in logic | 6 |
| § 4. Relation between grammatical and logical analysis | 8 |
| § 5. Connected functioning of substantive and adjective in the proposition | 9 |
| § 6. Criticism of the view that the essential nature of the proposition is a statement of identity | 13 |
| § 7. The proposition regarded as characterising the fact, and its analogies with the adjective | 14 |

CHAPTER II

THE PRIMITIVE PROPOSITION

| | |
|---|----|
| § 1. Account of the psychologically primitive form of assertion—exclamatory or impersonal | 18 |
| § 2. The need of separate presentment for the most elementary forms of judgment | 20 |
| § 3. Criticism of Mr Bradley's dictum 'distinction implies difference' | 22 |
| § 4. Illustrations of less primitive forms of assertion | 23 |

CHAPTER III

COMPOUND PROPOSITIONS

| | PAGE |
|--|------|
| § 1. Definition of compound and simple as applied to propositions | 26 |
| § 2. Distinction between the different forms of compound propositions; the logical conjunctions; distinction between the enumerative and conjunctive 'and' | 27 |
| § 3. The first law of thought called 'The Law of Double Negation' | 29 |
| § 4. The laws of conjunctive propositions | 29 |
| § 5. The four forms of composite proposition and their immediate implications | 30 |
| § 6. Complementary and supplementary propositions and their rules | 35 |
| § 7. Criticism of the paradoxical forms of the composite, and preliminary explanation of the solution of the paradox | 38 |
| § 8. The bearing of the distinction between hypothesis and assertion upon the paradox | 44 |
| § 9. Interpretation of compound propositions as expressing possible conjunctives and necessary composites. Table of all the possible relations of one proposition to another | 47 |

CHAPTER IV

SECONDARY PROPOSITIONS AND MODALITY

| | |
|---|----|
| § 1. Definition of primary and secondary propositions and of pre-propositional adjectives | 50 |
| § 2. Modal adjectives as a species of pre-propositional. The adjectives 'true' and 'false' in this connection | 51 |
| § 3. The opposition of modal adjectives | 53 |
| § 4. Antithesis between 'certified' and 'uncertified' | 55 |
| § 5. The rule that holds universally between two antithetical modals | 56 |
| § 6. Threefold meanings of 'necessary' and 'possible' | 59 |
| § 7. Antithesis of 'certified' and 'uncertified' as epistemic; antithesis of 'nomic' and 'contingent' as constitutive | 61 |
| § 8. Comparison of Mill's distinction between verbal and real propositions, with Kant's distinction between analytic and synthetic propositions | 62 |
| § 9. Summary of the treatment of modal adjectives in connection with secondary propositions | 65 |

CHAPTER V

NEGATION

| | |
|---|----|
| § 1. The nature of pure negation and its different degrees of significance | 66 |
| § 2. The importance of negation determined by its relevance to a specific purpose | 69 |
| § 3. Can there be a proposition where there is nothing corresponding to the subject-term? With illustrations of elementary forms of obversion and contradiction | 70 |
| § 4. An account of the incomplete proposition 'S is' | 73 |
| § 5. The hypothetical element in the proposition 'S is' | 77 |
| § 6. Summary: determinateness of fact contrasted with indeterminateness of knowledge | 79 |

CONTENTS

ix

CHAPTER VI

THE PROPER NAME AND THE ARTICLE

| | PAGE |
|--|------|
| § 1. Distinction between the proper and the descriptive name | 80 |
| § 2. Proper name explained in terms of the introductory and referential articles | 82 |
| § 3. Distinction between the connotation and the etymology of a name | 84 |
| § 4. Distinctions between the four kinds of article or applicatives | 85 |
| § 5. The demonstrative applicatives | 88 |
| § 6. Ostensive definition | 89 |

CHAPTER VII

GENERAL NAMES, DEFINITION AND ANALYSIS

| | |
|--|-----|
| § 1. Connection between general names and the applicatives | 97 |
| § 2. Distinction between connotation and comprehension. Can adjectives be predicated of abstract names? | 100 |
| § 3. The nature of the analysis involved in definition. Definition by means of substitution of phrase. The indefinable | 103 |
| § 4. Analytic and synthetic modes of definition | 106 |
| § 5. Partition, resolution and analysis | 109 |

