HEAT TRANSFER

The single objective of this book is to provide engineers with the capability, tools, and confidence to solve real-world heat transfer problems. The textbook includes many advanced topics, such as Bessel functions, Laplace transforms, separation of variables, Duhamel's theorem, and complex combination, as well as high-order explicit and implicit numerical integration algorithms. These analytical and numerical solution methods are applied to topics not considered in most textbooks. Examples are heat exchangers involving fluids with varying specific heats or phase changes, regenerators, semi-gray surface radiation exchange, and numerical solutions to internal flow problems. To improve readability, derivations of important results are presented completely, without skipping steps, which reduces student frustration and improves retention. The examples in the book are ubiquitous, not trivial “textbook” exercises. They are rather complex and timely real-world problems that are inherently interesting. This textbook integrates the computational software packages Maple, MATLAB, FEHT, and Engineering Equation Solver (EES) directly with the heat transfer material.

Gregory Nellis is an Associate Professor of Mechanical Engineering at the University of Wisconsin–Madison. He received his M.S. and Ph.D. at the Massachusetts Institute of Technology and is a member of the American Society of Heating, Refrigeration, and Air-Conditioning Engineers (ASHRAE), the American Society of Mechanical Engineers (ASME), the International Institute of Refrigeration (IIR), and the Cryogenic Society of America (CSA). Professor Nellis carries out applied research that is related to energy systems with a focus on refrigeration technology and he has published more than 40 journal papers. Professor Nellis’s focus has been on graduate and undergraduate education, and he has received the Polygon, Pi Tau Sigma, and Woodburn awards for excellence in teaching as well as the Boom Award for excellence in cryogenic research.

Sanford Klein is the Bascom Ouweneel Professor of Mechanical Engineering at the University of Wisconsin–Madison. He has been on the faculty at Wisconsin since 1977. He is associated with the Solar Energy Laboratory and has been involved in many studies of solar and other types of energy systems. He is the author or co-author of more than 160 publications relating to the analysis of energy systems. Professor Klein’s current research interests are in solar energy systems and applied thermodynamics and heat transfer. In addition, he is also actively involved in the development of engineering computer tools for both instruction and research. He is the primary author of a modular simulation program (TRNSYS), a solar energy system design program (F-CHART), a finite element heat transfer program (FEHT), and a general engineering equation solving program (EES). Professor Klein is a Fellow of ASME, ASHRAE, and the American Solar Energy Society (ASES).
Heat Transfer

GREGORY NELLIS
University of Wisconsin–Madison

SANFORD KLEIN
University of Wisconsin–Madison
This book is dedicated to Stephen H. Nellis...thanks Dad.
CONTENTS

Preface xix
Acknowledgments xxiii
Study guide xxvii
Nomenclature xxix

1 ONE-DIMENSIONAL, STEADY-STATE CONDUCTION 1

1.1 Conduction Heat Transfer 1
   1.1.1 Introduction 1
   1.1.2 Thermal Conductivity 1
      *Thermal Conductivity of a Gas* (E1) 5

1.2 Steady-State 1-D Conduction without Generation 5
   1.2.1 Introduction 5
   1.2.2 The Plane Wall 5
   1.2.3 The Resistance Concept 9
   1.2.4 Resistance to Radial Conduction through a Cylinder 10
   1.2.5 Resistance to Radial Conduction through a Sphere 11
   1.2.6 Other Resistance Formulae 13
      *Convection Resistance* 14
      *Contact Resistance* 14
      *Radiation Resistance* 16

EXAMPLE 1.2-1: LIQUID OXYGEN DEWAR 17

1.3 Steady-State 1-D Conduction with Generation 24
   1.3.1 Introduction 24
   1.3.2 Uniform Thermal Energy Generation in a Plane Wall 24
   1.3.3 Uniform Thermal Energy Generation in Radial Geometries 29
      EXAMPLE 1.3-1: MAGNETIC ABLATION 31
   1.3.4 Spatially Non-Uniform Generation 37
      EXAMPLE 1.3-2: ABSORPTION IN A LENS 38

1.4 Numerical Solutions to Steady-State 1-D Conduction Problems (EES) 44
   1.4.1 Introduction 44
   1.4.2 Numerical Solutions in EES 45
   1.4.3 Temperature-Dependent Thermal Conductivity 55
   1.4.4 Alternative Rate Models 60
      EXAMPLE 1.4-1: FUEL ELEMENT 62

1.5 Numerical Solutions to Steady-State 1-D Conduction Problems using MATLAB 68
   1.5.1 Introduction 68
   1.5.2 Numerical Solutions in Matrix Format 69
   1.5.3 Implementing a Numerical Solution in MATLAB 71

* Section can be found on the website that accompanies this book (www.cambridge.org/nellisandklein)
1.5.4 Functions
1.5.5 Sparse Matrices
1.5.6 Temperature-Dependent Properties

EXAMPLE 1.5-1: THERMAL PROTECTION SYSTEM

1.6 Analytical Solutions for Constant Cross-Section Extended Surfaces
1.6.1 Introduction
1.6.2 The Extended Surface Approximation
1.6.3 Analytical Solution
1.6.4 Fin Behavior
1.6.5 Fin Efficiency and Resistance

EXAMPLE 1.6-1: SOLDERING TUBES
1.6.6 Finned Surfaces

EXAMPLE 1.6-2: THERMOELECTRIC HEAT SINK
1.6.7 Fin Optimization* (E2)

1.7 Analytical Solutions for Advanced Constant Cross-Section Extended Surfaces
1.7.1 Introduction
1.7.2 Additional Thermal Loads

EXAMPLE 1.7-1: BENT-BEAM ACTUATOR
1.7.3 Moving Extended Surfaces

EXAMPLE 1.7-2: DRAWING A WIRE

1.8 Analytical Solutions for Non-Constant Cross-Section Extended Surfaces
1.8.1 Introduction
1.8.2 Series Solutions
1.8.3 Bessel Functions
1.8.4 Rules for Using Bessel Functions

EXAMPLE 1.8-1: PIPE IN A ROOF
EXAMPLE 1.8-2: MAGNETIC ABLATION WITH BLOOD PERFUSION

1.9 Numerical Solution to Extended Surface Problems
1.9.1 Introduction

EXAMPLE 1.9-1: TEMPERATURE SENSOR ERROR DUE TO MOUNTING & SELF HEATING
EXAMPLE 1.9-2: CRYOGENIC CURRENT LEADS

Problems

References

2 TWO-DIMENSIONAL, STEADY-STATE CONDUCTION • 202

2.1 Shape Factors

EXAMPLE 2.1-1: MAGNETIC ABLATIVE POWER MEASUREMENT

2.2 Separation of Variables Solutions
2.2.1 Introduction
2.2.2 Separation of Variables

Requirements for using Separation of Variables

Separate the Variables

Solve the Eigenproblem

Solve the Non-homogeneous Problem for each Eigenvalue

Obtain Solution for each Eigenvalue

Create the Series Solution and Enforce the Remaining Boundary Conditions

Summary of Steps

* Section can be found on the website that accompanies this book (www.cambridge.org/nellisandklein)
Contents

2.2.3 Simple Boundary Condition Transformations 224
   EXAMPLE 2.2-1: TEMPERATURE DISTRIBUTION IN A 2-D FIN 225
   EXAMPLE 2.2-2: CONSTRICITION RESISTANCE 236

2.3 Advanced Separation of Variables Solutions* (E3) 242

2.4 Superposition 242
   2.4.1 Introduction 242
   2.4.2 Superposition for 2-D Problems 245

2.5 Numerical Solutions to Steady-State 2-D Problems with EES 250
   2.5.1 Introduction 250
   2.5.2 Numerical Solutions with EES 251

2.6 Numerical Solutions to Steady-State 2-D Problems with MATLAB 260
   2.6.1 Introduction 260
   2.6.2 Numerical Solutions with MATLAB 260
   2.6.3 Numerical Solution by Gauss-Seidel Iteration* (E4) 268

2.7 Finite Element Solutions 269
   2.7.1 Introduction to FEHT* (E5) 269
   2.7.2 The Galerkin Weighted Residual Method* (E6) 269

2.8 Resistance Approximations for Conduction Problems 269
   2.8.1 Introduction 269
   EXAMPLE 2.8-1: RESISTANCE OF A BRACKET 270
   2.8.2 Isothermal and Adiabatic Resistance Limits 272
   2.8.3 Average Area and Average Length Resistance Limits 275
   EXAMPLE 2.8-2: RESISTANCE OF A SQUARE CHANNEL 276

2.9 Conduction through Composite Materials 278
   2.9.1 Effective Thermal Conductivity 278
   EXAMPLE 2.9-1: FIBER OPTIC BUNDLE 282
   Problems 290
   References 301

