

HEAT TRANSFER

The single objective of this book is to provide engineers with the capability, tools, and confidence to solve real-world heat transfer problems. The textbook includes many advanced topics, such as Bessel functions, Laplace transforms, separation of variables, Duhamel's theorem, and complex combination, as well as high-order explicit and implicit numerical integration algorithms. These analytical and numerical solution methods are applied to topics not considered in most textbooks. Examples are heat exchangers involving fluids with varying specific heats or phase changes, regenerators, semi-gray surface radiation exchange, and numerical solutions to internal flow problems. To improve readability, derivations of important results are presented completely, without skipping steps, which reduces student frustration and improves retention. The examples in the book are ubiquitous, not trivial "textbook" exercises. They are rather complex and timely real-world problems that are inherently interesting. This textbook integrates the computational software packages Maple, MATLAB, FEHT, and Engineering Equation Solver (EES) directly with the heat transfer material.

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Heat Transfer

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PREFACE

The single objective of this book is to provide engineers with the capability, tools, and confidence to solve real-world heat transfer problems. This objective has resulted in a textbook that differs from existing heat transfer textbooks in several ways. First, this textbook includes many topics that are typically not covered in undergraduate heat transfer textbooks. Examples are the detailed presentations of mathematical solution methods such as Bessel functions, Laplace transforms, separation of variables, Duhamel's theorem, and Monte Carlo methods as well as high order explicit and implicit numerical integration algorithms. These analytical and numerical solution methods are applied to advanced topics that are ordinarily not considered in a heat transfer textbook.

Judged by its content, this textbook should be considered as a graduate text. There is sufficient material for two-semester courses in heat transfer. However, the presentation does not presume previous knowledge or expertise. This book can be (and has been) successfully used in a single-semester undergraduate heat transfer course by appropriately selecting from the available topics. Our recommendations on what topics can be included in a first heat transfer course are provided in the suggested syllabus. The reason that this book can be used for a first course (despite its expanded content) and the reason it is also an effective graduate-level textbook is that all concepts and methods are presented in detail, starting at the beginning. The derivation of important results is presented completely, without skipping steps, in order to improve readability, reduce student frustration, and improve retention. You will not find many places in this textbook where it states that “it can be shown that . . .” The use of examples, solved and explained in detail, is ubiquitous in this textbook. The examples are not trivial, “textbook” exercises, but rather complex and timely real-world problems that are of interest by themselves. As with the presentation, the solutions to these examples are complete and do not skip steps.

Another significant difference between this textbook and most existing heat transfer textbooks is its integration of modern computational tools. The engineering student and practicing engineer of today is expected to be proficient with engineering computer tools. Engineering education must evolve accordingly. Most real engineering problems cannot be solved using a sequential set of calculations that can be accomplished with a pencil or hand calculator. Engineers must have the ability to quickly solve problems using the powerful computational tools that are available and essential for design, parametric study, and optimization of real-world systems. This book integrates the computational software packages Maple, MATLAB, FEHT, and Engineering Equation Solver (EES) directly with the heat transfer material. The specific commands and output associated with these software packages are presented as the theory is developed so that the integration is seamless rather than separated.

The computational software tools used in this book share some important characteristics. They are used in industry and have existed for more than a decade; therefore, while this software will certainly continue to evolve, it is not likely to disappear. Educational versions of these software packages are available, and therefore the use of these

tools should not represent an economic hardship to any academic institution or student. Useful versions of EES and FEHT are provided on the website that accompanies this textbook (www.cambridge.org/nellisandklein). With the help provided in the book, these tools are easy to learn and use. Students can become proficient with all of them in a reasonable amount of time. Learning the computer tools will not detract significantly from material coverage. To facilitate this learning process, tutorials for each of the software packages are provided on the companion website. The book itself is structured so that more advanced features of the software are introduced progressively, allowing students to become increasingly proficient using these tools as they progress through the text.

Most (if not all) of the tables and charts that have traditionally been required to solve heat transfer problems (for example, to determine properties, view factors, shape factors, convection relations, etc.) have been made available as functions and procedures in the EES software so that they can be easily accessed and used to solve problems. Indeed, the library of heat transfer functions that has been developed and integrated with EES as part of the preparation of this textbook enables a profound shift in the focus of the educational process. It is trivial to obtain, for example, a shape factor, a view factor, or a convection heat transfer coefficient using the heat transfer library. Therefore, it is possible to assign problems involving design and optimization studies that would be computationally impossible without the computer tools.