CHAPTER VIII

ENUMERATIONS AND CLASSES

| | |
|---|-----|
| § 1. The relation 'comprising' as defining the nature of an enumeration and an item. The three operators for enumerations | 113 |
| § 2. Enumerations of different orders | 116 |
| § 3. Distinction between 'comprised' and 'included' | 120 |
| § 4. Connection between enumerations and classes and the question whether 'class' is a genuine entity | 121 |
| § 5. The nature and reality of the class as determined by the type of adjective significantly predicable of it | 126 |
| § 6. Attempted proof of the genuineness of 'class' by the so-called principle of Abstraction. Charge of <i>petitio principii</i> and <i>ignoratio elenchi</i> against the alleged proof | 128 |

CHAPTER IX

THE GENERAL PROPOSITION AND ITS IMPLICATIONS

| | |
|--|-----|
| § 1. Pure general propositions. Thing | 130 |
| § 2. Analysis of the general proposition, showing how adjectives alone function in the predicate | 130 |
| § 3. Different interpretations of the universal and particular proposition, and the resultant modifications in their relations of implication and opposition | 135 |

J. L.

b

CONTENTS

| | PAGE |
|--|------|
| § 4. Comparison with traditional scheme | 139 |
| § 5. Development of the immediate implications of the universal proposition as ordinarily interpreted ; with tables of immediate inferences | 140 |
| § 6. Development of the analogies between universals and particulars on the one side and necessary composites and possible conjunctions on the other | 143 |
| § 7. Formulation of general propositions in terms of classes. The principle underlying Euler's and Venn's diagrams | 146 |
| § 8. Summary | 155 |

CHAPTER X

EXISTENTIAL, SUBSISTENTIAL AND NARRATIVE PROPOSITIONS

| | |
|--|-----|
| § 1. The proper principle for the Classification of Propositions | 156 |
| § 2. Philosophical distinction between Existential and Subsistential | 158 |
| § 3. The so-called existential formulation of Propositions : Two meanings of the Universe of Discourse | 159 |
| § 4. The interpretation of mythical propositions as elliptically secondary | 164 |
| § 5. The Narrative Proposition | 166 |
| § 6. Distinction between fictitious and historical narratives | 167 |
| § 7. The relation between the existence of a class and the existence of an individual | 170 |

CHAPTER XI

THE DETERMINABLE

| | |
|---|-----|
| § 1. The fundamentum divisionis, for which the name Determinable is to stand. The analogy and distinction between the relation of an individual to its class and that of an adjectival determinate to its determinable | 173 |
| § 2. Can abstracts be divided into singular and general? The corresponding distinction is properly between the comparatively determinate and the comparatively indeterminate. The notion of a determinable as generating its determinates | 177 |
| § 3. The increase of intension that determines a decrease of extension to be supplemented by the superdeterminateness of adjectival characterisation. Classification as starting from a summum genus constituted by mere determinables and terminating in an ultima species, constituted by absolute determinates. Illustration from Botany | 178 |
| § 4. Relations between determinates under a given determinable. Incompatibility. Order of qualitative betweenness, continuous or discrete. Complex determinables | 181 |
| § 5. Important consequences of the distinction between the absolutely determinate and the comparatively indeterminate | 183 |

CONTENTS

xi

CHAPTER XII

THE RELATION OF IDENTITY

| | PAGE |
|--|------|
| § 1. Identity goes along with Otherness, the two being co-opponent relations. Verbal and Factual Identification involve the same kind of identity, the nature of the propositions only differing | 186 |
| § 2. False sense in which identity has been said to imply difference | 188 |
| § 3. Relations like and unlike; similar and dissimilar; agreeing and disagreeing | 189 |
| § 4. Adjectives comparable and disparate. Distensive magnitude. Difference properly adjectival | 190 |
| § 5. Adjectival comparison implies substantival otherness. False view of the notion of 'numerical' difference | 192 |
| § 6. Criticism of the Leibnizian dictum: the identity of indiscernibles | 193 |
| § 7. Can identity be defined? Identity carries co-implication and substitutability. Exception for secondary proposition | 195 |
| § 8. Identity and the Continuant, psychical or physical. Relation of Occurrent to Continuant. The causal bond that constitutes the unity of Experience | 199 |