3 TRANSIENT CONDUCTION • 302

3.1 Analytical Solutions to 0-D Transient Problems 302
   3.1.1 Introduction 302
   3.1.2 The Lumped Capacitance Assumption 302
   3.1.3 The Lumped Capacitance Problem 303
   3.1.4 The Lumped Capacitance Time Constant 304
   EXAMPLE 3.1-1: DESIGN OF A CONVEYOR BELT 307
   EXAMPLE 3.1-2: SENSOR IN AN OSCILLATING TEMPERATURE ENVIRONMENT 310

3.2 Numerical Solutions to 0-D Transient Problems 317
   3.2.1 Introduction 317
   3.2.2 Numerical Integration Techniques 317
      Euler's Method 318
      Heun's Method 322
      Runge-Kutta Fourth Order Method 326
      Fully Implicit Method 328
      Crank-Nicolson Method 330
      Adaptive Step-Size and EES' Integral Command 332
      MATLAB's Ordinary Differential Equation Solvers 335
   EXAMPLE 3.2-1(A): OVEN BRAZING (EES) 339
   EXAMPLE 3.2-1(B): OVEN BRAZING (MATLAB) 344

* Section can be found on the website that accompanies this book (www.cambridge.org/nellisandklein)
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.3 Semi-Infinite 1-D Transient Problems</td>
<td>348</td>
</tr>
<tr>
<td>3.3.1 Introduction</td>
<td>348</td>
</tr>
<tr>
<td>3.3.2 The Diffusive Time Constant</td>
<td>348</td>
</tr>
<tr>
<td><strong>EXAMPLE 3.3-1: TRANSIENT RESPONSE OF A TANK WALL</strong></td>
<td>351</td>
</tr>
<tr>
<td>3.3.3 The Self-Similar Solution</td>
<td>354</td>
</tr>
<tr>
<td>3.3.4 Solutions to other Semi-Infinite Problems</td>
<td>361</td>
</tr>
<tr>
<td><strong>EXAMPLE 3.3-2: QUENCHING A COMPOSITE STRUCTURE</strong></td>
<td>363</td>
</tr>
<tr>
<td>3.4 The Laplace Transform</td>
<td>369</td>
</tr>
<tr>
<td>3.4.1 Introduction</td>
<td>369</td>
</tr>
<tr>
<td>3.4.2 The Laplace Transformation</td>
<td>370</td>
</tr>
<tr>
<td>Laplace Transformations with Tables</td>
<td>371</td>
</tr>
<tr>
<td>Laplace Transformations with Maple</td>
<td>371</td>
</tr>
<tr>
<td>3.4.3 The Inverse Laplace Transform</td>
<td>372</td>
</tr>
<tr>
<td>Inverse Laplace Transform with Tables and the Method of Partial Fractions</td>
<td>373</td>
</tr>
<tr>
<td>Inverse Laplace Transformation with Maple</td>
<td>376</td>
</tr>
<tr>
<td>3.4.4 Properties of the Laplace Transformation</td>
<td>378</td>
</tr>
<tr>
<td>3.4.5 Solution to Lumped Capacitance Problems</td>
<td>380</td>
</tr>
<tr>
<td>3.4.6 Solution to Semi-Infinite Body Problems</td>
<td>386</td>
</tr>
<tr>
<td><strong>EXAMPLE 3.4-1: QUENCHING OF A SUPERCONDUCTOR</strong></td>
<td>391</td>
</tr>
<tr>
<td>3.5 Separation of Variables for Transient Problems</td>
<td>395</td>
</tr>
<tr>
<td>3.5.1 Introduction</td>
<td>395</td>
</tr>
<tr>
<td>3.5.2 Separation of Variables Solutions for Common Shapes</td>
<td>396</td>
</tr>
<tr>
<td>The Plane Wall</td>
<td>396</td>
</tr>
<tr>
<td>The Cylinder</td>
<td>401</td>
</tr>
<tr>
<td>The Sphere</td>
<td>403</td>
</tr>
<tr>
<td><strong>EXAMPLE 3.5-1: MATERIAL PROCESSING IN A RADIANT OVEN</strong></td>
<td>405</td>
</tr>
<tr>
<td>3.5.3 Separation of Variables Solutions in Cartesian Coordinates</td>
<td>408</td>
</tr>
<tr>
<td>Requirements for using Separation of Variables</td>
<td>409</td>
</tr>
<tr>
<td>Separate the Variables</td>
<td>410</td>
</tr>
<tr>
<td>Solve the Eigenproblem</td>
<td>411</td>
</tr>
<tr>
<td>Solve the Non-homogeneous Problem for each Eigenvector</td>
<td>413</td>
</tr>
<tr>
<td>Obtain a Solution for each Eigenvector</td>
<td>414</td>
</tr>
<tr>
<td>Create the Series Solution and Enforce the Initial Condition</td>
<td>414</td>
</tr>
<tr>
<td>Limits of the Separation of Variables Solution</td>
<td>417</td>
</tr>
<tr>
<td><strong>EXAMPLE 3.5-2: TRANSIENT RESPONSE OF A TANK WALL (REVISITED)</strong></td>
<td>420</td>
</tr>
<tr>
<td>3.5.4 Separation of Variables Solutions in Cylindrical Coordinates*</td>
<td>427</td>
</tr>
<tr>
<td>(E7)</td>
<td>427</td>
</tr>
<tr>
<td>3.5.5 Non-homogeneous Boundary Conditions*</td>
<td>428</td>
</tr>
<tr>
<td>(E8)</td>
<td>428</td>
</tr>
<tr>
<td>3.6 Duhamel's Theorem*</td>
<td>428</td>
</tr>
<tr>
<td>(E9)</td>
<td>428</td>
</tr>
<tr>
<td>3.7 Complex Combination*</td>
<td>428</td>
</tr>
<tr>
<td>(E10)</td>
<td>428</td>
</tr>
<tr>
<td>3.8 Numerical Solutions to 1-D Transient Problems</td>
<td>428</td>
</tr>
<tr>
<td>3.8.1 Introduction</td>
<td>428</td>
</tr>
<tr>
<td>3.8.2 Transient Conduction in a Plane Wall</td>
<td>429</td>
</tr>
<tr>
<td>Euler's Method</td>
<td>432</td>
</tr>
<tr>
<td>Fully Implicit Method</td>
<td>438</td>
</tr>
<tr>
<td>Heun's Method</td>
<td>442</td>
</tr>
<tr>
<td>Runge-Kutta 4th Order Method</td>
<td>445</td>
</tr>
<tr>
<td>Crank-Nicolson Method</td>
<td>449</td>
</tr>
<tr>
<td>EES' Integral Command</td>
<td>452</td>
</tr>
<tr>
<td>MATLAB's Ordinary Differential Equation Solvers</td>
<td>453</td>
</tr>
</tbody>
</table>

* Section can be found on the website that accompanies this book (www.cambridge.org/nellisandklein)
Contents

3.9 Reduction of Multi-Dimensional Transient Problems* (E11) 468

4 EXTERNAL FORCED CONVECTION  483

4.1 Introduction to Laminar Boundary Layers 483
  4.1.1 Introduction 483
  4.1.2 The Laminar Boundary Layer 484
    A Conceptual Model of the Laminar Boundary Layer 485
    A Conceptual Model of the Friction Coefficient and Heat Transfer Coefficient 488
    The Reynolds Analogy 492
  4.1.3 Local and Integrated Quantities 494

4.2 The Boundary Layer Equations 495
  4.2.1 Introduction 495
  4.2.2 The Governing Equations for Viscous Fluid Flow 495
    The Continuity Equation 495
    The Momentum Conservation Equations 496
    The Thermal Energy Conservation Equation 498
  4.2.3 The Boundary Layer Simplifications 500
    The Continuity Equation 500
    The x-Momentum Equation 501
    The y-Momentum Equation 502
    The Thermal Energy Equation 503

4.3 Dimensional Analysis in Convection 506
  4.3.1 Introduction 506
  4.3.2 The Dimensionless Boundary Layer Equations 508
    The Dimensionless Continuity Equation 508
    The Dimensionless Momentum Equation in the Boundary Layer 509
    The Dimensionless Thermal Energy Equation in the Boundary Layer 509
  4.3.3 Correlating the Solutions of the Dimensionless Equations 511
    The Friction and Drag Coefficients 511
    The Nusselt Number 513

EXAMPLE 4.3-1: SUB-SCALE TESTING OF A CUBE-SHAPED MODULE 515
  4.3.4 The Reynolds Analogy (revisited) 520