Integrating the study of heat transfer with computer tools does not diminish the depth of understanding of the underlying physics. Conversely, our experience indicates that the innate understanding of the subject matter is enhanced by appropriate use of these tools for several reasons. First, the software allows the student to tackle practical and relevant problems as opposed to the comparatively simple problems that must otherwise be assigned. Real-world engineering problems are more satisfying to the student. Therefore, the marriage of computer tools with theory motivates students to understand the governing physics as well as learn how to apply the computer tools. The use of these tools allows for coverage of more advanced material and more interesting and relevant problems. When a solution is obtained, students can carry out a more extensive investigation of its behavior and therefore obtain a more intuitive and complete understanding of the subject of heat transfer.

This book is unusual in its linking of classical theory and modern computing tools. It fills an obvious void that we have encountered in teaching both undergraduate and graduate heat transfer courses. The text was developed over many years from our experiences teaching Introduction to Heat Transfer (an undergraduate course) and Heat Transfer (a first-year graduate course) at the University of Wisconsin. It is our hope that this text will not only be useful during the heat transfer course, but also provide a life-long resource for practicing engineers.

G. F. Nellis
S. A. Klein
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We have had the extreme good fortune to have had dedicated and insightful teachers. These include Glen Myers, John Mitchell, Bill Beckman, Joseph Smith Jr., John Brisson, Borivoje Mikic, and John Lienhard V. These individuals, among others, have provided us with an indication of the importance of teaching and provided an inspiration to us for writing this book.

Preparing this book has necessarily reduced the “quality time” available to spend with our families. We are most grateful to them for this indulgence. In particular, we wish to thank Jill, Jacob, and Spencer and Sharon Nellis and Jan Klein. We could have not completed this book without their continuous support.

Finally, we are indebted to Cambridge University Press and in particular Peter Gordon for giving us this opportunity and for helping us with the endless details needed to bring our original idea to this final state.

STUDY GUIDE

This book has been developed for use in either a graduate or undergraduate level course in heat transfer. A sample program of study is laid out below for a one-semester graduate course (consisting of 45 class sessions).

Graduate heat transfer class

<i>Day</i>	<i>Sections in Book</i>	<i>Topic</i>
1	1.1	Conduction heat transfer
2	1.2	1-D steady conduction and resistance concepts
3	2.8	Resistance approximations
4	1.3	1-D steady conduction with generation
5	1.4, 1.5	Numerical solutions with EES and MATLAB
6	1.6	Fin solution, fin efficiency, and finned surfaces
7	1.7	Other constant cross-section extended surface problems
8	1.8	Bessel function solutions
9	2.2	2-D conduction, separation of variables
10	2.2	2-D conduction, separation of variables
11	2.4	Superposition
12	3.1	Transient, lumped capacitance problems – analytical solutions
13	3.2	Transient, lumped capacitance problems – numerical solutions
14	3.3	Semi-infinite bodies, diffusive time constant
15	3.3	Semi-infinite bodies, self-similar solution
16	3.4	Laplace transform solutions to lumped capacitance problems
17	3.4	Laplace transform solutions to 1-D transient problems
18	3.5	Separation of variables for 1-D transient problems
19	3.8	Numerical solutions to 1-D transient problems
20	4.1	Laminar boundary layer concepts
21	4.2, 4.3	The boundary layer equations & dimensionless parameters
22	4.4	Blasius solution for flow over a flat plate
23	4.5, 4.6	Turbulent boundary layer concepts, Reynolds averaged equations
24	4.7	Mixing length models and the laws of the wall
25	4.8	Integral solutions
26	4.8, 4.9	Integral solutions, external flow correlations
27	5.1, 5.2	Internal flow concepts and correlations
28	5.3	The energy balance
29	5.4	Analytical solutions to internal flow problems
30	5.5	Numerical solutions to internal flow problems
31	6.1, 6.2	Natural convection concepts and correlations

