CHAPTER I

ELEMENTARY LAWS OF OPTICS

1. Introduction. Optical calculations are based upon the laws of reflexion and refraction. When we seek reasons for these laws, we must turn to some theory of light and endeavour to deduce the mathematical laws from the assumptions of the theory. We shall adopt Maxwell's electromagnetic theory as being most nearly in agreement with all that is known upon the subject.

2. Maxwell's electromagnetic theory of light. On Maxwell's theory, light is an electromagnetic disturbance involving both electric and magnetic forces. The electric displacement (or the electric polarisation) and the magnetic induction are in the wave front, and thus the vibrations which occur are transverse. The theory accounts for reflexion and refraction and leads to Fresnel's wave surface in crystals. The velocity of electromagnetic waves, calculated in terms of quantities found by electrical experiments, agrees closely with the observed velocity of light.

3. The electromagnetic vectors. Maxwell's theory involves four vectors. (1) The electric force E. (2) The magnetic force H. (3) The electric displacement D. (4) The magnetic induction B. In an isotropic medium, D is in the same direction as E and

$$D = KE/4\pi, \quad \text{.......................(1)}$$

where K is the specific inductive capacity. When the electric displacement varies, it acts as an electric current whose strength is equal to the rate of increase of the displacement. In an isotropic medium, B is in the same direction as H and

$$B = \mu H, \quad \text{..........................(2)}$$

where μ is the magnetic permeability.

4. The electromagnetic relations. These are:

I. The total flux of displacement outwards from any closed surface is equal to the electric charge within it.

II. The total flux of magnetic induction outwards from any closed surface is zero.

III. The work done upon a unit positive charge by the field, when the charge is taken once round any circuit, is equal to the rate of diminution of the flux of induction through the circuit, the positive direction round the circuit being connected with the positive direction of the induction in the same way as the rotation and translation of a right-handed screw working in a fixed nut.

IV. The work done upon a unit positive pole by the field, when the pole is taken once round any circuit, is 4π times the total current through the circuit, the positive directions being connected as in III.

5. Plane waves. Heaviside's method. We now consider a plane wave of the type devised by Dr Oliver Heaviside*. When the wave reaches any point, the electric and magnetic vectors at that point suddenly jump from zero to definite constant values E, H, D and B. Let any one of the vectors be represented by a line drawn perpendicular to the axis OX (Fig. 1), the direction of propagation of the wave. Then the

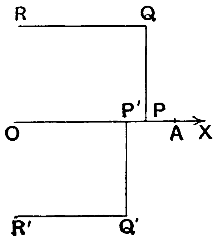

FIG. 1

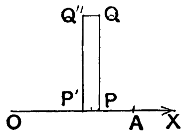

FIG. 2

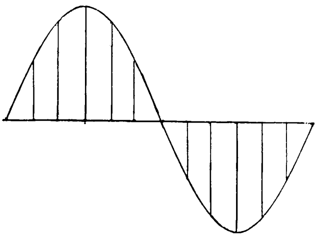

FIG. 3

wave is represented by $XPQR$, and the vector jumps from zero to the constant value PQ when the wave front reaches A. If, very soon after the first wave is started, a second wave of the

* "On the electromagnetic wave surface," *Electrical Papers*, Vol. II, p. 1, or *Phil. Mag.* June 1885, p. 397.

same character be started, the vectors in this wave being equal in magnitude but opposite in direction to those in the first wave, the second wave may be represented by $XP'Q'R'$. The second wave will neutralise all but a very small portion of the first wave, and we shall be left with the very thin wave represented by $XPQQ''P'O$ in Fig. 2. This wave produces only a momentary disturbance as it passes over A. Any sort of wave, as for instance the sinusoidal wave represented in Fig. 3, can now be built up of a succession of infinitely thin waves similar to $PQQ''P'$, and hence, at every part of a train of plane waves of any description, the relations between the vectors will be the same as in the Heaviside wave.

6. Directions of D and B. Take a fixed cylinder with its curved surface parallel to the direction of propagation of the wave and its plane ends p, q parallel to the wave front, as shown in section in Fig. 4, and suppose that the wave front lies between the ends of the cylinder and is moving towards p. By relation I, the total outward flux of displacement from the cylinder is zero, since it encloses no charge. There is no flux through the end p, for the wave has not yet reached it, and therefore D is zero there. If there be any outward flux through one part of the curved surface, it is exactly compensated by a flux inwards through the remainder of that surface. Thus the outward flux of displacement through the end p and through the curved surface is zero, and hence D can have no component perpendicular to the end q. Hence D is parallel to the wave front.

