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Karen Morrison and Lucille Dunne
Excerpt
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1 Reviewing number concepts

1.1 Different types of numbers

- Real numbers can be divided into rational and irrational numbers. You will deal with rational numbers in this chapter. Irrational numbers are covered in chapter 9.
- Rational numbers can be written as fractions in the form of $\frac{a}{b}$ where a and b are integers and $b \neq 0$. (Integers are negative and positive whole numbers, and zero.)
- Integers, fractions and terminating decimals are all rational numbers.

Tip

Make sure you know what the following sets of numbers are: natural numbers, integers, odd and even numbers and prime numbers.

Exercise 1.1

1 Tick the correct columns in the table to classify each number.

Number	Natural	Integer	Prime	Fraction
−0.2				
−57				
3.142				
0				
$0.\dot{3}$				
1				
51				
10 270				
$-\frac{1}{4}$				
$\frac{2}{7}$				
11				
$\sqrt[3]{512}$				

2 List:

(a) four square numbers greater than 100.
(b) four rational numbers smaller than $\frac{1}{3}$.
(c) two prime numbers that are > 80.
(d) the prime numbers < 10.

1.2 Multiples and factors

- A multiple of a number is the product obtained when multiplying that number and an integer. The lowest common multiple (LCM) of two or more numbers is the lowest number that is a multiple of both (or all) of the numbers.
- A factor of a number is any number that will divide into the number exactly.
- The highest common factor (HCF) of two or more numbers is the highest number that is a factor of all the given numbers.

To find the LCM of a set of numbers, you can list the multiples of each number until you find the first multiple that is in the lists for all of the numbers in the set.

FAST FORWARD

You will use LCM again when you work with fractions to find the lowest common denominator of two or more fractions. See chapter 5.

You need to work out whether to use LCM or HCF to find the answers. Problems involving LCM usually include repeating events. Problems involving HCF usually involve splitting things into smaller pieces or arranging things in equal groups or rows.

Exercise 1.2 A

1 Find the LCM of the given numbers.

(a) 9 and 18 **(b)** 12 and 18 **(c)** 15 and 18 **(d)** 24 and 12
(e) 36 and 9 **(f)** 4, 12 and 8 **(g)** 3, 9 and 24 **(h)** 12, 16 and 32

2 Find the HCF of the given numbers.

(a) 12 and 18 **(b)** 18 and 36 **(c)** 27 and 90 **(d)** 12 and 15
(e) 20 and 30 **(f)** 19 and 45 **(g)** 60 and 72 **(h)** 250 and 900

Exercise 1.2 B

1 Amira has two rolls of cotton fabric. One roll has 72 metres on it and the other has 90 metres on it. She wants to cut the fabric to make as many equal length pieces as possible of the longest possible length. How long should each piece be?

2 In a shopping mall promotion every 30th shopper gets a $10 voucher and every 120th shopper gets a free meal. How many shoppers must enter the mall before one receives a voucher and a free meal?

3 Amanda has 40 pieces of fruit and 100 sweets to share amongst the students in her class. She is able to give each student an equal number of pieces of fruit and an equal number of sweets. What is the largest possible number of students in her class?

4 Francesca, Ayuba and Claire are Olympic and Paralympic contenders. They share a training slot on a running track. Francesca cycles and completes a lap in 20 seconds, Ayuba runs the lap in 84 seconds and Claire, in her wheelchair, takes 105 seconds. They start training together. After how long will all three be at the same point again and how many laps will each have completed?

5 Mr Smit wants to tile a rectangular veranda with dimensions 3.2 m × 6.4 m with a whole number of identical square tiles. Mrs Smit wants the tiles to be as large as possible.

(a) Find the area of the largest possible tiles in cm^2.
(b) How many 3.2 m × 3.2 m tiles will Mr Smit need to tile the veranda?