4.4 Self-Similar Solution for Laminar Flow over a Flat Plate 521
  4.4.1 Introduction 521
  4.4.2 The Blasius Solution 522
    The Problem Statement 522
    The Similarity Variables 522
    The Problem Transformation 526
    Numerical Solution 530
  4.4.3 The Temperature Solution 535
    The Problem Statement 535
    The Similarity Variables 536
    The Problem Transformation 536
    Numerical Solution 538
  4.4.4 The Falkner-Skan Transformation* (E12) 542

* Section can be found on the website that accompanies this book (www.cambridge.org/nellisandklein)
## Contents

### 4.5 Turbulent Boundary Layer Concepts 542

#### 4.5.1 Introduction 542

#### 4.5.2 A Conceptual Model of the Turbulent Boundary Layer 543

### 4.6 The Reynolds Averaged Equations 548

#### 4.6.1 Introduction 548

#### 4.6.2 The Averaging Process 549

- *The Reynolds Averaged Continuity Equation* 550

- *The Reynolds Averaged Momentum Equation* 551

- *The Reynolds Averaged Thermal Energy Equation* 554

### 4.7 The Laws of the Wall 556

#### 4.7.1 Introduction 556

#### 4.7.2 Inner Variables 557

#### 4.7.3 Eddy Diffusivity of Momentum 560

#### 4.7.4 The Mixing Length Model 561

#### 4.7.5 The Universal Velocity Profile 562

#### 4.7.6 Eddy Diffusivity of Momentum Models 565

#### 4.7.7 Wake Region 566

#### 4.7.8 Eddy Diffusivity of Heat Transfer 567

#### 4.7.9 The Thermal Law of the Wall 568

### 4.8 Integral Solutions 571

#### 4.8.1 Introduction 571

#### 4.8.2 The Integral Form of the Momentum Equation 571

- Derivation of the Integral Form of the Momentum Equation 571

- Application of the Integral Form of the Momentum Equation 575

- **EXAMPLE 4.8-1: PLATE WITH TRANSPIRATION** 580

#### 4.8.3 The Integral Form of the Energy Equation 584

- Derivation of the Integral Form of the Energy Equation 584

- Application of the Integral Form of the Energy Equation 587

#### 4.8.4 Integral Solutions for Turbulent Flows 591

### 4.9 External Flow Correlations 593

#### 4.9.1 Introduction 593

#### 4.9.2 Flow over a Flat Plate

- Friction Coefficient 593

- Nusselt Number 598

- **EXAMPLE 4.9-1: PARTIALLY SUBMERGED PLATE** 603

  - Unheated Starting Length 606

  - Constant Heat Flux 606

  - Flow over a Rough Plate 607

#### 4.9.3 Flow across a Cylinder

- Drag Coefficient 609

- Nusselt Number 611

- **EXAMPLE 4.9-2: HOT WIRE ANEMOMETER** 615

  - Flow across a Bank of Cylinders 617

  - Non-Circular Extrusions 617

#### 4.9.4 Flow past a Sphere 618

- **EXAMPLE 4.9-3: BULLET TEMPERATURE** 620

### Problems 624

### References 633

* Section can be found on the website that accompanies this book (www.cambridge.org/nellisandklein)
5 INTERNAL FORCED CONVECTION • 635

5.1 Internal Flow Concepts 635
   5.1.1 Introduction 635
   5.1.2 Momentum Considerations 635
      The Mean Velocity 637
      The Laminar Hydrodynamic Entry Length 638
      Turbulent Internal Flow 638
      The Turbulent Hydrodynamic Entry Length 640
      The Friction Factor 641
   5.1.3 Thermal Considerations 644
      The Mean Temperature 644
      The Heat Transfer Coefficient and Nusselt Number 645
      The Laminar Thermal Entry Length 646
      Turbulent Internal Flow 648

5.2 Internal Flow Correlations 649
   5.2.1 Introduction 649
   5.2.2 Flow Classification 650
   5.2.3 The Friction Factor 650
      Laminar Flow 651
      Turbulent Flow 654
      EES’ Internal Flow Convection Library 656
   EXAMPLE 5.2-1: FILLING A WATERING TANK 657
   5.2.4 The Nusselt Number 661
      Laminar Flow 662
      Turbulent Flow 667
   EXAMPLE 5.2-2: DESIGN OF AN AIR HEATER 668

5.3 The Energy Balance 671
   5.3.1 Introduction 671
   5.3.2 The Energy Balance 671
   5.3.3 Prescribed Heat Flux 673
      Constant Heat Flux 674
   5.3.4 Prescribed Wall Temperature 674
      Constant Wall Temperature 674
   5.3.5 Prescribed External Temperature 675
   EXAMPLE 5.3-1: ENERGY RECOVERY WITH AN ANNULAR JACKET 677

5.4 Analytical Solutions for Internal Flows 686
   5.4.1 Introduction 686
   5.4.2 The Momentum Equation 686
      Fully Developed Flow between Parallel Plates 687
      The Reynolds Equation* (E13) 689
      Fully Developed Flow in a Circular Tube* (E14) 689
   5.4.3 The Thermal Energy Equation 689
      Fully Developed Flow through a Round Tube with a Constant Heat Flux 691
      Fully Developed Flow through Parallel Plates with a Constant Heat Flux 695

5.5 Numerical Solutions to Internal Flow Problems 697
   5.5.1 Introduction 697
   5.5.2 Hydrodynamically Fully Developed Laminar Flow 698
      EES’ Integral Command 702

* Section can be found on the website that accompanies this book (www.cambridge.org/nellisandklein)
## Contents

The Euler Technique ........................... 704
The Crank-Nicolson Technique ................. 706
MATLAB’s Ordinary Differential Equation Solvers ........... 710

5.5.3 Hydrodynamically Fully Developed Turbulent Flow 712
Problems .................................. 723
References .................................. 734

### 6 NATURAL CONVECTION • 735

6.1 Natural Convection Concepts .......... 735
6.1.1 Introduction .......................... 735
6.1.2 Dimensionless Parameters for Natural Convection 
   Identification from Physical Reasoning ........... 736
   Identification from the Governing Equations ......... 739

6.2 Natural Convection Correlations .... 741
6.2.1 Introduction ......................... 741
6.2.2 Plate .................................. 741
   Heated or Cooled Vertical Plate ................. 742
   Horizontal Heated Upward Facing or Cooled Downward Facing Plate ........... 744
   Horizontal Heated Downward Facing or Cooled Upward Facing Plate .......... 745
   Plate at an Arbitrary Tilt Angle ................. 747

**EXAMPLE 6.2-1: AIRCRAFT FUEL ULLAGE HEATER** 748
6.2.3 Sphere .................................. 752
**EXAMPLE 6.2-2: FRUIT IN A WAREHOUSE** 753
6.2.4 Cylinder .................................. 757
   Horizontal Cylinder ......................... 757
   Vertical Cylinder .......................... 758
6.2.5 Open Cavity ........................... 760
   Vertical Parallel Plates ..................... 761

**EXAMPLE 6.2-3: HEAT SINK DESIGN** 763
6.2.6 Enclosures ........................... 766
6.2.7 Combined Free and Forced Convection ........... 768
**EXAMPLE 6.2-4: SOLAR FLUX METER** 769
6.3 Self-Similar Solution* (E15) ............. 772
6.4 Integral Solution* (E16) .................. 772
Problems .................................. 773
References .................................. 777

### 7 BOILING AND CONDENSATION • 778

7.1 Introduction ............................ 778
7.2 Pool Boiling ............................. 779
7.2.1 Introduction .......................... 779
7.2.2 The Boiling Curve ..................... 780
7.2.3 Pool Boiling Correlations .......... 784

**EXAMPLE 7.2-1: COOLING AN ELECTRONICS MODULE USING NUCLEATE BOILING** 786
7.3 Flow Boiling ............................ 790
7.3.1 Introduction .......................... 790
7.3.2 Flow Boiling Correlations .......... 791

**EXAMPLE 7.3-1: CARBON DIOXIDE EVAPORATING IN A TUBE** 794

* Section can be found on the website that accompanies this book (www.cambridge.org/nellisandklein)
Contents

7.4 Film Condensation 798
  7.4.1 Introduction 798
  7.4.2 Solution for Inertia-Free Film Condensation on a Vertical Wall 799
  7.4.3 Correlations for Film Condensation 805
    Vertical Wall 805
  EXAMPLE 7.4-1: WATER DISTILLATION DEVICE 807
    Horizontal, Downward Facing Plate 810
    Horizontal, Upward Facing Plate 811
    Single Horizontal Cylinder 811
    Bank of Horizontal Cylinders 811
    Single Horizontal Finned Tube 811