CHAPTER XIII

RELATIONS OR TRANSITIVE ADJECTIVES

| | |
|--|-----|
| § 1. Monadic, diadic, triadic, etc. adjectives and propositions. Every relative implies its correlative | 203 |
| § 2. The substantive-couple characterised by an adjective-couple | 205 |
| § 3. Illustrations of 'analogy' extending beyond arithmetical ratio | 207 |
| § 4. The coupling-tie and the characterising-tie, expressed in the grammatical rules of governance and accordance | 209 |
| § 5. The paradox of relation. Distinction between tie and relation. The relation 'characterised' as unit relation entering into all adjectives and relations | 211 |
| § 6. How adjectives, relations and propositions may function as substantives | 214 |
| § 7. Relational propositions of triadic and higher orders. The cognates of a relation: permutations and bracketing. Relations between terms and possibilia | 216 |
| § 8. The problem of the relation of assertion to the proposition. How <i>ideas</i> are involved in propositions | 219 |

CHAPTER XIV

LAWS OF THOUGHT

| | |
|---|-----|
| § 1. The logical meaning of consistency as distinguished from truth. The Laws of Thought may possibly be not enumerable as independents | 222 |
| § 2. The value and functions of truisms | 223 |
| § 3. In what sense are the principles of logic imperatives? The distinction between normative and positive sciences dealing with psychological material | 224 |

b 2

| | PAGE |
|---|------|
| § 4. The fundamental formulae as used implicitly in building up the logical system itself | 226 |
| § 5. The three laws of identity: Transitive, Symmetrical, Reflexive | 227 |
| § 6. The four Principles of Propositional Determination; and their import. Expression of the same for Adjectival Determination. These in generalised form. The distinction between false and not-true, true and not-false. These compared with male and female | 228 |
| § 7. Defence of the first Principle of Propositional Determination. Truth as temporally unalterable and as capable of only one meaning | 233 |
| § 8. The four Principles of Adjectival Determination, as bringing out the relation of an adjective to its determinable. Postulate of the absolutely determinate character of the real. Substantival Categories distinguished by the Adjectival Determinables by which they are characterisable. The possibility of exhaustively comprehending the range of variation of a determinable. Summary | 237 |
| § 9. The four Principles of Connectional Determination. These embody the purely formal properties of the causal relation. Its complementary aspects—Agreement and Difference. The fundamental postulate involved in the existence of Laws of Nature. The plurality and uniqueness both of cause and of effect | 242 |
| § 10. The bearing of these principles upon the problem of internal and external relations | 249 |
| INDEX | 253 |

INTRODUCTION

§ I. LOGIC is most comprehensively and least controversially defined as the analysis and criticism of thought. This definition involves the least possible departure from the common understanding of the term and is not intended to restrict or extend its scope in any unusual way. The scope of logic has tended to expand in two directions—backwards into the domain of metaphysics, and forwards into that of science. These tendencies show that no rigid distinction need be drawn on the one side between logic and metaphysics, nor on the other between logic and science. The limits imposed by any writer are justified so far as his exposition exhibits unity; it is, in fact, much more important to remove confusions and errors within the subjects discussed under the head of logic, than to assign precise limits to its scope. It is, I hold, of less importance to determine the line of demarcation between logic and philosophy than that between logic and science; so that my treatment of logic might be called philosophical in comparison with that of those who implicitly or explicitly separate their criticism and analysis from what in their view should be relegated to epistemology and ontology.

This account of the scope of logic does not differ in any essential respects from that given, for example, in Mill's long introductory chapter. The special feature of Mill's logic is the great prominence given to the theory of induction, in contrast to most of his predecessors

and contemporaries, including Whately. Whately does not omit reference to induction any more than Mill omits syllogism: where they differ is that Whately asserts that in order to be valid any inductive inference must be formulated syllogistically, and that therefore the principle for induction is dependent on the principle of syllogism. Mill opposes this view; but as regards the scope of logic there is no disagreement between them: they differ simply on the question of the relations of deduction to induction.