1.3 Prime numbers

- Prime numbers only have two factors: 1 and the number itself.
- Prime factors are factors of a number that are also prime numbers.
- You can write any number as a product of prime factors. But remember the number 1 itself is *not* a prime number so you cannot use it to write a number as the product of its prime factors.
- You can use the product of prime factors to find the HCF or LCM of two or more numbers.

You can use a tree diagram or division to find the prime factors of a composite whole number.

Exercise 1.3

1 Identify the prime numbers in each set.

(a) 1, 2, 3, 4, 5, 6, 7, 8, 9, 10
(b) 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60
(c) 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105

2 Express the following numbers as a product of their prime factors.

(a) 36 **(b)** 65 **(c)** 64 **(d)** 84
(e) 80 **(f)** 1000 **(g)** 1270 **(h)** 1963

3 Find the LCM and the HCF of the following numbers by means of prime factors.

(a) 27 and 14 **(b)** 85 and 15 **(c)** 96 and 27 **(d)** 53 and 16
(e) 674 and 72 **(f)** 234 and 66 **(g)** 550 and 128 **(h)** 315 and 275

1.4 Powers and roots

- A number is squared (n^2) when it is multiplied by itself ($n \times n$).
- The square root ($\sqrt{n}$) of a number is the number that is multiplied by itself to get the number.
- A number is cubed (n^3) when it is multiplied by itself and then multiplied by itself again ($n \times n \times n$).
- The cube root ($\sqrt[3]{n}$) of a number is the number that is multiplied by itself twice to get the number.

FAST FORWARD

Powers greater than 3 are dealt with in chapter 2. See topic 2.5 indices. ▶

Exercise 1.4

1 Find all the square and cube numbers between 100 and 300.

2 Simplify.

(a) $\sqrt{9}+\sqrt{16}$ **(b)** $\sqrt{9+16}$ **(c)** $\sqrt{64}+\sqrt{36}$ **(d)** $\sqrt{64+36}$
(e) $\sqrt{\frac{36}{4}}$ **(f)** $\left(\sqrt{25}\right)^2$ **(g)** $\frac{\sqrt{9}}{\sqrt{16}}$ **(h)** $\sqrt{169-144}$
(i) $\sqrt[3]{27}-\sqrt[3]{1}$ **(j)** $\sqrt{100\div 4}$ **(k)** $\sqrt{1}+\sqrt{\frac{9}{16}}$ **(l)** $\sqrt{16}\times\sqrt[3]{27}$
(m) $\sqrt{(-5)^2}\times\sqrt[3]{-1}$ **(n)** $\sqrt{\frac{1}{4}}+\sqrt{\left(\frac{1}{3}\right)^2}$ **(o)** $\sqrt[3]{1}-\sqrt[3]{-125}$

3 A cube has a volume of 12 167 cm^3. Calculate:

(a) the height of the cube.
(b) the area of one face of the cube.

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1.5 Working with directed numbers

- Integers are directed whole numbers.
- Negative integers are written with a minus (–) sign. Positive integers may be written with a plus (+) sign, but usually they are not.
- In real life, negative numbers are used to represent temperatures below zero; movements downwards or left; depths; distances below sea level; bank withdrawals and overdrawn amounts, and many more things.

Exercise 1.5

1 If the temperature is 4 °C in the evening and it drops 7 °C overnight, what will the temperature be in the morning?

2 Which is colder in each pair of temperatures?

(a) 0 °C or –2 °C **(b)** 9 °C or –9 °C **(c)** –4 °C or –12 °C

Draw a number line to help you.

3 An office block has three basement levels (level –1, –2 and –3), a ground floor and 15 floors above the ground floor (1 to 15). Where will the lift be in the following situations?

(a) Starts on ground and goes down one floor then up five?
(b) Starts on level –3 and goes up 10 floors?
(c) Starts on floor 12 and goes down 13 floors?
(d) Starts on floor 15 and goes down 17 floors?
(e) Starts on level –2, goes up seven floors and then down eight?