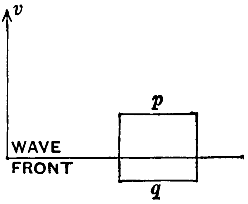

FIG. 4

If we use relation II, a similar argument shows that B is parallel to the wave front.

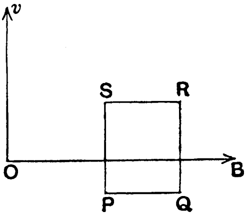

FIG. 5

7. Directions of E and H. Let O be a point on the wave front, let Ov (Fig. 5) be the normal to the wave front through O, and let OB, at right angles to Ov, be the direction of the

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magnetic induction B in the wave. Let $PQRS$ be a rectangle fixed in space with two sides parallel to Ov and two to OB, the line OB lying between PQ and RS. Since, at any point in the wave, B is parallel to the plane of this rectangle, there is no flux of induction through it, and therefore the flux does not vary with the time as the wave front advances. Hence, by relation III, no work is done in taking a charge round the rectangle. But along RS the electric force is zero, since the wave has not reached RS, and the amounts of work done along QR and SP neutralise each other. Hence the work done along PQ is zero, for the work done in the whole circuit is zero. Thus the electric force parallel to B is zero, or E is perpendicular to B.

By considering a rectangle with two sides parallel to D and two parallel to the normal to the wave front, we find, by relation IV, that H is perpendicular to D.

Thus, D and B are perpendicular to the wave normal and E is perpendicular to B and H to D. But it does not follow that, in general, E and H are perpendicular to the wave normal, for in a crystalline medium D is not necessarily in the same direction as E, nor B in the same direction as H.

In an isotropic medium, D is in the same direction as E, and B in the same direction as H, and then E and H are perpendicular to the wave normal and each is perpendicular to the other.

8. Relations between E and H in an isotropic medium. Take a rectangle $abcd$ (Fig. 6) with two sides parallel to H and two to the wave normal Ov, and let ab be l cm., this side being ahead of the wave front. Since E is perpendicular to Ov and to OH, it is normal to the plane of the rectangle; we shall suppose the positive direction of E to be towards the reader. If the velocity of the wave be v cm. sec.$^{-1}$, the disturbed area increases by lv square cm. per sec. Hence the flux of displacement increases at the rate lvD, or the current through the circuit is lvD. It is only along cd that there is any magnetic force and hence the work done by the field when a unit pole is taken once round the circuit in the direction $abcd$ is lH.

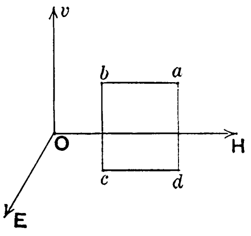

FIG. 6

By relation IV, $lH = 4\pi lvD$. Thus, by (1),

$$H = KvE. \quad \text{.........................(3)}$$

If we take a rectangle with two sides parallel to E and two to Ov, we find, by relation III, that

$$E = vB = \mu vH. \quad \text{.........................(4)}$$

Right-handed rotation through $\frac{1}{2}\pi$ about the forward direction of propagation turns E round to H.

9. Velocity of propagation. From (3) and (4), we obtain

$$v = 1/\sqrt{K\mu}. \quad \text{.........................(5)}$$

It can be shown that $\{K_{air} \times \mu_{air}\}^{-\frac{1}{2}}$ is numerically equal to the ratio of the electromagnetic unit to the electrostatic unit of electricity, the medium "air" being specified because the units are defined in connexion with that medium. Hence the velocity of an electromagnetic wave through air is numerically equal to the ratio of the units.

10. Periodic wave motion. When a single electron executes regular vibrations about a mean position O, it generates a succession of waves. At any fixed point the disturbance goes through cycles in a regular manner. At any instant the disturbance is not the same at all points on a straight line drawn from O. If we take a point P, where the disturbance is zero at any instant, there will at the same instant be zero disturbance at all those points on OP whose distances from O differ from OP by integral multiples of λ, the distance traversed by the waves during one cycle of the electron.

11. Periodic plane waves. When the distance from the electron is very great compared with the greatest width of the part of the wave under consideration, that part may be treated as a portion of a plane wave. In a train of plane waves there is no change in the amplitude as the waves advance; thus the disturbance will be periodic with respect to the distance measured along the wave normal as well as regards the time.

At every point of a fixed plane parallel to the planes of the waves the electric force is in the plane and has the same magnitude and direction. Let the electric force in this origin plane be

$$E_0 = a \sin pt. \quad \text{.........................(6)}$$

At a point at distance x measured from the origin plane in the direction in which the waves advance, the electric force at time t is the same as it was in the origin plane at the time $t - x/v$, and thus

$$E = a \sin p\,(t - x/v). \qquad \text{..................(7)}$$

The electric force at time t has the same value at all points for which x is a multiple of $2\pi v/p$, and hence this distance is the wave length λ. Thus, $\lambda = 2\pi v/p$. Hence,

$$E = a \sin \frac{2\pi}{\lambda}(vt - x). \qquad \text{.....................(8)}$$

The quantity $(2\pi/\lambda)(vt - x)$ is called the phase of the vibration at the time t at a point defined by x.