If any writer deliberately or on principle dismisses from logic the theory of inductive inference, it must be on one of three grounds: either (*a*) that no inductive inference is valid; or (*b*) that different criteria of validity apply to different sciences; or (*c*) that the problem of the validity of induction constitutes a topic to be included in some study other than that named logic. As regards (*a*), this is the view which seems to be held by Venn in his *Empirical Logic* where, in the chapter on the subjective foundations of induction, he acknowledges that as a matter of fact human beings do make directly inductive inferences, even with a feeling of conviction, but that no warrant for such conviction can be found. Another aspect of his view of induction is expounded in the chapter on the objective foundations of induction, in which he classifies the different kinds of uniformity—such as sequence, co-existence, permanence, rhythm—which are used as major premisses, expressive of actual fact, by means of which specific uniformities under each general head are established as valid. When then he is asked what reasonable ground there is for accepting these major premisses as true, he

INTRODUCTION

xv

maintains in effect that they have to be assumed, in order to give security to the conclusions inductively inferred. In using the word assumption, there seems to be some ambiguity, namely whether it is to be understood to mean 'assumed to be true although known to be false' or 'assumed to be true although unprovable.' I take Venn to mean the latter, and that the attitude towards this assumption is merely one of felt certainty—felt, indeed, by all human beings, but having no root in our rational nature, and only exhibiting a common psychological disposition or character. This view, that there is no inductive principle that is self-evidently or demonstrably true, seems to be held by many other logicians, though none of them, I think, put it as explicitly as Venn. So while he and others include induction in their logical exposition, they neglect what I take to be the essential justification for its inclusion, namely as affording a systematic criticism of the question of its validity. As regards (*b*), many excellent text-books have been written in these days treating of the principles and methods peculiar to different sciences; it is not denied by their authors that this treatment is logical; but, if not explicitly stated, yet it seems to be suggested that in comparing the logic of one science with that of another the sole result is to exhibit differences, and that no one set of principles applies to all the different sciences. If this were the fact there would be some excuse for excluding the treatment of induction from the scope of logic, on the ground that the discussion of each of the separate principles should be relegated to its own department of science. But if, as I hold in agreement with most

other logicians, there must be a community of principle discoverable in all sciences, then the discussion of this must be included in logic. As regards (*c*) the question raised seems to be: 'Given the topic induction, what name shall be given to the science that includes it in its treatment?' rather than the converse question 'Given the name logic, shall it be defined so as to include, or so as to exclude, induction?' If we put the question in the first form, the answer is of course purely arbitrary; we might give it the name Epagogics. But if the question is put in the second form, the answer is not in the same sense arbitrary, assuming that there is general unanimity as regards the usage of the name logic to denote a science whose central or essential function is to criticise thought as valid or invalid. That induction should be included in logic thus defined follows from the undeniable fact that we do infer inductively, and that some persons in reference to some problems do infer invalidly. Even if this were not the fact, it is certainly of scientific importance to render explicit what everyone implicitly recognises in their inferences—as much for the case of induction as for that of syllogism or other formal types of inference. It has even been hinted that nobody makes mistakes in formal inference; and yet—in despite of this, if true—no one questions the value of systematising the principles under which people may unconsciously reason; and what holds of formal inference would certainly hold *à fortiori* of the processes of inductive inference which present many more serious opportunities for fallacy.

§ 2. As regards the term 'thought' which enters

INTRODUCTION

xvii

into my definition, its application is intended to include perceptual judgments which are commonly contrasted with rather than subsumed under thought, for the reason that thought is conceived as purely abstract while perception contains an element of concreteness. But properly speaking even in perceptual judgment there is an element of abstraction; and on the other hand no thought involves mere abstraction. It follows, therefore, that the processes of thinking and of perceptual judgment have an essential identity of character which justifies their treatment in a single systematic whole. It is the distinction between sense-experience and perceptual judgment, and not that between perceptual judgment and thought, that must be emphasised. The essential feature of perceptual judgment in contrast to mere sense-experience is that it involves activity, and that this activity is controlled by the purpose of attaining truth; further it is the presence of this purpose which distinguishes thought from other forms of mental activity. Thought may therefore be defined as mental activity controlled by a single purpose, the attainment of truth.