1.6 Order of operations

- When there is more than one operation to be done in a calculation you must work out the parts in brackets first. Then do any division or multiplication (from left to right) before adding and subtracting (from left to right).
- The word 'of' means × and a fraction line means divide.
- Long fraction lines and square or cube root signs act like brackets, indicating parts of the calculation that have to be done first.

Remember the order of operations using BODMAS:
*B*rackets
*O*f
*D*ivide
*M*ultiply
*A*dd
*S*ubtract

Exercise 1.6

Tip
Most modern scientific calculators apply the rules for order of operations automatically. If there are brackets, fractions or roots in your calculation you need to enter these correctly on the calculator. When there is more than one term in the denominator, the calculator will divide by the first term only unless you enter brackets.

FAST FORWARD
The next section will remind you of the rules for rounding numbers. ▶

1 Calculate and give your answer correct to two decimal places.

(a) $8 + 3 \times 6$ **(b)** $(8 + 3) \times 6$ **(c)** $8 \times 3 - 4 \div 5$

(d) $12.64 + 2.32 \times 1.3$ **(e)** $6.5 \times 1.3 - 5.06$ **(f)** $(6.7 \div 8) + 1.6$

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(g) $1.453+\frac{7.6}{3.2}$ (h) $\frac{5.34+3.315}{4.03}$ (i) $\frac{6.54}{2.3}-1.08$

(j) $\frac{5.27}{1.4\times1.35}$ (k) $\frac{11.5}{2.9-1.43}$ (l) $\frac{0.23\times4.26}{1.32+3.43}$

(m) $8.9-\frac{8.9}{10.4}$ (n) $\frac{12.6}{8.3}-\frac{1.98}{4.62}$ (o) $12.9-2.03^2$

(p) $(9.4-2.67)^3$ (q) $12.02^2-7.05^2$ (r) $\left(\frac{16.8}{9.3}-1.01\right)^2$

(s) $\frac{4.07^2}{8.2-4.09}$ (t) $6.8+\frac{1.4}{6.9}-\frac{1.2}{9.3}$ (u) $4.3+\left(1.2+\frac{1.6}{5}\right)^2$

(v) $\frac{6.1}{2.8}+\left(\frac{2.1}{1.6}\right)^2$ (w) $6.4-(1.2^2+1.9^2)^2$ (x) $\left(4.8-\frac{1}{9.6}\right)\times4.3$

1.7 Rounding numbers

- You may be asked to round numbers to a given number of decimal places or to a given number of significant figures.
- To round to a decimal place:
  - look at the value of the digit to the right of the place you are rounding to
  - if this value is ≥ 5 then you round up (add 1 to the digit you are rounding to)
  - if this value is ≤ 4 then leave the digit you are rounding to as it is.
- To round to a significant figure:
  - the first non-zero digit (before or after the decimal place in a number) is the first significant figure
  - find the correct digit and then round off from that digit using the rules above.

Exercise 1.7

FAST FORWARD

Rounding is very useful when you have to estimate an answer. You will deal with this in more detail in chapter 5. ▶

1 Round these numbers to:

(i) two decimal places
(ii) one decimal place
(iii) the nearest whole number.

(a) 5.6543 (b) 9.8774 (c) 12.8706
(d) 0.0098 (e) 10.099 (f) 45.439
(g) 13.999 (h) 26.001

2 Round each of these numbers to three significant figures.

(a) 53 217 (b) 712 984 (c) 17.364 (d) 0.007279

3 Round the following numbers to two significant figures.

(a) 35.8 (b) 5.234 (c) 12 345 (d) 0.00875
(e) 432 128 (f) 120.09 (g) 0.00456 (h) 10.002

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Mixed exercise

1 State whether each number is natural, rational, an integer and/or a prime number.

$-\frac{3}{4}$ 24 0.65 −12 $3\frac{1}{2}$ 0 0.66 17

2 List the factors of 36.

(a) How many of these factors are prime numbers?
(b) Express 36 as the product of its prime factors.
(c) List two numbers that are factors of both 36 and 72.
(e) What is the highest number that is a factor of both 36 and 72?