The number of complete vibrations per second is called the frequency and is denoted by n.

Thus $$n\lambda = v. \qquad \text{...........................(9)}$$

The magnetic force is at right angles both to the electric force and to the wave normal. By (3),

$$H = KvE = Kva \sin \frac{2\pi}{\lambda}(vt - x). \qquad \text{...........(10)}$$

In the train of plane waves which we have considered, the electric force is everywhere parallel to a fixed straight line which is perpendicular to the wave normal. This character is expressed by saying that the train of waves is polarised; the "plane of polarisation" is perpendicular to the electric force.

12. Rays. A luminous point sends out a succession of waves, which, in their progress, may be reflected or refracted. When the medium is isotropic, a straight line PN drawn normal to the wave front which is passing over a point P at any instant is called the ray through P.

When we consider doubly refracting media, such as some crystals, it is necessary to distinguish between the wave normal and another straight line which is then called a "ray." With isotropic media, the expressions "wave normal" and "ray" are interchangeable.

13. Laws of reflexion. Let v_1 be the velocity of propagation of waves in the medium [1] containing the incident wave, and

v_2 the velocity in the second medium [2]. Let the plane of Fig. 7 cut the surface of separation at right angles in AB, and

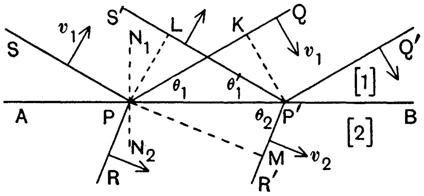

FIG. 7

also be perpendicular to the incident wave fronts. Let PQ be the section of the front of the incident wave at time $t = 0$, and let $P'Q'$ correspond to time t. Then, if $P'K$ be the perpendicular from P' on PQ, $P'K = v_1 t$.

If θ_1 be the angle QPB between the surface and the front of the incident wave, $PP' = P'K/\sin\theta_1 = v_1 t/\sin\theta_1$. If θ_1' be the angle SPA between the surface and the front of the reflected wave, $PP' = v_1 t/\sin\theta_1'$, since the perpendicular PL from P on $P'S'$ is $v_1 t$. Hence

$$\sin\theta_1' = \sin\theta_1. \quad \text{.....................(11)}$$

Thus the reflected wave is inclined to the surface at the same angle as the incident wave but is on the other side of N_1PN_2, the normal to the surface at P, as indicated by PS and PQ in Fig. 7.

The incident ray, or the normal to the incident wave at P, is perpendicular to PQ, while the reflected ray, or the normal to the reflected wave at P, is perpendicular to PS, and each is in the plane of the paper. The plane containing the incident ray and the normal to the surface is called the plane of incidence. The acute angle between the incident ray and the normal to the surface is called the angle of incidence, and the acute angle between the reflected ray and the normal to the surface is called the angle of reflexion. These angles are equal to those between the surface and the two wave fronts. The result, in terms of the rays, is as follows:

Laws of reflexion. (1) The reflected ray is in the plane containing the incident ray and the normal to the surface at the point of incidence. (2) The incident and reflected rays make equal angles with the normal: or the angles of incidence and reflexion are equal.

14. Laws of refraction; Snell's law. The perpendicular PM from P on the refracted wave front $P'R'$ is v_2t. Hence, if θ_2 be the angle RPA between the surface and the front of the refracted wave,

$$v_2t/\sin\theta_2 = PP' = v_1t/\sin\theta_1.$$

Thus, $$\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2}. \quad \text{.....................(12)}$$

The incident ray at P is perpendicular to PQ while the refracted ray is perpendicular to PR, and each is in the plane of the paper. The acute angle between the refracted ray and the normal N_1PN_2 to the surface is called the angle of refraction. It is clear that the angle of incidence is equal to θ_1, and that the angle of refraction is equal to θ_2. Hence the result is as follows:

Laws of refraction. (1) The refracted ray is in the plane containing the incident ray and the normal to the surface at the point of incidence. (2) The sine of the angle of refraction bears a constant ratio to the sine of the angle of incidence (Snell's law).