§ 3. Now it is true, as often urged, that thought is motivated not solely by the purpose of attaining truth, but rather by the intention of realising a particular end in some specific form and under certain specific circumstances. But I have to maintain that any other or further purpose which may prompt us to undertake the activity of thinking is irrelevant to the nature of thought as such, this other purpose serving only to determine the direction of activity. When such activity is actually in operation its course is wholly independent of the

prompting motive and guided by the single purpose of attaining truth. For instance, our desire for food may prompt us to search for it ; but this resolve, once taken, leads to a thinking process the purpose of which is to come to some conclusion as to where food is likely to be found, and the sole aim of this process is to discover what is true on the matter in hand. This being so, the logical treatment of thought must be disencumbered from all reference to any ulterior purpose.

Whether truth is ever pursued without any ulterior purpose is a psychological question which may fairly be asked ; and if introspection is to be trusted must certainly be answered in the affirmative, although the enquiry whether true knowledge has intrinsic value or not belongs to ethics. That the attainment of truth for its own sake constitutes a genuine motive force is further confirmed by recognising the fact that people do actually attach value to true knowledge, as is incontestably proved by their willingness to defy the prospect of social disapprobation, persecution, and even martyrdom incurred by the utterance and promulgation of what they hold to be true. At the same time, it must be pointed out that the aim of the thinking process is not the attainment of truth in general, but always of truth in regard to some determinate question under consideration. This is closely analogous to the psychological fact that what we desire is never pleasure in general, but always—if the doctrine of psychological hedonism is to be accepted—some specific experience which is represented as pleasurable.

Any thinking process is normally initiated by a question and terminated by an answer ; what dis-

INTRODUCTION

xix

tinguishes one thinking process from another is the difference of the question proposed. The bond of unity amongst the phases of a single process does not necessarily entail unbroken temporal continuity, but only identity of the question proposed. Indeed any thought process may be temporarily interrupted before the proposed question has been answered. It must be left as a topic for psychology to investigate the causes of such suspension, and how far the advance made serves as a starting point for further advances. Logic, on the other hand, is concerned with the nature of the advance as an advance and criticises the process from the point of view of validity or invalidity.

§ 4. The above definition of logic as the analysis and criticism of thought should be compared with that of the Scholastics, who laid emphasis on the point that logic is concerned with the art of thinking, where art is nearly equivalent to the modern term technique, and has an understood reference to activity with an end in view. The study of the art of thinking as thus understood is of use in instructing us how to proceed when thinking out any problem: for instance, it lays down rules of classification and division for the clearing up of obscurities and inconsistencies in thought; rules for the recall and selection of knowledge appropriate to any given problem; etc. Descartes' *Discourse on Method* is a classical illustration of this species of science. Modern examples of excellent treatises on these lines are to be found in Alfred Sidgwick, and other neo-pragmatists. It is a science of the highest value, and need only be separated from logic on the ground of the difference of purpose; inasmuch as its direct

purpose is the attainment of valid thought, whereas logic is the study of the conditions of valid thought, and as such it does not exclude the study of the art.

§ 5. Alongside of the use of the term 'art' to mean technique, there is a more modern usage where it implies reference to aesthetic feelings and judgments. Nowadays discussions as to whether an objective standard for these feelings and judgments should be recognised are very prominent. The nature of the feelings and judgments that enter into aesthetic appreciation belongs to psychology; but if we agree that there is a discoverable objective standard, then the treatment of the subject of aesthetics is to be distinguished from the psychological treatment, precisely as the treatment of thought in logic is distinguished from that in psychology.