3 Write each number as a product of its prime factors.

(a) 196 **(b)** 1845 **(c)** 8820

4 Amira starts an exercise programme on the 3rd of March. She decides she will swim every 3 days and cycle every 4 days. On which dates in March will she swim and cycle on the same day?

5 State whether each equation is true or false.

(a) $18 \div 6 + (5 + 3 \times 4) = 20$
(b) $6 \times (5 - 4) + 3 = 9$
(c) $\frac{30+10}{30} - 10 = 1$
(d) $(6 + 3)^2 = 45$

6 Simplify:

(a) $\sqrt{100} \div \sqrt{4}$ **(b)** $\sqrt{100 \div 4}$ **(c)** $\left(\sqrt[3]{64}\right)^3$ **(d)** $4^3 + 9^2$

7 Calculate. Give your answer correct to two decimal places.

(a) $\frac{5.4 \times 12.2}{4.1}$ **(b)** $\frac{12.2^2}{3.9^2}$ **(c)** $\frac{12.65}{2.04} + 1.7 \times 4.3$

(d) $\frac{3.8 \times 12.6}{4.35}$ **(e)** $\frac{2.8 \times 4.2^2}{3.3^2 \times 6.2^2}$ **(f)** $2.5 - \left(3.1 + \frac{0.5}{5}\right)^2$

8 Round each number to three significant figures.

(a) 1235.6 **(b)** 0.76513 **(c)** 0.0237548 **(d)** 31.4596

9 A building supply store is selling tiles with an area of 790 cm^2.

(a) Is it possible to have square tiles whose area is not a square number? Explain.
(b) Find the length of each side of the tile correct to 3 significant figures.
(c) What is the minimum number of tiles you would need to tile a rectangular floor 3.6 m long and 2.4 m wide?

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2 Making sense of algebra

2.1 Using letters to represent unknown values

- Letters in algebra are called variables because they can have many different values (the value varies). Any letter can be used as a variable, but x and y are used most often.
- A number on its own is called a constant.
- A term is a group of numbers and/or variables combined by the operations multiplying and/or dividing only.
- An algebraic expression links terms by using the + and – operation signs. An expression does not have an equals sign (unlike an equation). An expression could have just one term.

Exercise 2.1

Tip
An expression in terms of x means that the variable letter used in the expression is x.

1 Write expressions, in terms of x, to represent:

(a) 3 times the sum of a number and 2
(b) 6 times the difference of a number and 1
(c) twice the sum of 11 and a number
(d) a number times the difference of 12 and –6
(e) 4 added to 3 times the square of a number
(f) a number squared added to 4 times the difference of 7 and 5
(g) a number subtracted from the result of 4 divided by 20
(h) a number added to the result of 3 divided 9
(i) the sum of 8 times $\frac{1}{2}$ and a number times 3
(j) the difference of a number times –5 and 6 times –2

2 A boy is p years old.

(a) How old will the boy be in five years' time?
(b) How old was the boy four years ago?
(c) His father is four times the boy's age. How old is the father?

3 Three people win a prize of \$$x$.

(a) If they share the prize equally, how much will each of them receive?
(b) If the prize is divided so that the first person gets half as much money as the second person and the third person gets three times as much as the second person, how much will each receive?

2.2 Substitution

- Substitution involves replacing variables with given numbers to work out the value of an expression. For example, you may be told to evaluate $5x$ when $x = -2$. To do this you work out $5 \times (-2) = -10$

Exercise 2.2

1 The formula for finding the area (A) of a triangle is $A = \frac{1}{2}bh$, where b is the length of the base and h is the perpendicular height of the triangle.
Find the area of a triangle if:

(a) the base is 12 cm and the height is 9 cm
(b) the base is 2.5 m and the height is 1.5 m
(c) the base is 21 cm and the height is half as long as the base
(d) the height is 2 cm and the base is the cube of the height.