7.5 Flow Condensation 812
  7.5.1 Introduction 812
  7.5.2 Flow Condensation Correlations 813

8 HEAT EXCHANGERS • 823

8.1 Introduction to Heat Exchangers 823
  8.1.1 Introduction 823
  8.1.2 Applications of Heat Exchangers 823
  8.1.3 Heat Exchanger Classifications and Flow Paths 824
  8.1.4 Overall Energy Balances 828
  8.1.5 Heat Exchanger Conductance 831
    Fouling Resistance 831
  EXAMPLE 8.1-1: CONDUCTANCE OF A CROSS-FLOW HEAT EXCHANGER 832
  8.1.6 Compact Heat Exchanger Correlations 838
  EXAMPLE 8.1-2: CONDUCTANCE OF A CROSS-FLOW HEAT EXCHANGER (REVISED) 841

8.2 The Log-Mean Temperature Difference Method 841
  8.2.1 Introduction 841
  8.2.2 LMTD Method for Counter-Flow and Parallel-Flow Heat Exchangers 842
  8.2.3 LMTD Method for Shell-and-Tube and Cross-Flow Heat Exchangers 847
  EXAMPLE 8.2-1: PERFORMANCE OF A CROSS-FLOW HEAT EXCHANGER 848

8.3 The Effectiveness-NTU Method 851
  8.3.1 Introduction 851
  8.3.2 The Maximum Heat Transfer Rate 852
  8.3.3 Heat Exchanger Effectiveness 853
  EXAMPLE 8.3-1: PERFORMANCE OF A CROSS-FLOW HEAT EXCHANGER (REVISED) 858
  8.3.4 Further Discussion of Heat Exchanger Effectiveness 861
    Behavior as C<sub>r</sub> Approaches Zero 862
    Behavior as NTU Approaches Zero 863
    Behavior as NTU Becomes Infinite 864
    Heat Exchanger Design 865

8.4 Pinch Point Analysis 867
  8.4.1 Introduction 867
  8.4.2 Pinch Point Analysis for a Single Heat Exchanger 867
  8.4.3 Pinch Point Analysis for a Heat Exchanger Network 872

8.5 Heat Exchangers with Phase Change 876

* Section can be found on the website that accompanies this book (www.cambridge.org/nellisandklein)
## Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.5.1</td>
<td>Introduction</td>
<td>876</td>
</tr>
<tr>
<td>8.5.2</td>
<td>Sub-Heat Exchanger Model for Phase-Change</td>
<td>876</td>
</tr>
<tr>
<td>8.6</td>
<td>Numerical Model of Parallel- and Counter-Flow Heat Exchangers</td>
<td>888</td>
</tr>
<tr>
<td>8.6.1</td>
<td>Introduction</td>
<td>888</td>
</tr>
<tr>
<td>8.6.2</td>
<td>Numerical Integration of Governing Equations</td>
<td>888</td>
</tr>
<tr>
<td></td>
<td>Parallel-Flow Configuration</td>
<td>889</td>
</tr>
<tr>
<td></td>
<td>Counter-Flow Configuration* (E17)</td>
<td>896</td>
</tr>
<tr>
<td>8.6.3</td>
<td>Discretization into Sub-Heat Exchangers</td>
<td>897</td>
</tr>
<tr>
<td></td>
<td>Parallel-Flow Configuration</td>
<td>897</td>
</tr>
<tr>
<td></td>
<td>Counter-Flow Configuration* (E18)</td>
<td>902</td>
</tr>
<tr>
<td>8.6.4</td>
<td>Solution with Axial Conduction* (E19)</td>
<td>902</td>
</tr>
<tr>
<td>8.7</td>
<td>Axial Conduction in Heat Exchangers</td>
<td>903</td>
</tr>
<tr>
<td>8.7.1</td>
<td>Introduction</td>
<td>903</td>
</tr>
<tr>
<td>8.7.2</td>
<td>Approximate Models for Axial Conduction</td>
<td>905</td>
</tr>
<tr>
<td></td>
<td>Approximate Model at Low $\lambda$</td>
<td>907</td>
</tr>
<tr>
<td></td>
<td>Approximate Model at High $\lambda$</td>
<td>907</td>
</tr>
<tr>
<td></td>
<td>Temperature Jump Model</td>
<td>909</td>
</tr>
<tr>
<td>8.8</td>
<td>Perforated Plate Heat Exchangers</td>
<td>911</td>
</tr>
<tr>
<td>8.8.1</td>
<td>Introduction</td>
<td>911</td>
</tr>
<tr>
<td>8.8.2</td>
<td>Modeling Perforated Plate Heat Exchangers</td>
<td>913</td>
</tr>
<tr>
<td>8.9</td>
<td>Numerical Modeling of Cross-Flow Heat Exchangers</td>
<td>919</td>
</tr>
<tr>
<td>8.9.1</td>
<td>Introduction</td>
<td>919</td>
</tr>
<tr>
<td>8.9.2</td>
<td>Finite Difference Solution</td>
<td>920</td>
</tr>
<tr>
<td></td>
<td>Both Fluids Unmixed with Uniform Properties</td>
<td>920</td>
</tr>
<tr>
<td></td>
<td>Both Fluids Unmixed with Temperature-Dependent Properties</td>
<td>927</td>
</tr>
<tr>
<td></td>
<td>One Fluid Mixed, One Fluid Unmixed* (E20)</td>
<td>936</td>
</tr>
<tr>
<td></td>
<td>Both Fluids Mixed* (E21)</td>
<td>936</td>
</tr>
<tr>
<td>8.10</td>
<td>Regenerators</td>
<td>937</td>
</tr>
<tr>
<td>8.10.1</td>
<td>Introduction</td>
<td>937</td>
</tr>
<tr>
<td>8.10.2</td>
<td>Governing Equations</td>
<td>939</td>
</tr>
<tr>
<td>8.10.3</td>
<td>Balanced, Symmetric Flow with No Entrained Fluid Heat Capacity</td>
<td>942</td>
</tr>
<tr>
<td></td>
<td>Utilization and Number of Transfer Units</td>
<td>942</td>
</tr>
<tr>
<td></td>
<td>Regenerator Effectiveness</td>
<td>944</td>
</tr>
<tr>
<td>8.10.4</td>
<td>Correlations for Regenerator Matrices</td>
<td>948</td>
</tr>
<tr>
<td></td>
<td>Packed Bed of Spheres</td>
<td>950</td>
</tr>
<tr>
<td></td>
<td>Screens</td>
<td>951</td>
</tr>
<tr>
<td></td>
<td>Triangular Passages</td>
<td>952</td>
</tr>
<tr>
<td>EXAMPLE 8.10-1: AN ENERGY RECOVERY WHEEL</td>
<td>953</td>
<td></td>
</tr>
<tr>
<td>8.10.5</td>
<td>Numerical Model of a Regenerator with No Entrained Heat Capacity* (E22)</td>
<td>962</td>
</tr>
<tr>
<td>Problems</td>
<td>962</td>
<td></td>
</tr>
<tr>
<td>References</td>
<td>973</td>
<td></td>
</tr>
</tbody>
</table>

9 MASS TRANSFER* (E23) • 974

Problems | 974

10 RADIATION • 979

10.1 Introduction to Radiation | 979
10.1.1 Radiation | 979

* Section can be found on the website that accompanies this book (www.cambridge.org/nellisandklein)
Contents

10.1.2 The Electromagnetic Spectrum 980
10.2 Emission of Radiation by a Blackbody 981
  10.2.1 Introduction 981
  10.2.2 Blackbody Emission 982
    Planck’s Law 982
    Blackbody Emission in Specified Wavelength Bands 985
    EXAMPLE 10.2-1: UV RADIATION FROM THE SUN 987
10.3 Radiation Exchange between Black Surfaces 989
  10.3.1 Introduction 989
  10.3.2 View Factors 989
    The Enclosure Rule 990
    Reciprocity 991
    Other View Factor Relationships 992
    The Crossed and Uncrossed String Method 992
    EXAMPLE 10.3-1: CROSSED AND UNCROSSED STRING METHOD 993
    View Factor Library 996
    EXAMPLE 10.3-2: THE VIEW FACTOR LIBRARY 998
  10.3.3 Blackbody Radiation Calculations 1001
    The Space Resistance 1001
    EXAMPLE 10.3-3: APPROXIMATE TEMPERATURE OF THE EARTH 1002
    N-Surface Solutions 1006
    EXAMPLE 10.3-4: HEAT TRANSFER IN A RECTANGULAR ENCLOSURE 1007
    EXAMPLE 10.3-5: DIFFERENTIAL VIEW FACTORS: RADIATION EXCHANGE BETWEEN PARALLEL PLATES 1009
10.4 Radiation Characteristics of Real Surfaces 1012
  10.4.1 Introduction 1012
  10.4.2 Emission of Real Materials 1012
    Intensity 1012
    Spectral, Directional Emissivity 1014
    Hemispherical Emissivity 1014
    Total Hemispherical Emissivity 1015
    The Diffuse Surface Approximation 1016
    The Diffuse Gray Surface Approximation 1016
    The Semi-Gray Surface 1016
  10.4.3 Reflectivity, Absorptivity, and Transmittivity 1018
    Diffuse and Specular Surfaces 1019
    Hemispherical Reflectivity, Absorptivity, and Transmittivity 1020
    Kirchoff’s Law 1020
    Total Hemispherical Values 1022
    The Diffuse Surface Approximation 1023
    The Diffuse Gray Surface Approximation 1023
    The Semi-Gray Surface 1023
    EXAMPLE 10.4-1: ABSORPTIVITY AND EMISSIVITY OF A SOLAR SELECTIVE SURFACE 1024
10.5 Diffuse Gray Surface Radiation Exchange 1027
  10.5.1 Introduction 1027
  10.5.2 Radiosity 1028
  10.5.3 Gray Surface Radiation Calculations 1029
    EXAMPLE 10.5-1: RADIATION SHIELD 1032
    EXAMPLE 10.5-2: EFFECT OF OVEN SURFACE PROPERTIES 1037