32	8.1	Introduction to heat exchangers
33	8.2, 8.3	The LMTD and ε -NTU forms of the solutions
34	8.5	Heat exchangers with phase change
35	8.7	Axial conduction in heat exchangers
36	8.8, 8.10	Perforated plate heat exchangers and regenerators
37	10.1, 10.2	Introduction to radiation, Blackbody emissive power
38	10.3	View factors and the space resistance
39	10.3	Blackbody radiation exchange
40	10.4	Real surfaces, Kirchoff's law
41	10.5	Gray surface radiation exchange
42	10.5	Gray surface radiation exchange
43	10.5	Semi-gray surface radiation exchange
44	10.7	Introduction to Monte Carlo techniques
45	10.7	Introduction to Monte Carlo techniques

A sample program of study is laid out below for a one-semester undergraduate course (consisting of 45 class sessions).

Undergraduate heat transfer class

<i>Day</i>	<i>Sections in Book</i>	<i>Topic</i>
1	A.1	Review of thermodynamics, Using EES
2	1.2.2-1.2.3	1-D steady conduction, resistance concepts and circuits
3	1.2.4-1.2.6	1-D steady conduction in radial systems, other thermal resistance
4		More thermal resistance problems
5	1.3.1-1.3.3	1-D steady conduction with generation
6	1.4	Numerical solutions with EES
7	1.6.1-1.6.3	The extended surface approximation and the fin solution
8	1.6.4-1.6.6	Fin behavior, fin efficiency, and finned surfaces
9	1.9.1	Numerical solutions to extended surface problems
10	2.1	2-D steady-state conduction, shape factors
11	2.8.1-2.8.2	Resistance approximations
12	2.9	Conduction through composite materials
13	2.5	Numerical solution to 2-D steady-state problems with EES
14	3.1	Lumped capacitance assumption, the lumped time constant
15	3.2.1, 3.2.2	Numerical solution to lumped problems (Euler's, Heun's, Crank-Nicolson)
16	3.3.1-3.3.2	Semi-infinite body, the diffusive time constant
17	3.3.2, 3.3.4	Approximate models of diffusion, other semi-infinite solutions
18	3.5.1-3.5.2	Solutions to 1-D transient conduction in a bounded geometry
19	3.8.1-3.8.2	Numerical solution to 1-D transient conduction using EES
20	4.1	Introduction to laminar boundary layer concepts
21	4.2, 4.3	Dimensionless numbers
22	4.5	Introduction to turbulent boundary layer concepts
23	4.9.1-4.9.2	Correlations for external flow over a plate
24	4.9.3-4.9.4	Correlations for external flow over spheres and cylinders
25	5.1	Internal flow concepts

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26	5.2	Internal flow correlations
27	5.3	Energy balance for internal flows
28		Internal flow problems
29	6.1	Introduction to natural convection
30	6.2.1-6.2.3	Natural convection correlations
31	6.2.4-6.2.7	Natural convection correlations and combined forced/free convection
32	7.1, 7.2	Pool boiling
33	7.3, 7.4.3, 7.5	Correlations for flow boiling, flow condensation, and film condensation
34	8.1	Introduction to heat exchangers, compact heat exchanger correlations
35	8.2	The LMTD Method
36	8.3.1-8.3.3	The ε - NTU Method
37	8.3.4	Limiting behaviors of the ε - NTU Method
38	8.10.1, 8.10.3-4	Regenerators, solution for balanced & symmetric regenerator, packings
39	10.1, 10.2	Introduction to radiation, blackbody emission
40	10.3.1-10.3.2	View factors
41	10.3.3	Blackbody radiation exchange
42	10.4	Real surfaces and Kirchoff's law
43	10.5.1-10.5.3	Gray surface radiation exchange
44		Gray surface radiation exchange
45	10.6	Radiation with other heat transfer mechanisms