Aesthetics, in this sense, raises very similar problems to those presented in Ethics; and it is frequently said that as normative Logic, Aesthetics and Ethics are related in the same way to the three psychological factors, thought, feeling and volition respectively. Each of the normative studies may be said to be based on a standard of value, the precise determination of which it is their function to formulate; in each, imperatives are laid down which are acknowledged by the individual, not on any external authority, but as self-imposed; and, in each, the ultimate appeal is to the individual's intuitive judgment. There is, however, a closer resemblance between Ethics and Aesthetics in their relations to volitions and feelings respectively, than between either of them and Logic; inasmuch as there are apparently fundamental differences of opinion

INTRODUCTION

xxi

as to the ultimate ethical and aesthetical standards, that give to the studies of Ethics and Aesthetics a controversial character absent from Logic about whose standards there is no genuine disagreement. As regards the relation of Ethics to Logic, the question sometimes arises as to which subject is supreme. The answer to this question depends entirely upon the nature of the supremacy intended: the imperatives for thought become imperatives for conduct only on condition that true judgments have intrinsic value and false judgments intrinsic disvalue; and thus, from the point of view of conduct, Logic is subordinate to Ethics. On the other hand, ethical enquiry—like any other scientific investigation—has to avoid violating logical principles, so that from the point of view of true thought Logic is supreme over Ethics.

§ 6. Our discussion so far has led us to consider the relations of Logic to Philosophy in general, Psychology, Aesthetics and Ethics. Another subject to which it is closely allied and from which it is yet distinct is Grammar, the alliance being *prima facie* accounted for by the common concern of the two studies with language. The connection between thought and language presents a problem for the science of Psychology; but, so far as thinking or the communication of thought involves the use of words, the provinces of Logic and Grammar coincide; that is to say universal Grammar, which excludes what pertains to different languages and includes only what is common to all languages, should be subsumed under Logic. For the modes in which words are combined—which constitute the subject matter for Grammar—cannot be expounded or

understood except as reflecting the modes in which thoughts are combined; and this combination is effected by means of such logical operations as negation, conjunction, disjunction, alternation, implication and so on, represented by the words *not*, *and*, *not both*, *or*, *if*, etc. To justify the subordination of Grammar to Logic we have only to realise that the analysis of the sentence in Grammar corresponds to the analysis of thought in Logic, and that grammatical criticism is confined to securing that the sentence precisely represents the thought, any further criticism of the proposition coming exclusively within the province of Logic. It may be pointed out in this connection as specially significant both for the linguist and for the logician, that languages differ in the degree of their capacity to exhibit through their structure intimacy between words and thoughts.

§ 7. Amongst all the sciences over which logic must rule, there is one that occupies a unique place. The constituents of thought which are in the most narrow sense logical are those which give form to the construct, connecting alien elements by modes which give specific significance to the whole. The first group of these is expressed by ties, conjunctive words, prepositional words, and modes of verbal inflection. But as the form of thought is further elaborated there enter new kinds of terms, namely specific adjectives which have a constant meaning definable in terms of pure thought, or else are to be admitted and understood as indefinables. The most generic form of such adjectives directly expresses the result of such mental acts of comparison as like, unlike, different from, agreeing with. Owing to the purely logical nature of these relations, universal

INTRODUCTION

xxiii

formulae in which they are introduced can be constructed by mere abstract thought. The preliminary condition for this construction is the separating of what is given to constitute a plurality, and thus to introduce a formal factor which can only be verbally expressed by the separations and juxtapositions of the substantial words. The very general relation that separation effects is that most indeterminate relation *otherness*. When the complementary notions of separateness and togetherness are joined to constitute a unity, there enters the idea of number, and we are in the domain of mathematics.

The extraordinary capacity for development that marks mathematics is due to the precision with which the relations of comparison are capable of being amplified. Through the substitutions that are thus rendered possible, the range of application of mathematical formulae is extended beyond the bounds which would otherwise delimit logic. Any material that might be presented to thought upon which the same precise operations of comparison could be performed, would lead to the same forms as mathematics. For example ideas, not only of difference, but of determinable degrees of difference, bring the material into relations of intrinsic order, and out of these relations emanate relations between relations, so that theoretically the science develops into a highly complicated system. The point then, where we may venture to say that logic actually passes into mathematics is where the specific indefinable adjectives above referred to give new material for further logical combinations.