REWIND
Remember that the BODMAS rules always apply in these calculations. ◀

2 Evaluate $3xy - 4(2x - 3y)$ when $x = 4$ and $y = -3$.

3 Given that $a = 3$, $b = -2$ and $c = -4$, evaluate $(a + 2b)^2 - 4c$.

Take special care when substituting negative numbers. If you replace x with -3 in the expression $4x$, you will obtain $4 \times -3 = -12$, but in the expression $-4x$, you will obtain $-4 \times -3 = 12$.

4 When $m = 2$ and $n = -3$, what is the value of $m^3 - \frac{n^3}{m^2} + mn + n^2$?

5 The number of games that can be played among x competitors in a chess tournament is given by the expression $\frac{1}{2}x^2 - \frac{1}{2}x$.

(a) How many games will be played if there are 4 competitors?
(b) How many games will be played if there are 14 competitors?

2.3 Simplifying expressions

- To simplify an expression you add or subtract like terms.
- Like terms are those that have exactly the same variables (including powers of variables).
- You can also multiply and divide to simplify expressions. Both like and unlike terms can be multiplied or divided.

Remember, like terms must have exactly the same variables with exactly the same indices. So $3x$ and $2x$ are like terms but $3x^2$ and $2x$ are not like terms.

Remember, multiplication can be done in any order so, although it is better to put variable letters in a term in alphabetical order, $ab = ba$. So, $3ab + 2ba$ can be simplified to $5ab$.

Remember,
$x \times x = x^2$
$y \times y \times y = y^3$
$x \div x = 1$

Exercise 2.3

1 Simplify the following expressions.

(a) $3x^2 + 6x - 8x + 3$ **(b)** $x^2y + 3x^2y - 2yx$ **(c)** $2ab - 4ac + 3ba$
(d) $x^2 + 2x - 4 + 3x^2 - y + 3x - 1$ **(e)** $-6m \times 5n$ **(f)** $3xy \times 2x$
(g) $-2xy \times -3y^2$ **(h)** $-2xy \times 2x^2$ **(i)** $12ab \div 3a$ **(j)** $12x \div 48xy$
(k) $\frac{33abc}{11ca}$ **(l)** $\frac{45mn}{20n}$ **(m)** $\frac{80xy^2}{12x^2y}$ **(n)** $\frac{-36x^3}{-12xy}$
(o) $\frac{y}{x} \times \frac{2y}{x}$ **(p)** $\frac{xy}{2} \times \frac{y}{x}$ **(q)** $5a \times \frac{3a}{4}$ **(r)** $7 \times \frac{-2y}{5}$
(s) $\frac{x}{4} \times \frac{2}{3y}$ **(t)** $\frac{3x}{5} \times \frac{9x}{2}$

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2.4 Working with brackets

- You can remove brackets from an expression by multiplying everything inside the brackets by the value (or values) in front of the bracket.
- Removing brackets is also called expanding the expression.
- When you remove brackets in part of an expression you may end up with like terms. Add or subtract any like terms to simplify the expression fully.
- In general terms $a(b + c) = ab + ac$

Exercise 2.4

Remember the rules for multiplying integers:
$+ \times + = +$
$- \times - = +$
$+ \times - = -$
If the quantity in front of a bracket is negative, the signs of the terms inside the bracket will change when the brackets are expanded.