NOMENCLATURE

a_i	i^{th} coefficient of a series solution
A_c	cross-sectional area (m^2)
A_{min}	minimum flow area (m^2)
A_p	projected area (m^2)
A_s	surface area (m^2)
$A_{s,fin}$	surface area of a fin (m^2)
A_{tot}	prime (total) surface area of a finned surface (m^2)
AR	aspect ratio of a rectangular duct
AR_{tip}	area ratio of fin tip to fin surface area
Att	attenuation (-)
B	parameter in the blowing factor (-)
BF	blowing factor (-)
Bi	Biot number (-)
Bo	boiling number (-)
Br	Brinkman number
c	specific heat capacity (J/kg-K) concentration (-) speed of light (m/s)
c''_a	specific heat capacity of an air-water mixture on a unit mass of air basis ($\text{J/kg}_a\text{-K}$)
$c''_{a,sat}$	specific heat capacity of an air-water mixture along the saturation line on a unit mass of air basis ($\text{J/kg}_a\text{-K}$)
c_{eff}	effective specific heat capacity of a composite (J/kg-K)
c_{ms}	ratio of the energy carried by a micro-scale energy carrier to its temperature (J/K)
c_v	specific heat capacity at constant volume (J/kg-K)
C	total heat capacity (J/K)
\dot{C}	capacitance rate of a flow (W/K)
C_1, C_2, \dots	undetermined constants
C_{crit}	dimensionless coefficient for critical heat flux correlation (-)
C_D	drag coefficient (-)
C_f	friction coefficient (-)
$\overline{C_f}$	average friction coefficient (-)
C_{lam}	coefficient for laminar plate natural convection correlation (-)
C_{nb}	dimensionless coefficient for nucleate boiling correlation (-)
C_R	capacity ratio (-)
$C_{turb,U}$	coefficient for turbulent, horizontal upward plate natural conv. correlation (-)
$C_{turb,V}$	coefficient for turbulent, vertical plate natural convection correlation (-)

Co	convection number (-)
CTE	coefficient of thermal expansion (1/K)
D	diameter (m)
	diffusion coefficient (m^2/s)
D_h	hydraulic diameter (m)
dx	differential in the x -direction (m)
dy	differential in the y -direction (m)
e	size of surface roughness (m)
err	convergence or numerical error
\dot{E}	rate of thermal energy carried by a mass flow (W)
E	total emissive power (W/m^2)
E_b	total blackbody emissive power (W/m^2)
E_λ	spectral emissive power ($W/m^2\text{-}\mu m$)
$E_{b,\lambda}$	blackbody spectral emissive power ($W/m^2\text{-}\mu m$)
Ec	Eckert number (-)
f	frequency (Hz)
	dimensionless stream function, for Blasius solution (-)
	friction factor (-)
\bar{f}	average friction factor (-)
f_l	friction factor for liquid-only flow in flow boiling (-)
F	force (N)
	correction-factor for log-mean temperature difference (-)
$F_{0-\lambda_1}$	external fractional function (-)
$F_{i,j}$	view factor from surface i to surface j (-)
$\hat{F}_{i,j}$	the “F-hat” parameter characterizing radiation from surface i to surface j (-)
fd	fractional duty for a pinch-point analysis (-)
Fo	Fourier number (-)
Fr	Froude number (-)
Fr_{mod}	modified Froude number (-)
g	acceleration of gravity (m/s^2)
\dot{g}	rate of thermal energy generation (W)
\dot{g}'''	rate of thermal energy generation per unit volume (W/m^3)
\dot{g}_{eff}'''	effective rate of generation per unit volume of a composite (W/m^3)
\dot{g}_v'''	rate of thermal energy generation per unit volume due to viscous dissipation (W/m^3)
G	mass flux or mass velocity ($kg/m^2\text{-}s$)
	total irradiation (W/m^2)
G_λ	spectral irradiation ($W/m^2\text{-}\mu m$)
Ga	Galileo number (-)
Gr	Grashof number (-)
Gz	Graetz number (-)
h	local heat transfer coefficient ($W/m^2\text{-}K$)
\bar{h}	average heat transfer coefficient ($W/m^2\text{-}K$)
\tilde{h}	dimensionless heat transfer coefficient for flow boiling correlation (-)
h_D	mass transfer coefficient (m/s)
\bar{h}_D	average mass transfer coefficient (m/s)
h_l	superficial heat transfer coefficient for the liquid phase ($W/m^2\text{-}K$)