* Section can be found on the website that accompanies this book (www.cambridge.org/nellisandklein)
## Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.5.4 The $\hat{F}$ Parameter</td>
<td>1043</td>
</tr>
<tr>
<td><strong>EXAMPLE 10.5-3: RADIATION HEAT TRANSFER BETWEEN PARALLEL PLATES</strong></td>
<td>1046</td>
</tr>
<tr>
<td>10.5.5 Radiation Exchange for Semi-Gray Surfaces</td>
<td>1050</td>
</tr>
<tr>
<td><strong>EXAMPLE 10.5-4: RADIATION EXCHANGE IN A DUCT WITH SEMI-GRAY SURFACES</strong></td>
<td>1051</td>
</tr>
<tr>
<td>10.6 Radiation with other Heat Transfer Mechanisms</td>
<td>1055</td>
</tr>
<tr>
<td>10.6.1 Introduction</td>
<td>1055</td>
</tr>
<tr>
<td>10.6.2 When Is Radiation Important?</td>
<td>1055</td>
</tr>
<tr>
<td>10.6.3 Multi-Mode Problems</td>
<td>1057</td>
</tr>
<tr>
<td>10.7 The Monte Carlo Method</td>
<td>1058</td>
</tr>
<tr>
<td>10.7.1 Introduction</td>
<td>1058</td>
</tr>
<tr>
<td>10.7.2 Determination of View Factors with the Monte Carlo Method</td>
<td>1058</td>
</tr>
<tr>
<td>* Select a Location on Surface 1</td>
<td>1060</td>
</tr>
<tr>
<td>* Select the Direction of the Ray</td>
<td>1060</td>
</tr>
<tr>
<td>* Determine whether the Ray from Surface 1 Strikes Surface 2</td>
<td>1061</td>
</tr>
<tr>
<td>10.7.3 Radiation Heat Transfer Determined by the Monte Carlo Method</td>
<td>1068</td>
</tr>
<tr>
<td>Problems</td>
<td>1077</td>
</tr>
<tr>
<td>References</td>
<td>1088</td>
</tr>
</tbody>
</table>

### Appendices

- A.1: Introduction to EES<sup>*</sup> (E24)
- A.2: Introduction to Maple<sup>*</sup> (E25)
- A.3: Introduction to MATLAB<sup>*</sup> (E26)
- A.4: Introduction to FEHT<sup>*</sup> (E27)
- A.5: Introduction to Economics<sup>*</sup> (E28)

### Index

1091

* Section can be found on the website that accompanies this book (www.cambridge.org/nellisandklein)
PREFACE

The single objective of this book is to provide engineers with the capability, tools, and confidence to solve real-world heat transfer problems. This objective has resulted in a textbook that differs from existing heat transfer textbooks in several ways. First, this textbook includes many topics that are typically not covered in undergraduate heat transfer textbooks. Examples are the detailed presentations of mathematical solution methods such as Bessel functions, Laplace transforms, separation of variables, Duhamel’s theorem, and Monte Carlo methods as well as high order explicit and implicit numerical integration algorithms. These analytical and numerical solution methods are applied to advanced topics that are ordinarily not considered in a heat transfer textbook.

Judged by its content, this textbook should be considered as a graduate text. There is sufficient material for two-semester courses in heat transfer. However, the presentation does not presume previous knowledge or expertise. This book can be (and has been) successfully used in a single-semester undergraduate heat transfer course by appropriately selecting from the available topics. Our recommendations on what topics can be included in a first heat transfer course are provided in the suggested syllabus. The reason that this book can be used for a first course (despite its expanded content) and the reason it is also an effective graduate-level textbook is that all concepts and methods are presented in detail, starting at the beginning. The derivation of important results is presented completely, without skipping steps, in order to improve readability, reduce student frustration, and improve retention. You will not find many places in this textbook where it states that “it can be shown that...” The use of examples, solved and explained in detail, is ubiquitous in this textbook. The examples are not trivial, “textbook” exercises, but rather complex and timely real-world problems that are of interest by themselves. As with the presentation, the solutions to these examples are complete and do not skip steps.

Another significant difference between this textbook and most existing heat transfer textbooks is its integration of modern computational tools. The engineering student and practicing engineer of today is expected to be proficient with engineering computer tools. Engineering education must evolve accordingly. Most real engineering problems cannot be solved using a sequential set of calculations that can be accomplished with a pencil or hand calculator. Engineers must have the ability to quickly solve problems using the powerful computational tools that are available and essential for design, parametric study, and optimization of real-world systems. This book integrates the computational software packages Maple, MATLAB, FEHT, and Engineering Equation Solver (EES) directly with the heat transfer material. The specific commands and output associated with these software packages are presented as the theory is developed so that the integration is seamless rather than separated.

The computational software tools used in this book share some important characteristics. They are used in industry and have existed for more than a decade; therefore, while this software will certainly continue to evolve, it is not likely to disappear. Educational versions of these software packages are available, and therefore the use of these
tools should not represent an economic hardship to any academic institution or student. Useful versions of EES and FEHT are provided on the website that accompanies this textbook (www.cambridge.org/nellisandklein). With the help provided in the book, these tools are easy to learn and use. Students can become proficient with all of them in a reasonable amount of time. Learning the computer tools will not detract significantly from material coverage. To facilitate this learning process, tutorials for each of the software packages are provided on the companion website. The book itself is structured so that more advanced features of the software are introduced progressively, allowing students to become increasingly proficient using these tools as they progress through the text.

Most (if not all) of the tables and charts that have traditionally been required to solve heat transfer problems (for example, to determine properties, view factors, shape factors, convection relations, etc.) have been made available as functions and procedures in the EES software so that they can be easily accessed and used to solve problems. Indeed, the library of heat transfer functions that has been developed and integrated with EES as part of the preparation of this textbook enables a profound shift in the focus of the educational process. It is trivial to obtain, for example, a shape factor, a view factor, or a convection heat transfer coefficient using the heat transfer library. Therefore, it is possible to assign problems involving design and optimization studies that would be computationally impossible without the computer tools.

Integrating the study of heat transfer with computer tools does not diminish the depth of understanding of the underlying physics. Conversely, our experience indicates that the innate understanding of the subject matter is enhanced by appropriate use of these tools for several reasons. First, the software allows the student to tackle practical and relevant problems as opposed to the comparatively simple problems that must otherwise be assigned. Real-world engineering problems are more satisfying to the student. Therefore, the marriage of computer tools with theory motivates students to understand the governing physics as well as learn how to apply the computer tools. The use of these tools allows for coverage of more advanced material and more interesting and relevant problems. When a solution is obtained, students can carry out a more extensive investigation of its behavior and therefore obtain a more intuitive and complete understanding of the subject of heat transfer.

This book is unusual in its linking of classical theory and modern computing tools. It fills an obvious void that we have encountered in teaching both undergraduate and graduate heat transfer courses. The text was developed over many years from our experiences teaching Introduction to Heat Transfer (an undergraduate course) and Heat Transfer (a first-year graduate course) at the University of Wisconsin. It is our hope that this text will not only be useful during the heat transfer course, but also provide a life-long resource for practicing engineers.

G. F. Nellis
S. A. Klein
May, 2008
Acknowledgments

The development of this book has taken several years and a substantial effort. This has only been possible due to the collegial and supportive atmosphere that makes the Mechanical Engineering Department at the University of Wisconsin such a unique and impressive place. In particular, we would like to acknowledge Tim Shedd, Bill Beckman, Doug Reindl, John Pfotenhauer, Roxann Engelstad, and Glen Myers for their encouragement throughout the process.

Several years of undergraduate and graduate students have used our initial drafts of this manuscript. They have had to endure carrying two heavy volumes of poorly bound paper with no index and many typographical errors. Their feedback has been invaluable to the development of this book.