15. Remarks on the laws of reflexion and refraction. The method by which we investigated the laws of reflexion and refraction does not give the relative magnitudes of the disturbances in the three waves; it does not even tell us that the reflected and the refracted waves exist. It merely tells us the directions they must have if they do exist. For a given angle of incidence, the relative magnitudes and the directions of the electric forces in the reflected and the refracted waves depend upon the direction of the electric force in the incident wave. By the electromagnetic theory, the directions and magnitudes of the electric forces in the reflected and refracted waves can be calculated when the direction and the magnitude of the electric force in the incident wave are given (see Chapter XII).

16. Refractive index. The ratio of the sine of the angle of incidence to that of refraction is called the refractive index of the second medium relative to the first; the ratio will be denoted by ${}_1\mu_2$. Thus,

$$\sin\theta_1/\sin\theta_2 = {}_1\mu_2. \quad \text{.....................(13)}$$

Hence, by (12), $${}_1\mu_2 = v_1/v_2. \quad \text{........................(14)}$$

If ${}_2\mu_1$ be the refractive index of the first medium relative to the second, i.e. the ratio of $\sin\theta_2$ to $\sin\theta_1$, we have, by (12)

$${}_2\mu_1 = \sin\theta_2/\sin\theta_1 = v_2/v_1,$$

and hence

$${}_2\mu_1 = 1/{}_1\mu_2. \quad \text{.......................(15)}$$

The refractive index of any medium relative to a vacuum is called the absolute refractive index, or simply the refractive index, of the medium. If v be the velocity of a wave in a vacuum and μ_1, μ_2 be the absolute refractive indices of the media [1], [2], then $\mu_1 = v/v_1$, $\mu_2 = v/v_2$. Hence, by (14),

$${}_1\mu_2 = v_1/v_2 = \mu_2/\mu_1. \quad \text{.................(16)}$$

Since $v_1/v_2 = \mu_2/\mu_1$, we have, by (12),

$$\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2, \quad \text{.................(17)}$$

the form in which Snell's law is usually employed.

In most experiments, it is the refractive index of the medium relative to air which is determined. By (16), we can find the absolute refractive index of the medium (M), since that of air (A) is known. Thus,

$$\mu_M = {}_A\mu_M \times \mu_A. \quad \text{....................(18)}$$

At 0° C. and 760 mm., the absolute refractive index, μ_A, of air for sodium light is 1·0002918.

17. Dispersion. In the case of glass, water and many other substances and within the range of visible light, the refractive index increases as the frequency increases. Since the frequency in blue is greater than in red light, the refractive indices of these media for blue are greater than for red light, and thus blue is often said to be more refrangible than red light. But it is not universally true that the refractive index increases with the frequency, for with some substances there is a narrow range of frequency within which the refractive index *diminishes* as the frequency increases.

The dependence of the refractive index upon the frequency of the disturbance is called *dispersion*.

CHAPTER II

SOME APPLICATIONS OF THE LAWS OF REFLEXION AND REFRACTION

18. Relations between incident and reflected rays. Let PA (Fig. 8) be a normal to the plane reflecting surface AHK, K any point on AHK and KN the normal at K. Let a plane HAP containing PA cut the surface in AH. Since PA is normal to AHK, HA, KA cut AP at right angles. Let the ray PK be reflected along KL. Since KN is parallel to AP, the plane $PKLN$, containing the rays and the normal KN, is identical with the plane KPA. Hence KL, produced backwards, cuts PA in Q. Draw KH perpendicular to AH. Since KH is at right angles to AH and to AP, it is normal to the plane HAP. Since KN is parallel to PQ and since $LKN = PKN$, we have $KQP = KPQ$. Hence KA bisects PQ at right angles in A, and thus $QA = PA$. Then, by symmetry, all corresponding distances and angles are equal. Hence HP and QHT, the *projections* on the plane HAP of the rays PK and KL, make equal angles with HM, the normal to the surface at H. Since KH is normal to HAP, the angles which KP, KQ make with the plane HAP are KPH, KQH, and by symmetry these are equal. Thus:

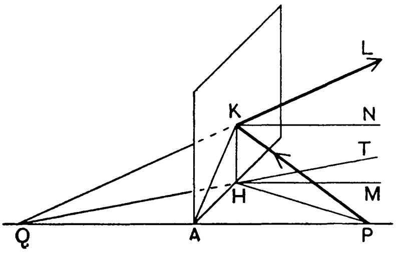

FIG. 8

(1) The *projections* of the incident and reflected rays on any *plane* normal to the reflecting surface obey the laws of reflexion.
(2) The incident and reflected rays are equally inclined to any *plane* normal to the reflecting surface.

19. Image of a point by reflexion at a plane surface. By § 18, the position of Q is independent of the direction of the ray PK. Hence, if P be a luminous point, *all* the reflected rays, if produced, pass *accurately* through Q. The point Q is called the image of P. Hence the image of a point formed by a plane