Nomenclature

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\bar{h}_{rad}	the equivalent heat transfer coefficient associated with radiation ($\text{W/m}^2\text{-K}$)
i	index of node (-) index of eigenvalue (-) index of term in a series solution (-) specific enthalpy (J/kg-K) square root of negative one, $\sqrt{-1}$
i''_a	specific enthalpy of an air-water mixture on a per unit mass of air basis (J/kg_a)
I	current (ampere)
I_e	intensity of emitted radiation ($\text{W/m}^2\text{-}\mu\text{m-steradian}$)
I_i	intensity of incident radiation ($\text{W/m}^2\text{-}\mu\text{m-steradian}$)
j	index of node (-) index of eigenvalue (-)
J	radiosity (W/m^2)
j_H	Colburn j_H factor (-)
k	thermal conductivity (W/m-K)
k_B	Boltzmann's constant (J/K)
k_c	contraction loss coefficient (-)
k_e	expansion loss coefficient (-)
k_{eff}	effective thermal conductivity of a composite (W/m-K)
Kn	Knudsen number (-)
l_1	Lennard-Jones 12-6 potential characteristic length for species 1 (m)
$l_{1,2}$	characteristic length of a mixture of species 1 and species 2 (m)
L	length (m)
L^+	dimensionless length for a hydrodynamically developing internal flow (-)
L^*	dimensionless length for a thermally developing internal flow (-)
L_{char}	characteristic length of the problem (m)
$L_{char,vs}$	the characteristic size of the viscous sublayer (m)
L_{cond}	length for conduction (m)
L_{flow}	length in the flow direction (m)
L_{ml}	mixing length (m)
L_{ms}	distance between interactions of micro-scale energy or momentum carriers (m)
Le	Lewis number (-)
M	number of nodes (-) mass (kg)
m	fin parameter ($1/\text{m}$)
\dot{m}	mass flow rate (kg/s)
\dot{m}''	mass flow rate per unit area ($\text{kg/m}^2\text{-s}$)
m_{ms}	mass of microscale momentum carrier (kg/carrier)
mf	mass fraction (-)
MW	molar mass (kg/kgmol)
n	number density ($\#/m^3$)
n_{ms}	number density of the micro-scale energy carriers ($\#/m^3$)
\dot{n}''	molar transfer rate per unit area ($\text{kgmol/m}^2\text{-s}$)
N	number of nodes (-) number of moles (kgmol)
N_s	number of species in a mixture (-)
Nu	Nusselt number (-)

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Nomenclature

\overline{Nu}	average Nusselt number (-)
NTU	number of transfer units (-)
p	pressure (Pa) pitch (m)
P	$LMTD$ effectiveness (-) probability distribution (-)
p_∞	free-stream pressure (Pa)
\tilde{p}	dimensionless pressure (-)
Pe	Peclet number (-)
per	perimeter (m)
Pr	Prandtl number (-)
Pr_{turb}	turbulent Prandtl number (-)
\dot{q}	rate of heat transfer (W)
$\dot{q}_{i \text{ to } j}$	rate of radiation heat transfer from surface i to surface j (W)
\dot{q}_{max}	maximum possible rate of heat transfer, for an effectiveness solution (W)
\dot{q}''	heat flux, rate of heat transfer per unit area (W/m^2)
\dot{q}_s''	surface heat flux (W/m^2)
$\dot{q}_{s,crit}''$	critical heat flux for boiling (W/m^2)
\underline{Q}	total energy transfer by heat (J)
\tilde{Q}	dimensionless total energy transfer by heat (-)
r	radial coordinate (m) radius (m)
\tilde{r}	dimensionless radial coordinate (-)
R	thermal resistance (K/W) ideal gas constant (J/kg-K) $LMTD$ capacitance ratio (-)
$R_{\bar{A}}$	thermal resistance approximation based on average area limit (K/W)
R_{ac}	thermal resistance to axial conduction in a heat exchanger (K/W)
R_{ad}	thermal resistance approximation based on adiabatic limit (K/W)
R_{bl}	thermal resistance of the boundary layer (K/W)
R_c	thermal resistance due to solid-to-solid contact (K/W)
R_{conv}	thermal resistance to convection from a surface (K/W)
R_{cyl}	thermal resistance to radial conduction through a cylindrical shell (K/W)
R_e	electrical resistance (ohm)
R_f	thermal resistance due to fouling (K/W)
R_{fin}	thermal resistance of a fin (K/W)
$R_{i,j}$	the radiation space resistance between surfaces i and j ($1/m^2$)
R_{iso}	thermal resistance approximation based on isothermal limit (K/W)
$R_{\bar{L}}$	thermal resistance approximation based on average length limit (K/W)
R_{pw}	thermal resistance to radial conduction through a plane wall (K/W)
R_{rad}	thermal resistance to radiation (K/W)
$R_{s,i}$	the radiation surface resistance for surface i ($1/m^2$)
$R_{semi-\infty}$	thermal resistance approximation for a semi-infinite body (K/W)
R_{sph}	thermal resistance to radial conduction through a spherical shell (K/W)
R_{tot}	thermal resistance of a finned surface (K/W)
R_{univ}	universal gas constant (8314 J/kgmol-K)
R_c''	area-specific contact resistance ($K\cdot m^2/W$)
R_f''	area-specific fouling resistance ($K\cdot m^2/W$)
Ra	Rayleigh number (-)