We have had the extreme good fortune to have had dedicated and insightful teachers. These include Glen Myers, John Mitchell, Bill Beckman, Joseph Smith Jr., John Brisson, Borivoje Mikic, and John Lienhard V. These individuals, among others, have provided us with an indication of the importance of teaching and provided an inspiration to us for writing this book.

Preparing this book has necessarily reduced the “quality time” available to spend with our families. We are most grateful to them for this indulgence. In particular, we wish to thank Jill, Jacob, and Spencer and Sharon Nellis and Jan Klein. We could have not completed this book without their continuous support.

Finally, we are indebted to Cambridge University Press and in particular Peter Gordon for giving us this opportunity and for helping us with the endless details needed to bring our original idea to this final state.
## STUDY GUIDE

This book has been developed for use in either a graduate or undergraduate level course in heat transfer. A sample program of study is laid out below for a one-semester graduate course (consisting of 45 class sessions).

### Graduate heat transfer class

<table>
<thead>
<tr>
<th>Day</th>
<th>Sections in Book</th>
<th>Topic</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.1</td>
<td>Conduction heat transfer</td>
</tr>
<tr>
<td>2</td>
<td>1.2</td>
<td>1-D steady conduction and resistance concepts</td>
</tr>
<tr>
<td>3</td>
<td>2.8</td>
<td>Resistance approximations</td>
</tr>
<tr>
<td>4</td>
<td>1.3</td>
<td>1-D steady conduction with generation</td>
</tr>
<tr>
<td>5</td>
<td>1.4, 1.5</td>
<td>Numerical solutions with EES and MATLAB</td>
</tr>
<tr>
<td>6</td>
<td>1.6</td>
<td>Fin solution, fin efficiency, and finned surfaces</td>
</tr>
<tr>
<td>7</td>
<td>1.7</td>
<td>Other constant cross-section extended surface problems</td>
</tr>
<tr>
<td>8</td>
<td>1.8</td>
<td>Bessel function solutions</td>
</tr>
<tr>
<td>9</td>
<td>2.2</td>
<td>2-D conduction, separation of variables</td>
</tr>
<tr>
<td>10</td>
<td>2.2</td>
<td>2-D conduction, separation of variables</td>
</tr>
<tr>
<td>11</td>
<td>2.4</td>
<td>Superposition</td>
</tr>
<tr>
<td>12</td>
<td>3.1</td>
<td>Transient, lumped capacitance problems – analytical solutions</td>
</tr>
<tr>
<td>13</td>
<td>3.2</td>
<td>Transient, lumped capacitance problems – numerical solutions</td>
</tr>
<tr>
<td>14</td>
<td>3.3</td>
<td>Semi-infinite bodies, diffusive time constant</td>
</tr>
<tr>
<td>15</td>
<td>3.3</td>
<td>Semi-infinite bodies, self-similar solution</td>
</tr>
<tr>
<td>16</td>
<td>3.4</td>
<td>Laplace transform solutions to lumped capacitance problems</td>
</tr>
<tr>
<td>17</td>
<td>3.4</td>
<td>Laplace transform solutions to 1-D transient problems</td>
</tr>
<tr>
<td>18</td>
<td>3.5</td>
<td>Separation of variables for 1-D transient problems</td>
</tr>
<tr>
<td>19</td>
<td>3.8</td>
<td>Numerical solutions to 1-D transient problems</td>
</tr>
<tr>
<td>20</td>
<td>4.1</td>
<td>Laminar boundary layer concepts</td>
</tr>
<tr>
<td>21</td>
<td>4.2, 4.3</td>
<td>The boundary layer equations &amp; dimensionless parameters</td>
</tr>
<tr>
<td>22</td>
<td>4.4</td>
<td>Blasius solution for flow over a flat plate</td>
</tr>
<tr>
<td>23</td>
<td>4.5, 4.6</td>
<td>Turbulent boundary layer concepts, Reynolds averaged equations</td>
</tr>
<tr>
<td>24</td>
<td>4.7</td>
<td>Mixing length models and the laws of the wall</td>
</tr>
<tr>
<td>25</td>
<td>4.8</td>
<td>Integral solutions</td>
</tr>
<tr>
<td>26</td>
<td>4.8, 4.9</td>
<td>Integral solutions, external flow correlations</td>
</tr>
<tr>
<td>27</td>
<td>5.1, 5.2</td>
<td>Internal flow concepts and correlations</td>
</tr>
<tr>
<td>28</td>
<td>5.3</td>
<td>The energy balance</td>
</tr>
<tr>
<td>29</td>
<td>5.4</td>
<td>Analytical solutions to internal flow problems</td>
</tr>
<tr>
<td>30</td>
<td>5.5</td>
<td>Numerical solutions to internal flow problems</td>
</tr>
<tr>
<td>31</td>
<td>6.1, 6.2</td>
<td>Natural convection concepts and correlations</td>
</tr>
</tbody>
</table>
### Study Guide

<table>
<thead>
<tr>
<th>Section</th>
<th>Topic</th>
</tr>
</thead>
<tbody>
<tr>
<td>32</td>
<td>8.1 Introduction to heat exchangers</td>
</tr>
<tr>
<td>33</td>
<td>8.2, 8.3 The LMTD and $\varepsilon$-NTU forms of the solutions</td>
</tr>
<tr>
<td>34</td>
<td>8.5 Heat exchangers with phase change</td>
</tr>
<tr>
<td>35</td>
<td>8.7 Axial conduction in heat exchangers</td>
</tr>
<tr>
<td>36</td>
<td>8.8, 8.10 Perforated plate heat exchangers and regenerators</td>
</tr>
<tr>
<td>37</td>
<td>10.1, 10.2 Introduction to radiation, Blackbody emissive power</td>
</tr>
<tr>
<td>38</td>
<td>10.3 View factors and the space resistance</td>
</tr>
<tr>
<td>39</td>
<td>10.3 Blackbody radiation exchange</td>
</tr>
<tr>
<td>40</td>
<td>10.4 Real surfaces, Kirchhoff’s law</td>
</tr>
<tr>
<td>41</td>
<td>10.5 Gray surface radiation exchange</td>
</tr>
<tr>
<td>42</td>
<td>10.5 Gray surface radiation exchange</td>
</tr>
<tr>
<td>43</td>
<td>10.5 Semi-gray surface radiation exchange</td>
</tr>
<tr>
<td>44</td>
<td>10.7 Introduction to Monte Carlo techniques</td>
</tr>
<tr>
<td>45</td>
<td>10.7 Introduction to Monte Carlo techniques</td>
</tr>
</tbody>
</table>

A sample program of study is laid out below for a one-semester undergraduate course (consisting of 45 class sessions).

#### Undergraduate heat transfer class

<table>
<thead>
<tr>
<th>Day</th>
<th>Sections in Book</th>
<th>Topic</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A.1</td>
<td>Review of thermodynamics, Using EES</td>
</tr>
<tr>
<td>2</td>
<td>1.2.2-1.2.3</td>
<td>1-D steady conduction, resistance concepts and circuits</td>
</tr>
<tr>
<td>3</td>
<td>1.2.4-1.2.6</td>
<td>1-D steady conduction in radial systems, other thermal resistance</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>More thermal resistance problems</td>
</tr>
<tr>
<td>5</td>
<td>1.3.1-1.3.3</td>
<td>1-D steady conduction with generation</td>
</tr>
<tr>
<td>6</td>
<td>1.4</td>
<td>Numerical solutions with EES</td>
</tr>
<tr>
<td>7</td>
<td>1.6.1-1.6.3</td>
<td>The extended surface approximation and the fin solution</td>
</tr>
<tr>
<td>8</td>
<td>1.6.4-1.6.6</td>
<td>Fin behavior, fin efficiency, and finned surfaces</td>
</tr>
<tr>
<td>9</td>
<td>1.9.1</td>
<td>Numerical solutions to extended surface problems</td>
</tr>
<tr>
<td>10</td>
<td>2.1</td>
<td>2-D steady-state conduction, shape factors</td>
</tr>
<tr>
<td>11</td>
<td>2.8.1-2.8.2</td>
<td>Resistance approximations</td>
</tr>
<tr>
<td>12</td>
<td>2.9</td>
<td>Conduction through composite materials</td>
</tr>
<tr>
<td>13</td>
<td>2.5</td>
<td>Numerical solution to 2-D steady-state problems with EES</td>
</tr>
<tr>
<td>14</td>
<td>3.1</td>
<td>Lumped capacitance assumption, the lumped time constant</td>
</tr>
<tr>
<td>15</td>
<td>3.2.1, 3.2.2</td>
<td>Numerical solution to lumped problems (Euler's, Heun's, Crank-Nicolson)</td>
</tr>
<tr>
<td>16</td>
<td>3.3.1-3.3.2</td>
<td>Semi-infinite body, the diffusive time constant</td>
</tr>
<tr>
<td>17</td>
<td>3.3.2, 3.3.4</td>
<td>Approximate models of diffusion, other semi-infinite solutions</td>
</tr>
<tr>
<td>18</td>
<td>3.5.1-3.5.2</td>
<td>Solutions to 1-D transient conduction in a bounded geometry</td>
</tr>
<tr>
<td>19</td>
<td>3.8.1-3.8.2</td>
<td>Numerical solution to 1-D transient conduction using EES</td>
</tr>
<tr>
<td>20</td>
<td>4.1</td>
<td>Introduction to laminar boundary layer concepts</td>
</tr>
<tr>
<td>21</td>
<td>4.2, 4.3</td>
<td>Dimensionless numbers</td>
</tr>
<tr>
<td>22</td>
<td>4.5</td>
<td>Introduction to turbulent boundary layer concepts</td>
</tr>
<tr>
<td>23</td>
<td>4.9.1-4.9.2</td>
<td>Correlations for external flow over a plate</td>
</tr>
<tr>
<td>24</td>
<td>4.9.3-4.9.4</td>
<td>Correlations for external flow over spheres and cylinders</td>
</tr>
<tr>
<td>25</td>
<td>5.1</td>
<td>Internal flow concepts</td>
</tr>
</tbody>
</table>
Study Guide

