

Contents

| | | |
|----------|---|----------------|
| | <i>Preface</i> | <i>page ix</i> |
| 1 | Introduction | 1 |
| | 1.1 Graphs and Weighted Graphs | 1 |
| | 1.2 Random Walks on a Weighted Graph | 6 |
| | 1.3 Transition Densities and the Laplacian | 11 |
| | 1.4 Dirichlet or Energy Form | 15 |
| | 1.5 Killed Process | 21 |
| | 1.6 Green's Functions | 22 |
| | 1.7 Harmonic Functions, Harnack Inequalities, and the Liouville Property | 26 |
| | 1.8 Strong Liouville Property for \mathbb{R}^d | 31 |
| | 1.9 Interpretation of the Liouville Property | 32 |
| 2 | Random Walks and Electrical Resistance | 38 |
| | 2.1 Basic Concepts | 38 |
| | 2.2 Transience and Recurrence | 42 |
| | 2.3 Energy and Variational Methods | 44 |
| | 2.4 Resistance to Infinity | 55 |
| | 2.5 Traces and Electrical Equivalence | 61 |
| | 2.6 Stability under Rough Isometries | 67 |
| | 2.7 Hitting Times and Resistance | 70 |
| | 2.8 Examples | 73 |
| | 2.9 The Sierpinski Gasket Graph | 75 |
| 3 | Isoperimetric Inequalities and Applications | 80 |
| | 3.1 Isoperimetric Inequalities | 80 |
| | 3.2 Nash Inequality | 85 |
| | 3.3 Poincaré Inequality | 91 |

| | | |
|----------|--|-----|
| 3.4 | Spectral Decomposition for a Finite Graph | 97 |
| 3.5 | Strong Isoperimetric Inequality and Spectral Radius | 101 |
| 4 | Discrete Time Heat Kernel | 106 |
| 4.1 | Basic Properties and Bounds on the Diagonal | 106 |
| 4.2 | Carne–Varopoulos Bound | 111 |
| 4.3 | Gaussian and Sub-Gaussian Heat Kernel Bounds | 116 |
| 4.4 | Off-diagonal Upper Bounds | 124 |
| 4.5 | Lower Bounds | 128 |
| 5 | Continuous Time Random Walks | 132 |
| 5.1 | Introduction to Continuous Time | 132 |
| 5.2 | Heat Kernel Bounds | 140 |
| 6 | Heat Kernel Bounds | 149 |
| 6.1 | Strongly Recurrent Graphs | 149 |
| 6.2 | Gaussian Upper Bounds | 155 |
| 6.3 | Poincaré Inequality and Gaussian Lower Bounds | 160 |
| 6.4 | Remarks on Gaussian Bounds | 168 |
| 7 | Potential Theory and Harnack Inequalities | 172 |
| 7.1 | Introduction to Potential Theory | 172 |
| 7.2 | Applications | 179 |
| | Appendix | 183 |
| A.1 | Martingales and Tail Estimates | 183 |
| A.2 | Discrete Time Markov Chains and the Strong Markov Property | 186 |
| A.3 | Continuous Time Random Walk | 190 |
| A.4 | Invariant and Tail σ -fields | 197 |
| A.5 | Hilbert Space Results | 202 |
| A.6 | Miscellaneous Estimates | 205 |
| A.7 | Whitney Type Coverings of a Ball | 206 |
| A.8 | A Maximal Inequality | 211 |
| A.9 | Poincaré Inequalities | 213 |
| | <i>References</i> | 219 |
| | <i>Index</i> | 224 |