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Re	Reynolds number (-)
Re_{crit}	critical Reynold number for transition to turbulence (-)
RH	relative humidity (-)
RR	radius ratio of an annular duct (-)
s	Laplace transformation variable (1/s)
S	shape factor (m)
	channel spacing (m)
Sc	Schmidt number (-)
\overline{Sh}	Sherwood number (-)
\overline{Sh}	average Sherwood number (-)
St	Stanton number (-)
t	time (s)
t_{sim}	simulated time (s)
th	thickness (m)
tol	convergence tolerance
T	temperature (K)
T_b	base temperature of fin (K)
T_{film}	film temperature (K)
T_m	mean or bulk temperature (K)
T_s	surface temperature (K)
T_{sat}	saturation temperature (K)
T_∞	free-stream or fluid temperature (K)
T^*	eddy temperature fluctuation (K)
T'	fluctuating component of temperature (K)
\overline{T}	average temperature (K)
TR	temperature solution that is a function of r , for separation of variables
Tt	temperature solution that is a function of t , for separation of variables
TX	temperature solution that is a function of x , for separation of variables
TY	temperature solution that is a function of y , for separation of variables
th	thickness (m)
U	internal energy (J)
	utilization (-)
u	specific internal energy (J/kg)
	velocity in the x -direction (m/s)
u_{char}	characteristic velocity (m/s)
u_f	frontal or upstream velocity (m/s)
u_m	mean or bulk velocity (m/s)
u_∞	free-stream velocity (m/s)
u^*	eddy velocity (m/s)
u^+	inner velocity (-)
\tilde{u}	dimensionless x -velocity (-)
u'	fluctuating component of x -velocity (m/s)
\bar{u}	average x -velocity (m/s)
UA	conductance (W/K)
v	velocity in the y - or r -directions (m/s)
v_δ	y -velocity at the outer edge of the boundary layer, approximate scale of y -velocity in a boundary layer (m/s)
v_{ms}	mean velocity of micro-scale energy or momentum carriers (m/s)

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\tilde{v}	dimensionless y -velocity (-)
v'	fluctuating component of y -velocity (m/s)
\bar{v}	average y -velocity (m/s)
V	volume (m^3)
	voltage (V)
\dot{V}	volume flow rate (m^3/s)
v^f	void fraction (-)
w	velocity in the z -direction (m/s)
\dot{w}	rate of work transfer (W)
W	width (m)
	total amount of work transferred (J)
x	x -coordinate (m)
	quality (-)
\tilde{x}	dimensionless x -coordinate (-)
X	particular solution that is only a function of x
$x_{fd,h}$	hydrodynamic entry length (m)
$x_{fd,t}$	thermal entry length (m)
X_{tt}	Lockhart Martinelli parameter (-)
y	y -coordinate (m)
	mole fraction (-)
y^+	inner position (-)
\tilde{y}	dimensionless y -coordinate (-)
Y	particular solution that is only a function of y
z	z -coordinate (m)