26  5.2  Internal flow correlations
27  5.3  Energy balance for internal flows
28  Internal flow problems
29  6.1  Introduction to natural convection
30  6.2.1-6.2.3  Natural convection correlations
31  6.2.4-6.2.7  Natural convection correlations and combined forced/free convection
32  7.1, 7.2  Pool boiling
33  7.3, 7.4.3, 7.5  Correlations for flow boiling, flow condensation, and film condensation
34  8.1  Introduction to heat exchangers, compact heat exchanger correlations
35  8.2  The LMTD Method
36  8.3.1-8.3.3  The $\varepsilon$-NTU Method
37  8.3.4  Limiting behaviors of the $\varepsilon$-NTU Method
38  8.10.1, 8.10.3-4  Regenerators, solution for balanced & symmetric regenerator, packings
39  10.1, 10.2  Introduction to radiation, blackbody emission
40  10.3.1-10.3.2  View factors
41  10.3.3  Blackbody radiation exchange
42  10.4  Real surfaces and Kirchoff's law
43  10.5.1-10.5.3  Gray surface radiation exchange
44  Gray surface radiation exchange
45  10.6  Radiation with other heat transfer mechanisms
NOMENCLATURE

\( a_i \) \( \text{\textsuperscript{th} coefficient of a series solution} \)
\( A_c \) cross-sectional area (m\(^2\))
\( A_{\text{min}} \) minimum flow area (m\(^2\))
\( A_p \) projected area (m\(^2\))
\( A_s \) surface area (m\(^2\))
\( A_{\text{fin}} \) surface area of a fin (m\(^2\))
\( A_{\text{sat}} \) prime (total) surface area of a finned surface (m\(^2\))
\( AR \) aspect ratio of a rectangular duct
\( AR_{\text{tip}} \) area ratio of fin tip to fin surface area
\( Att \) attenuation (-)
\( B \) parameter in the blowing factor (-)
\( BF \) blowing factor (-)
\( Bi \) Biot number (-)
\( Bo \) boiling number (-)
\( Br \) Brinkman number
\( c \) specific heat capacity (J/kg-K)
\( c_{\text{sat}} \) concentration (-)
\( c \) speed of light (m/s)
\( c_a'' \) specific heat capacity of an air-water mixture on a unit mass of air basis (J/kg\(_a\)-K)
\( c_a''_{\text{sat}} \) specific heat capacity of an air-water mixture along the saturation line on a unit mass of air basis (J/kg\(_a\)-K)
\( c_{\text{eff}} \) effective specific heat capacity of a composite (J/kg-K)
\( c_{\text{ms}} \) ratio of the energy carried by a micro-scale energy carrier to its temperature (J/K)
\( c_v \) specific heat capacity at constant volume (J/kg-K)
\( C \) total heat capacity (J/K)
\( C \) capacitance rate of a flow (W/K)
\( C_1, C_2, \ldots \) undetermined constants
\( C_{\text{crit}} \) dimensionless coefficient for critical heat flux correlation (-)
\( C_D \) drag coefficient (-)
\( C_f \) friction coefficient (-)
\( C_f^\prime \) average friction coefficient (-)
\( C_{\text{lum}} \) coefficient for laminar plate natural convection correlation (-)
\( C_{\text{nh}} \) dimensionless coefficient for nucleate boiling correlation (-)
\( C_R \) capacity ratio (-)
\( C_{\text{turb,}\text{U}} \) coefficient for turbulent, horizontal upward plate natural conv. correlation (-)
\( C_{\text{turb,}\text{V}} \) coefficient for turbulent, vertical plate natural convection correlation (-)
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Co</td>
<td>convection number (-)</td>
</tr>
<tr>
<td>CTE</td>
<td>coefficient of thermal expansion (1/K)</td>
</tr>
<tr>
<td>D</td>
<td>diameter (m)</td>
</tr>
<tr>
<td>( D_h )</td>
<td>hydraulic diameter (m)</td>
</tr>
<tr>
<td>dx</td>
<td>differential in the x-direction (m)</td>
</tr>
<tr>
<td>dy</td>
<td>differential in the y-direction (m)</td>
</tr>
<tr>
<td>e</td>
<td>size of surface roughness (m)</td>
</tr>
<tr>
<td>err</td>
<td>convergence or numerical error</td>
</tr>
<tr>
<td>E</td>
<td>rate of thermal energy carried by a mass flow (W)</td>
</tr>
<tr>
<td>E_\lambda</td>
<td>spectral emissive power (W/m²-μm)</td>
</tr>
<tr>
<td>E_{b,\lambda}</td>
<td>blackbody spectral emissive power (W/m²-μm)</td>
</tr>
<tr>
<td>Ec</td>
<td>Eckert number (-)</td>
</tr>
<tr>
<td>f</td>
<td>frequency (Hz)</td>
</tr>
<tr>
<td>( \bar{f} )</td>
<td>average friction factor (-)</td>
</tr>
<tr>
<td>( f_i )</td>
<td>friction factor for liquid-only flow in flow boiling (-)</td>
</tr>
<tr>
<td>F</td>
<td>force (N)</td>
</tr>
<tr>
<td>( F_{0-\lambda} )</td>
<td>external fractional function (-)</td>
</tr>
<tr>
<td>( F_{i,j} )</td>
<td>view factor from surface i to surface j (-)</td>
</tr>
<tr>
<td>( \hat{F}_{i,j} )</td>
<td>the “F-hat” parameter characterizing radiation from surface i to surface j (-)</td>
</tr>
<tr>
<td>fd</td>
<td>fractional duty for a pinch-point analysis (-)</td>
</tr>
<tr>
<td>Fo</td>
<td>Fourier number (-)</td>
</tr>
<tr>
<td>Fr</td>
<td>Froude number (-)</td>
</tr>
<tr>
<td>Fr_{mod}</td>
<td>modified Froude number (-)</td>
</tr>
<tr>
<td>g</td>
<td>acceleration of gravity (m/s²)</td>
</tr>
<tr>
<td>( \dot{g} )</td>
<td>rate of thermal energy generation (W)</td>
</tr>
<tr>
<td>( \dot{g}_{\text{eff}} )</td>
<td>effective rate of generation per unit volume of a composite (W/m³)</td>
</tr>
<tr>
<td>( \dot{g}_{\text{vis}} )</td>
<td>rate of thermal energy generation per unit volume due to viscous dissipation (W/m³)</td>
</tr>
<tr>
<td>G</td>
<td>mass flux or mass velocity (kg/m²-s)</td>
</tr>
<tr>
<td>( G_\lambda )</td>
<td>spectral irradiation (W/m²-μm)</td>
</tr>
<tr>
<td>Ga</td>
<td>Galileo number (-)</td>
</tr>
<tr>
<td>Gr</td>
<td>Grashof number (-)</td>
</tr>
<tr>
<td>Gz</td>
<td>Graetz number (-)</td>
</tr>
<tr>
<td>( h )</td>
<td>local heat transfer coefficient (W/m²-K)</td>
</tr>
<tr>
<td>( \overline{h} )</td>
<td>average heat transfer coefficient (W/m²-K)</td>
</tr>
<tr>
<td>( \bar{h} )</td>
<td>dimensionless heat transfer coefficient for flow boiling correlation (-)</td>
</tr>
<tr>
<td>( h_D )</td>
<td>mass transfer coefficient (m/s)</td>
</tr>
<tr>
<td>( \overline{h}_D )</td>
<td>average mass transfer coefficient (m/s)</td>
</tr>
<tr>
<td>( h_l )</td>
<td>superficial heat transfer coefficient for the liquid phase (W/m²-K)</td>
</tr>
</tbody>
</table>
Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{rad}$</td>
<td>the equivalent heat transfer coefficient associated with radiation (W/m²-K)</td>
</tr>
<tr>
<td>$i$</td>
<td>index of node (--)</td>
</tr>
<tr>
<td></td>
<td>index of eigenvalue (--)</td>
</tr>
<tr>
<td></td>
<td>index of term in a series solution (--)</td>
</tr>
<tr>
<td>$i''$</td>
<td>specific enthalpy (J/kg-K)</td>
</tr>
<tr>
<td>$i''_a$</td>
<td>specific enthalpy of an air-water mixture on a per unit mass of air basis</td>
</tr>
<tr>
<td>$I$</td>
<td>current (ampere)</td>
</tr>
<tr>
<td>$I_e$</td>
<td>intensity of emitted radiation (W/m²-µm-steradian)</td>
</tr>
<tr>
<td>$I_i$</td>
<td>intensity of incident radiation (W/m²-µm-steradian)</td>
</tr>
<tr>
<td>$j$</td>
<td>index of node (--)</td>
</tr>
<tr>
<td></td>
<td>index of eigenvalue (--)</td>
</tr>
<tr>
<td>$J$</td>
<td>radiosity (W/m²)</td>
</tr>
<tr>
<td>$j_H$</td>
<td>Colburn $j_H$ factor (-)</td>
</tr>
<tr>
<td>$k$</td>
<td>thermal conductivity (W/m-K)</td>
</tr>
<tr>
<td>$k_B$</td>
<td>Boltzmann’s constant (J/K)</td>
</tr>
<tr>
<td>$k_c$</td>
<td>contraction loss coefficient (--)</td>
</tr>
<tr>
<td>$k_e$</td>
<td>expansion loss coefficient (--)</td>
</tr>
<tr>
<td>$k_{eff}$</td>
<td>effective thermal conductivity of a composite (W/m-K)</td>
</tr>
<tr>
<td>$Kn$</td>
<td>Knudsen number (-)</td>
</tr>
<tr>
<td>$l_1$</td>
<td>Lennard-Jones 12-6 potential characteristic length for species 1 (m)</td>
</tr>
<tr>
<td>$l_{1,2}$</td>
<td>characteristic length of a mixture of species 1 and species 2 (m)</td>
</tr>
<tr>
<td>$L$</td>
<td>length (m)</td>
</tr>
<tr>
<td>$L^+$</td>
<td>dimensionless length for a hydrodynamically developing internal flow (-)</td>
</tr>
<tr>
<td>$L^*$</td>
<td>dimensionless length for a thermally developing internal flow (-)</td>
</tr>
<tr>
<td>$L_{char}$</td>
<td>characteristic length of the problem (m)</td>
</tr>
<tr>
<td>$L_{char,vs}$</td>
<td>the characteristic size of the viscous sublayer (m)</td>
</tr>
<tr>
<td>$L_{cond}$</td>
<td>length for conduction (m)</td>
</tr>
<tr>
<td>$L_{flow}$</td>
<td>length in the flow direction (m)</td>
</tr>
<tr>
<td>$L_{ml}$</td>
<td>mixing length (m)</td>
</tr>
<tr>
<td>$L_{ms}$</td>
<td>distance between interactions of micro-scale energy or momentum carriers</td>
</tr>
<tr>
<td></td>
<td>(m)</td>
</tr>
<tr>
<td>$Le$</td>
<td>Lewis number (-)</td>
</tr>
<tr>
<td>$M$</td>
<td>number of nodes (-)</td>
</tr>
<tr>
<td></td>
<td>mass (kg)</td>
</tr>
<tr>
<td>$m$</td>
<td>fin parameter (1/m)</td>
</tr>
<tr>
<td>$m'$</td>
<td>mass flow rate (kg/s)</td>
</tr>
<tr>
<td>$m''$</td>
<td>mass flow rate per unit area (kg/m²-s)</td>
</tr>
<tr>
<td>$m_{ms}$</td>
<td>mass of microscale momentum carrier (kg/carrier)</td>
</tr>
<tr>
<td>$mf$</td>
<td>mass fraction (-)</td>
</tr>
<tr>
<td>$MW$</td>
<td>molar mass (kg/kgmol)</td>
</tr>
<tr>
<td>$n$</td>
<td>number density (#/m³)</td>
</tr>
<tr>
<td>$n_{ms}$</td>
<td>number density of the micro-scale energy carriers (#/m³)</td>
</tr>
<tr>
<td>$n''$</td>
<td>molar transfer rate per unit area (kgmol/m²-s)</td>
</tr>
<tr>
<td>$N$</td>
<td>number of nodes (-)</td>
</tr>
<tr>
<td></td>
<td>number of moles (kgmol)</td>
</tr>
<tr>
<td>$N_s$</td>
<td>number of species in a mixture (-)</td>
</tr>
<tr>
<td>$Nu$</td>
<td>Nusselt number (-)</td>
</tr>
</tbody>
</table>
### Nomenclature