1 Remove the brackets and simplify where possible.

(a) $2x(x-2)$ **(b)** $(y-3)x$ **(c)** $(x-2)-3x$ **(d)** $-2x-(x-2)$
(e) $(x-3)(-2x)$ **(f)** $2(x+1)-(1-x)$ **(g)** $x(x^2-2x-1)$
(h) $-x(1-x)+2(x+3)-4$

2 Remove the brackets and simplify where possible.

(a) $2x(\frac{1}{2}x+\frac{1}{4})$ **(b)** $-3x(x-y)-2x(y-2x)$ **(c)** $-2x(4x^2-2x-1)$
(d) $(x+y)-(\frac{1}{2}x-\frac{1}{2}y)$ **(e)** $5x+x(3-2x)$ **(f)** $2x(2x-2)-x(x+2)$
(g) $x(1-x)+x(2x-5)-2x(1+3x)$

2.5 Indices

- An index (also called a power or exponent) shows how many times the base is multiplied by itself.
- x^2 means $x \times x$ and $(3y)^4$ means $3y \times 3y \times 3y \times 3y$.
- The laws of indices are used to simplify algebraic terms and expressions. Make sure you know the laws and understand how they work (see below).
- When an expression contains negative indices you apply the same laws as for other indices to simplify it.

Exercise 2.5 A

Tip
Memorise this summary of the **index laws:**
$x^m \times x^n = x^{m+n}$
$x^m \div x^n = x^{m-n}$
$(x^m)^n = x^{mn}$
$x^0 = 1$
$x^{-m} = \frac{1}{x^m}$
$x^{\frac{m}{n}} = \left(x^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{x}\right)^m$

1 Simplify.

(a) $\frac{x^4y \times y^2x^6}{x^4y^5}$ **(b)** $\frac{2x^2y^4 \times 3x^3y}{2xy^4}$ **(c)** $\frac{2x^5y^4 \times 2xy^3}{2x^2y^5 \times 3x^2y^3}$

(d) $\frac{x^3y^7}{xy^4} \times \frac{x^2y^8}{x^3y}$ **(e)** $\frac{2x^7y^2}{4x^3y^7} \times \frac{10x^8y^4}{2x^3y^2}$ **(f)** $\frac{x^9y^6}{x^4y^2} \div \frac{x^3y^2}{x^5y}$

(g) $\frac{10x^5y^2}{9x^6y^6} \div \frac{3x^3y}{5x^7y^4}$ **(h)** $\frac{7y^3x^2}{5y^5x^4} \div \frac{5x^6y^2}{7x^5y^3}$ **(i)** $\frac{(x^5y)^2 \times (x^3y^4)^2}{(x^3y^3)^3}$

(j) $\frac{(2x^4y^2)^3}{(y^3x^2)^3} \times \frac{(x^4y^4)^2}{3(x^2y)^2}$ **(k)** $\left(\frac{x^2}{y^4}\right)^3 \times \left(\frac{x^5}{y^2}\right)^2$ **(l)** $\frac{(5x^3y^2)^3}{4x^7y^6} \div \left(\frac{2xy^3}{5x^2y^4}\right)^2$

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> **Tip**
> Some exam questions will accept simplified expressions with negative indices, such as $5x^{-4}$.
> If, however, the question states positive indices only, you can use the law $x^{-m} = \frac{1}{x^m}$ so that $5x^{-4} = \frac{5}{x^4}$.
> Similarly, $\frac{y}{x^{-2}} = x^2 y$.

2 Simplify each expression and give your answer using positive indices only.

(a) $\frac{x^5 y^{-4}}{x^{-3} y^{-2}}$ (b) $\frac{x^{-4} y^3}{x^2 y^{-1}} \times \frac{x^7 y^{-5}}{x^{-4} y^3}$ (c) $\frac{\left(2x^{-3} y^{-1}\right)^3}{\left(y^2 x^{-2}\right)^2}$

(d) $\left(\frac{x}{y^3}\right)^{-1} \div \frac{\left(x^2\right)^4}{y^{-3}}$ (e) $\frac{x^{-10}}{\left(y^{-4}\right)^2} \div \left(\frac{y^2}{x^3}\right)^{-4}$ (f) $\left(\frac{x^4 y^{-1}}{x^5 y^{-3}}\right)^2 \times \frac{\left(x^{-2} y^6\right)^2}{2\left(xy^3\right)^{-2}}$