Greek Symbols

α	thermal diffusivity (m^2/s)
	absorption coefficient (1/m)
	absorptivity or absorptance (-), total hemispherical absorptivity (-)
	surface area per unit volume (1/m)
α_{eff}	effective thermal diffusivity of a composite (m^2/s)
α_λ	hemispherical absorptivity (-)
$\alpha_{\lambda,\theta,\phi}$	spectral directional absorptivity (-)
β	volumetric thermal expansion coefficient (1/K)
δ	film thickness for condensation (m)
	boundary layer thickness (m)
δ_d	mass transfer diffusion penetration depth (m)
	concentration boundary layer thickness (m)
δ_m	momentum diffusion penetration depth (m)
	momentum boundary layer thickness (m)
δ_{vs}	viscous sublayer thickness (m)
δ_t	energy diffusion penetration depth (m)
	thermal boundary layer thickness (m)
Δi_{fus}	latent heat of fusion (J/kg)
Δi_{vap}	latent heat of vaporization (J/kg)
Δp	pressure drop (N/m^2)
Δr	distance in r -direction between adjacent nodes (m)

Nomenclature

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ΔT	temperature difference (K)
ΔT_e	excess temperature (K)
ΔT_{lm}	log-mean temperature difference (K)
Δt	time step (s) time period (s)
Δt_{crit}	critical time step (s)
Δx	distance in x -direction between adjacent nodes (m)
Δy	distance in y -direction between adjacent nodes (m)
ε	heat exchanger effectiveness (-) emissivity or emittance (-), total hemispherical emissivity (-)
ε_{fin}	fin effectiveness (-)
ε_H	eddy diffusivity for heat transfer (m^2/s)
ε_λ	hemispherical emissivity (-)
$\varepsilon_{\lambda,\theta,\phi}$	spectral, directional emissivity (-)
ε_M	eddy diffusivity of momentum (m^2/s)
ε_1	Lennard-Jones 12-6 potential characteristic energy for species 1 (J)
$\varepsilon_{1,2}$	characteristic energy parameter for a mixture of species 1 and species 2 (J)
ϕ	porosity (-) phase angle (rad) spherical coordinate (rad)
η	similarity parameter (-) efficiency (-)
η_{fin}	fin efficiency (-)
η_o	overall efficiency of a finned surface (-)
κ	von Kármán constant
λ	dimensionless axial conduction parameter (-) wavelength of radiation (μm)
λ_i	i^{th} eigenvalue of a solution (1/m)
μ	viscosity ($N\cdot s/m^2$)
ν	frequency of radiation (1/s)
θ	temperature difference (K) angle (rad) spherical coordinate (rad)
$\tilde{\theta}$	dimensionless temperature difference (-)
θ^+	inner temperature difference (-)
θ_R	temperature difference solution that is only a function of r , for separation of variables
θ_t	temperature difference solution that is only a function of t , for separation of variables
θ_X	temperature difference solution that is only a function of x , for separation of variables
θ_{Xt}	temperature difference solution that is only a function of x and t , for reduction of multi-dimensional transient problems
θ_Y	temperature difference solution that is only a function of y , for separation of variables
θ_{Yt}	temperature difference solution that is only a function of y and t , for reduction of multi-dimensional transient problems
θ_{Zt}	temperature difference solution that is only a function of z and t , for reduction of multi-dimensional transient problems

ρ	density (kg/m^3)
	reflectivity or reflectance (-), total hemispherical reflectivity (-)
ρ_e	electrical resistivity (ohm-m)
ρ_{eff}	effective density of a composite (kg/m^3)
ρ_λ	hemispherical reflectivity (-)
$\rho_{\lambda,\theta,\varphi}$	spectral, directional reflectivity (-)
σ	surface tension (N/m), molecular radius (m)
	ratio of free-flow to frontal area (-)
	Stefan-Boltzmann constant ($5.67 \times 10^{-8} \text{ W/m}^2\text{-K}^4$)
τ	time constant (s)
	shear stress (Pa)
	transmittivity or transmittance (-), total hemispherical transmittivity (-)
τ_{diff}	diffusive time constant (s)
τ_{lumped}	lumped capacitance time constant (s)
τ_λ	hemispherical transmittivity (-)
$\tau_{\lambda,\theta,\varphi}$	spectral, directional transmittivity (-)
τ_s	shear stress at surface (N/m^2)
ν	kinematic viscosity (m^2/s)
ω	angular velocity (rad/s)
	humidity ratio (kg_v/kg_a)
	solid angle (steradian)
Ω_D	dimensionless collision integral for diffusion (-)
Ψ	stream function (m^2/s)
ζ	tilt angle (rad)
	curvature parameter for vertical cylinder, natural convection correlation (-)
ζ_i	the i^{th} dimensionless eigenvalue (-)