Here it is of great importance to point to the

relative nature of the distinction between form and matter. Logic begins with a sharp contrast between matter, as what is given as merely shapeless, and form, as that which thought imposes. But as we advance to mathematics, we impose a new element of form in introducing the relation otherness and its developments; and this being operated on by thought takes the place of new matter: in short, what is introduced as matter is form in the making. All this could be summed up by saying that for elemental logic, mathematical notions would constitute matter; whereas when the step into mathematics is once taken these same elements are just those in accordance with which thought advances in constructing more and more complicated forms. This view of the relation of logic to mathematics will be worked out in Part II of the present work under 'Demonstration,' where the procedure of building up mathematical science is shown to involve the very same principles as are used in the logical structure.

All the sciences, including mathematics, over which logic has supreme control, have been properly described as applied logic. But mathematics is applied logic in a certain very unique sense, for mathematics is nothing but an extension of logical formulae introducing none but purely logical factors; while every other science borrows its material from experiential sources, and can only use logical principles when or after such material is supplied. Within mathematics we have again the same kind of distinction, namely that between pure and applied mathematics, as it has been called. In pure mathematics, the mathematician can give free

INTRODUCTION

xxv

play to his imagination in constructing forms that are restricted only by principles of logical consistency, and he develops the implications that are derivable from what may be indifferently regarded either as definitions of his fictitious constructs or as hypothetically entertained first axioms. In order that these axioms and the theorems therefrom derived may be considered as true, recourse must be had to the real world, and if applicable, the axioms come to be assertorically entertained as premisses, and the derived propositions as the developed conclusions. This application of mathematics to reality constitutes applied mathematics. Taking geometry as our first example, while there is no limit to constructing conceived spaces other than Euclidian, their application to reality demands the enquiry whether our space is or is not Euclidian. This is answered by an appeal to our immediate intuitions directed to our spatial experiences, and it is this appeal that is outside the range of pure mathematics. Again the merely logical conception of betweenness, which develops into that of serial orders of lower or higher forms of complexity, is in the first instance a product of pure logical constructiveness, and would yield implications from which a system of implicates could be developed. But such a hypothetically conceived body of propositions would have no basis in the real but for the applicability of the defined conceptions to what is given in non-mathematical intuition. This applicability holds not only in the domain of spatial order, but also in that of the qualitative relations of difference which impose serial order amongst sense impressions.

Regarded in the light of its control over all sciences

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logic has been called by the name 'Methodology'; that is to say while the forms of logic implicitly control the conclusions of science, logic itself includes the study which renders explicit the ways according to which its authority is exercised. The department of logic known as methodology constitutes the third part of the present work, which is entitled 'The logical foundations of Science.'

Another illustration of applied mathematics is to quantity. Quantity is not a mere direct development from number, since a new conception, namely that of equality of units, enters as a distinctive factor which is not purely logical. It is true that equality for merely formal developments could be defined as a certain relation having the formal properties of symmetry and transitivity, and if to this conception is added the fundamental operation plus (+), definable as a certain relation having the formal properties commutative and associative, the whole system of quantitative science could be developed without recourse to any but pure mathematical principles. But even in this range of thought quantities of different types would need recognition. For example, given the notion of length as the first spatial quantity, a new quantity is derived by multiplying length by length, which is called area; here 'multiplied' need not be more specifically defined than a certain relation having the formal properties commutative and associative. Again where a quantum of space is divided by a quantum of time, we have velocity, and in this way a totally new type of quantity is constructed and we pass from geometry to kinematics. Another quantity called mass is such that when multi-

INTRODUCTION

xxvii

plied by velocity there is engendered the new quantity called momentum, and when multiplied by velocity squared, energy; and in the introduction of these new species of quantity we pass from kinematics to dynamics. This is the terminus on these lines of applied mathematics; and dynamics may be defined as the science that uses the three independently definable species of quantity time, space and mass. In every extension, then, of mathematics no new idea or mode of thought need accompany the work of the calculus. It is only when the formulae have to be applied to reality, and thus to be entertained categorically, that a process of thought other than merely mathematical enters in, and intuition is directed to what is given in some form of experience. The ideas which enter into the mathematical sciences thus constructed have a form which renders them amenable to purely logical processes of indefinite degrees of complexity; this distinguishes them from the non-mathematical or 'natural' sciences that introduce ideas dependent simply upon brute matter, unamenable to logical analysis, logic entering only in the application to these ideas of classification, and the principles of inductive inference.

