CAMBRIDGE

Cambridge University Press
978-1-107-68531-4 — The Riemann Hypothesis for Function Fields
Machiel van Frankenhuijsen
Table of Contents
More Information

Contents

List of illustrations *page* x
Preface xi
Introduction 1

1 Valuations 9
1.1 Trace and norm 9
1.1.1 The canonical pairing 11
1.2 Valued fields 12
1.2.1 Norms on a vector space 13
1.2.2 Discrete valuations 16
1.2.3 Different and ramification 18
1.2.4 Inseparable extensions 20
1.3 Valuations of $\mathbb{F}_q(T)$ 21
1.4 Global fields 24
1.4.1 Constants and nonconstant functions 26
1.4.2 Ramification 28

2 The local theory 31
2.1 Additive character and measure 31
2.1.1 Characters of $\mathbb{F}_q(T)$ 32
2.1.2 Characters of K 35
2.1.3 Fourier transform 36
2.2 Multiplicative character and measure 38
2.3 Local zeta function 40
2.4 Functional equation 40

3 The zeta function 43
3.1 Additive theory 43
3.1.1 Divisors 45
3.1.2 Riemann–Roch 46

© in this web service Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-1-107-68531-4 — The Riemann Hypothesis for Function Fields
Machiel van Frankenhuijsen
Table of Contents
More Information

3.2 Multiplicative theory 48
3.2.1 Divisor classes 48
3.2.2 Coarse idele classes 50
3.3 The zeta function 51
3.3.1 Constant field extensions 54
3.3.2 Shifted zeta function 56
3.4 Computation 56
3.5 Semi-local theory 59
3.6 Two-variable zeta function 63
3.6.1 The polynomial $L_{\mathcal{C}}(X, Y)$ 65

4 Weil positivity 69
4.1 Functions on the coarse idele classes 69
4.2 Zeros of $\Lambda_{\mathcal{C}}$ 70
4.3 Explicit formula 72

5 The Frobenius flow 75
5.1 Heuristics for the Riemann hypothesis 75
5.2 Orbits of Frobenius 79
5.2.1 The projective line 79
5.2.2 Example: elliptic curves 84
5.2.3 The curve $\mathcal{C}$ 85
5.3 Galois covers 89
5.4 The Riemann hypothesis for $\mathcal{C}$ 91
5.4.1 The Frobenius flow 92
5.4.2 Frobenius as symmetries 97
5.5 Comparison with the Riemann hypothesis 100

6 Shift operators 107
6.1 The Hilbert spaces $\mathcal{Z}$ and $\mathcal{H}$ 108
6.1.1 The truncated shift on $\mathcal{Z}$ 110
6.1.2 The trace using a kernel 111
6.2 Shift operators 112
6.2.1 Averaging spaces 115
6.3 Local trace 117
6.3.1 Order of the cutoffs 119
6.3.2 Direct computation 120
6.3.3 Trace using the kernel 121
6.4 How to prove the Riemann hypothesis for $\mathcal{C}$? 124
6.5 The operators M, A, C, $\mathcal{F}^*$, and E 126
6.5.1 Restricting the support 126
6.5.2 Making the measure additive 127

© in this web service Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-1-107-68531-4 — The Riemann Hypothesis for Function Fields
Machiel van Frankenhuijsen
Table of Contents
More Information

6.5.3 Smoothing the oscillations 128
6.5.4 Restricting to the coarse idele classes 130
6.6 Semi-local and global trace 130
6.7 The kernel on the analytic space 133

7 Epilogue 137
7.1 Archimedean translation 138
7.2 Global translation 140
7.3 The space of adele classes 141

References 143
Index of notation 149
Index 151

© in this web service Cambridge University Press www.cambridge.org