Superscripts

o at infinite dilution

Subscripts

a air
 abs absorbed
 ac axial conduction (in heat exchangers)
 an analytical
 app apparent
 approximate
 b blackbody
 bl boundary layer
 $bottom$ bottom
 c condensate film
 corrected
 C cold
 cold-side of a heat exchanger
 cc complex conjugate, for complex combination problems
 $char$ characteristic
 cf counter-flow heat exchanger

Nomenclature

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<i>cond</i>	conduction, conductive
<i>conv</i>	convection, convective
<i>crit</i>	critical
<i>CTHB</i>	cold-to-hot blow process
<i>dc</i>	dry coil
<i>df</i>	downward facing
<i>diff</i>	diffusive transfer
<i>eff</i>	effective
<i>emit</i>	emitted
<i>evap</i>	evaporative
<i>ext</i>	external
<i>f</i>	fluid
<i>fc</i>	forced convection
<i>fd,h</i>	hydrodynamically fully developed
<i>fd,t</i>	thermally fully developed
<i>fin</i>	fin, finned
<i>h</i>	homogeneous solution
<i>H</i>	hot
	hot-side of a heat exchanger
	constant heat flux boundary condition
<i>hs</i>	on a hemisphere
<i>HTCB</i>	hot-to-cold blow process
<i>i</i>	node <i>i</i>
	surface <i>i</i>
	species <i>i</i>
<i>in</i>	inner
	inlet
<i>ini</i>	initial
<i>int</i>	internal
	interface
	integration period
<i>j</i>	node <i>j</i>
	surface <i>j</i>
<i>l</i>	liquid
<i>lam</i>	laminar
<i>LHS</i>	left-hand side
<i>lumped</i>	lumped-capacitance
<i>m</i>	mean or bulk
	melting
<i>max</i>	maximum or maximum possible
<i>min</i>	minimum or minimum possible
<i>mod</i>	modified
<i>ms</i>	micro-scale carrier
<i>n</i>	normal
<i>nac</i>	without axial conduction (in heat exchangers)
<i>nb</i>	nucleate boiling
<i>nc</i>	natural convection
<i>no-fin</i>	without a fin
<i>out</i>	outer
	outlet
<i>p</i>	particular (or non-homogeneous) solution

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Nomenclature

<i>pf</i>	parallel-flow heat exchanger
<i>pp</i>	pinch-point
<i>r</i>	regenerator matrix at position r
<i>rad</i>	radiation, radiative
<i>ref</i>	reference
<i>RHS</i>	right-hand side
<i>s</i>	at the surface
<i>sat</i>	saturated
	saturated section of a heat exchanger
<i>sat,l</i>	saturated liquid
<i>sat,v</i>	saturated vapor
<i>sc</i>	sub-cooled section of a heat exchanger
<i>semi-∞</i>	semi-infinite
<i>sh</i>	super-heated section of a heat exchanger
<i>sph</i>	sphere
<i>sur</i>	surroundings
<i>sus</i>	sustained solution
<i>T</i>	constant temperature boundary condition at temperature T
<i>top</i>	top
<i>tot</i>	total
<i>turb</i>	turbulent
<i>uf</i>	upward-facing
<i>unfin</i>	not finned
<i>v</i>	vapor vertical viscous dissipation
<i>w</i>	water
<i>wb</i>	wet-bulb
<i>wc</i>	wet coil
<i>x</i>	at position x in the x -direction
x^-	in the negative x -direction
x^+	in the positive x -direction
<i>y</i>	at position y in the y -direction
∞	free-stream, fluid
90°	solution that is 90° out of phase, for complex combination problems

Other notes

A	arbitrary variable
A'	fluctuating component of variable A
	value of variable A on a unit length basis
A''	value of variable A on a unit area basis
A'''	value of variable A on a unit volume basis
\tilde{A}	dimensionless form of variable A
\hat{A}	a guess value or approximate value for variable A
\bar{A}	Laplace transform of the function A

Nomenclature

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\bar{A}	average of variable A
\underline{A}	denotes that variable A is a vector
\overline{A}	denotes that variable A is a matrix
dA	differential change in the variable A
δA	uncertainty in the variable A
ΔA	finite change in the variable A
$O(A)$	order of magnitude of the variable A