§ 8. Having considered logic in its relation to the different sciences, we may now pass to a discussion of its more philosophical aspects. Logicians have been classified as nominalists, conceptualists, and realists or materialists, according as they think it worth while to discuss words, thoughts or things. Names that are apt to be understood as synonyms for these have been applied to different philosophical opinions; and this fact is indicative of the change which has occurred in

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the course of the history of philosophy, where the ground has been shifted from ontology to psychology, and later from psychology to logic. To take realism first. It is the name given to the Platonic view which formed the basis of Aristotle's controversy with Plato. Plato in discussing the relation between the universal and the individual, attributes *real* existence in the truest or most ultimate sense to the universal, holding that the particular individual has reality only so far as it partakes of the nature of the universal, towards which it strives as the end (*ἐντελέκη*) of its existence. Aristotle, opposing this view, holds that the universal exists not apart from the particular but in it.

A new psychological significance came to be attached to the term Realism, when the question of reality was raised not about the *thing*, but about the possible *idea* of the thing, these two concepts being taken to be equivalent. The so-called nominalist school of philosophers maintained the psychological view that we had no idea corresponding to a general name, along with the ontological view according to which the particular individual or concrete alone existed, and no existence could be attributed to the universal; generality, for them, attached only to names in use, and had no objective application. On the psychological point at issue the opponents of this view have been known as conceptualists, and in maintaining their opposition were led to make a psychological distinction of great importance between images and ideas. In common with the nominalists, they held that images are necessarily concrete, particular or individual, but they maintained that we can also frame ideas which can properly be

INTRODUCTION

xxix

called abstract or general. Both schools assumed that images were equivalent to or at least resembled perceptions, and further that the latter were obviously concrete and particular. Berkeley represents the nominalist school, and his subtle difference from Locke—who definitely held that we can frame general ideas, though with difficulty—comes out clearly when he disputes the possibility of a general idea of a triangle (instanced by Locke) which shall be neither equilateral nor isosceles nor scalene, and from which we can in thought abstract the shape from variations of colour. In my view Locke and Berkeley were both wrong, even where they agreed; inasmuch as neither images nor perceptions reflect the concreteness and particularity of the individual thing, which should be described as determinate, in contrast to the indeterminateness of the mental processes. In fact there has been a confusion in the description of our thoughts, images and percepts, between the distinction of the universal from the particular, and that of the indeterminate from the determinate. The modern term ‘generic,’ which has been applied to images, should be extended also to percepts, on the ground that they share with images the character of indeterminateness—a character which must be rigidly distinguished from general or universal as properly applied to ideas or concepts.

Nominalism has yet another meaning when applied as a special logical theory; in this sense it denotes the theory according to which the proposition is an indication of the names that have been arbitrarily chosen to denote things or classes of things, and predicates merely what follows from the consistent use of these names.

Propositions are thus used as mere formulae and repeated in thought when necessary, without demanding any consideration of their meaning; so that the only ultimate foundations or premisses of knowledge are definitions, no other propositions of the nature of axioms being required. This view still clings to some modern philosophical expositions of arithmetic and pure logic, and is rather subtly akin to the view that the first premisses for science are nothing but postulates or hypotheses which, if *consistently* held, lead to the discovery of truth.

As regards Conceptualism, it is doubtful whether, as applied to the work of such writers as Hamilton and Sigwart, it can be properly regarded as a distinctive logical theory. For the prominent use of the word concept and its associate judgment points not necessarily to any difference of logical theory between those who use these words, and those who prefer the words 'term' or 'name' and 'proposition,' but merely to the common recognition that thought has form as well as verbal expression. If, however, the conceptualist proceeds to limit the scope of logic to the consideration of the forms of thought alone, then he must maintain that the truth of a judgment is tested by the form that connects the content as conceived; and conceptualism becomes equivalent to formalism. The criterion for the formalist is indeed mere consistency or coherence in fact; that for the conceptualist proper, clearness or distinctness in thought. The latter is expressed negatively by Herbert Spencer: what is clearly not conceivable is false; positively by Descartes: what is clearly conceivable is true. It follows immediately from this view that truth concerns only conceived content;