

## Index

- adele class space, 6
- adeles, 4, 5, 43, 51
- archimedean valuation, 1, 4, 100
- arithmetic coordinate, 5, 104
- ARTIN, Emil, 3
- BOMBIERI, Enrico, 3, 5, 75
- canonical divisor, 45
- canonical exponent, 36
- canonical pairing, 11
- CAUCHY, Augustin-Louis, 1
- character
  - additive, 32
  - multiplicative, 38
- class number, 49
- coarse idele class group, 50
- combined shift, 109, 121, 131
- completion, 24, 79
- conjugate, 70, 108, 109
- CONNES, Alain, 4–6, 75, 137, 141
- CONSANI, Katia, 141
- convolution product, 38, 70, 109
- criterion for ramification, 18
- critical strip, 1
- decomposition group, 90
- degree of a divisor, 45
- degree of a valuation, **79**
- degree of inertia, 17
- DELIGNE, Pierre, 4
- DENINGER, Christopher, 5
- description of a valuation
  - algebraic, dynamic, geometric, 83, 89
- diagonal, 4
- different, 18
- differential exponent, 18
- dilation, 113, 127
- divisor, 45
  - positive, 47
  - principal, 46
- divisor class group, 49
- equivalence of norms, 14
- EULER, Leonhard, 33
- Euler product, 3, 52, 54, 57
  - Riemann zeta function, 1
- extension of a valuation, 15
- flow on a curve, 5
- Fourier transform, 37, 44, 60
  - on ideles, 128, 131
- Frobenius
  - action, 3–6
  - automorphism, 6, 81, 91
  - flow, 5, 83, 88, 92
  - graph of, 3, 4, 92, 98
- functional equation
  - for  $L_C(X)$ , 59
  - for  $\Lambda_C(s)$ , 57
  - for  $\zeta_C(f, s)$ , 55
  - for  $\zeta_C(s)$ , 3, 57
  - for  $\zeta_v(s)$ , 40
  - for  $\zeta_C(s, t)$ , 64
  - for the Riemann zeta function, 1
- fundamental group, 97
- Galois cover, 89
- Galois extension, 90
- Gaussian measure, 101
- genus of  $C$ , 2, 45
- geometric coordinate, 5, 104
- global field, 24
- Haar measure
  - additive, 36, 43
  - multiplicative, 39, 48

HADAMARD, Jacques Salomon, 69  
 HARAN, Shai M. J., 4, 52, 137, 139  
 HASSE, Helmut, 3  
 Hensel's lemma, 81  
 hyperring, 100, 106, 141  
 idele class group, 6, 49  
 ideles, 43  
 inertia group, 91  
 infrared cutoff, 117, 126  
 inseparable  
   element, 20  
   extension, 20  
 integers of  $K_v$ , 16  
 inverse different, 18  
 inversion, 109, 110  
 IWASAWA, Kenkichi, 31  
 Jacobian of a curve, 3, 4  
 kernel, 111–117  
 KMS states, 141  
 local trace, 26  
 locally constant, 37  
 MARCOLLI, Matilde, 141  
 Mellin transform, 40, 56, **70**, 110  
   on  $\mathbb{R}^+$ , 70  
 Möbius function, 132  
 multiplication map, 9  
 multiplicative structure, 38  
 NAUMANN, Niko, 66  
 Nevanlinna Theory, 103  
 Newton's algorithm, 81  
 noncommutative space, 7  
 norm, **10**  
   archimedean, 12  
   nonarchimedean, 12  
   on a field, 12  
   on a vector space, 13  
 norm of an idele, 46  
 orbit of Frobenius, 83, 88  
 order of ramification, 17  
 order of vanishing, 13, **31**  
 Ostrowski's theorem, 24  
 $P$ -adic valuation, 22  
 partial fraction decomposition, 44  
 PELLIKAAN, Ruud, 63  
 Poisson summation formula, 46, 62  
 Poisson–Jensen formula, 104  
 principal divisor, 46  
 purely inseparable, 20

quasi-character, 38  
 real prime, 4  
 regular function, 16  
 residue class field, 17  
 residue of a function, 34  
 restriction, 40, 50  
 RIEMANN, Georg Friedrich Bernhard, 1, 69  
 Riemann hypothesis, 137  
   for curves, 2, 3, 100  
   for integers, 1, 100  
 Riemann zeta function, 1  
 Riemann–Roch formula  
   for curves, 3–5, **46**, 100  
   for integers, 1  
 ROQUETTE, Peter, 3  
 SCHMIDT, Friedrich Karl, 2, 3  
 SCHMIDT, Wolfgang M., 3  
 Selberg's trace formula, 75, 124  
 separable closure, 21  
 separable extension, 10, 79  
 shift operator, 4, 5, 7, 109–124, 130–136  
 shifted zeta function, 56, 57  
 $S$ -integers, 60  
 SOULÉ, Christophe, 139  
 STEPANOV, Sergei Aleksandrovich, 3  
 $S$ -units, 61  
 TATE, John, 1, 5, 31, 140  
 Tate's thesis, 78  
 theta-function, 1  
 trace, **10**  
 trace pairing, 11  
 triangle inequality, 12  
   for a valuation, 12  
 trivial different, 18  
 ultraviolet cutoff, 117, 128  
 uniformizer, 17  
 unramified extension, 17, 18  
 unramified quasi-character, 38  
 DE LA VALLÉE-POUSSIN, Charles-Jean  
   Étienne Gustave Nicolas, 69  
 valuation, 12  
 WEIL, André, 3, 4, 137  
 Weil distribution, 73, 121  
 zeta function  
   of  $\mathcal{C}$ , 2  
   two-variable, 63