3 Simplify.

(a) $x^{\frac{1}{4}} \times x^{\frac{1}{4}}$ (b) $x^{\frac{1}{3}} \times x^{\frac{1}{5}}$ (c) $\frac{x^{\frac{1}{2}}}{x^{\frac{1}{3}}}$ (d) $\left(x^{\frac{1}{3}}\right)^{\frac{1}{3}}$

(e) $\left(64x^6\right)^{\frac{1}{2}}$ (f) $\left(8x^9 y\right)^{\frac{1}{3}}$ (g) $\sqrt{xy^8}$ (h) $\left(\frac{x^6}{y^2}\right)^{\frac{1}{2}}$

(i) $\left(x^{\frac{1}{2}}\right)^8 \times \frac{x^2}{x^3}$ (j) $\left(x^6 y^3\right)^{\frac{1}{3}} \times \left(x^{-8} y^{-10}\right)^{\frac{1}{2}}$ (k) $\left(xy^3\right)^{\frac{1}{2}} \times \frac{x^{\frac{1}{2}} y^{\frac{1}{2}}}{xy^4}$

4 Simplify.

(a) $\left(x^{\frac{2}{3}}\right)^{\frac{1}{2}} \times \frac{x}{x^{\frac{2}{3}}}$ (b) $\left(x^{\frac{5}{2}}\right)^{\frac{3}{5}} \times \frac{x}{x^{\frac{1}{2}}}$ (c) $\left(x^{\frac{1}{2}} y^2\right)^{\frac{1}{2}} \times \left(x^{-\frac{3}{4}} y^4\right)^{\frac{1}{3}}$

(d) $\left(x^{\frac{2}{3}} y^{\frac{1}{3}}\right)^4 \times \frac{x^{\frac{1}{3}} y^{\frac{2}{3}}}{xy^2}$ (e) $\frac{y^{\frac{1}{3}}}{x^{\frac{1}{2}}} \div \left(\frac{x^{\frac{1}{2}} y^{\frac{2}{3}}}{x^3 y^4}\right)^{\frac{1}{2}}$ (f) $\frac{x^{\frac{1}{4}}}{y^{\frac{3}{2}}} \times \left(\frac{xy^{\frac{3}{4}}}{x^3 y^2}\right)^{\frac{1}{4}}$

> **Tip**
> Apply the index laws and work in this order:
> - simplify any terms in brackets
> - apply the multiplication law to numerators and then to denominators
> - cancel numbers if you can
> - apply the division law if the same letter appears in the numerator and denominator
> - express your answer using positive indices

Exercise 2.5 B

1 Evaluate:

(a) $(-3^4)(-4)^2$ (b) $\frac{-2^4}{(-2)^4}$ (c) $\frac{6^3}{(-3)^4}$ (d) $8^{\frac{1}{3}}$

(e) $256^{-\frac{1}{4}}$ (f) $125^{-\frac{4}{3}}$ (g) $\left(\frac{1}{4}\right)^{-\frac{5}{2}}$ (h) $\left(\frac{1}{8}\right)^{-\frac{2}{3}}$

(i) $\left(\frac{8}{27}\right)^{-\frac{1}{3}}$ (j) $\left(\frac{8}{18}\right)^{-\frac{1}{2}}$

2 Calculate.

(a) $5 - 7(23 - 5^2) - 16 \div 2^3$
(b) $3(5^2) - 6(-3^2 - 4^2) \div -15$
(c) $-2(-3^2) + 24 \div (-2)^3$
(d) $-2(3)^4 - (6 - 7)^6$

3 Solve for x.

(a) $81^x = 3$ (b) $3^x = 81$ (c) $3^x = \frac{1}{81}$ (d) $16^x = 8$

(e) $16^x = 256$ (f) $5^{-x} = \frac{1}{25}$ (g) $3^{2x-4} = 1$ (h) $2^{2x+1} = 16$