- $\bar{\nu}$: average Nusselt number (-)
- $NTU$: number of transfer units (-)
- $p$: pressure (Pa)
- $P$: $LMTD$ effectiveness (-)
- $p_\infty$: free-stream pressure (Pa)
- $\bar{p}$: dimensionless pressure (-)
- $Pe$: Peclet number (-)
- $per$: perimeter (m)
- $Pr$: Prandtl number (-)
- $Pr_{turb}$: turbulent Prandtl number (-)
- $\dot{q}$: rate of heat transfer (W)
- $\dot{q}_{ij}$: rate of radiation heat transfer from surface $i$ to surface $j$ (W)
- $\dot{q}_{\text{max}}$: maximum possible rate of heat transfer, for an effectiveness solution (W)
- $\dot{q}_{\text{r}}$: heat flux, rate of heat transfer per unit area (W/m$^2$)
- $\dot{q}_{\text{r,s}}$: surface heat flux (W/m$^2$)
- $\dot{q}_{\text{r,crit}}$: critical heat flux for boiling (W/m$^2$)
- $\dot{Q}$: total energy transfer by heat (J)
- $\tilde{Q}$: dimensionless total energy transfer by heat (-)
- $r$: radial coordinate (m)
- $\tilde{r}$: dimensionless radial coordinate (-)
- $R$: thermal resistance (K/W)
- $R_T$: ideal gas constant (J/kgmol-K)
- $R_{\text{LMTD}}$: $LMTD$ capacitance ratio (-)
- $R_{\text{A}}$: thermal resistance approximation based on average area limit (K/W)
- $R_{ac}$: thermal resistance to axial conduction in a heat exchanger (K/W)
- $R_{\text{ad}}$: thermal resistance approximation based on adiabatic limit (K/W)
- $R_{\text{bd}}$: thermal resistance of the boundary layer (K/W)
- $R_c$: thermal resistance due to solid-to-solid contact (K/W)
- $R_{\text{conv}}$: thermal resistance to convection from a surface (K/W)
- $R_{\text{cyl}}$: thermal resistance to radial conduction through a cylindrical shell (K/W)
- $R_e$: electrical resistance (ohm)
- $R_f$: thermal resistance due to fouling (K/W)
- $R_{\text{fin}}$: thermal resistance of a fin (K/W)
- $R_{ij}$: the radiation space resistance between surfaces $i$ and $j$ (1/m$^2$)
- $R_{\text{iso}}$: thermal resistance approximation based on isothermal limit (K/W)
- $R_{\text{T}}$: thermal resistance approximation based on average length limit (K/W)
- $R_{\text{pw}}$: thermal resistance to radial conduction through a plane wall (K/W)
- $R_{\text{rad}}$: thermal resistance to radiation (K/W)
- $R_{ki}$: the radiation surface resistance for surface $i$ (1/m$^2$)
- $R_{\text{semi-}}$: thermal resistance approximation for a semi-infinite body (K/W)
- $R_{\text{ph,l}}$: thermal resistance to radial conduction through a spherical shell (K/W)
- $R_{\text{tot}}$: thermal resistance of a finned surface (K/W)
- $R_{\text{univ}}$: universal gas constant (8314 J/kgmol-K)
- $R_{c}''$: area-specific contact resistance (K-m$^2$/W)
- $R_{f}''$: area-specific fouling resistance (K-m$^2$/W)
- $Ra$: Rayleigh number (-)
Nomenclature

Re \hspace{5cm} \text{Reynolds number (-)}
Re_{crit} \hspace{5cm} \text{critical Reynolds number for transition to turbulence (-)}
RH \hspace{5cm} \text{relative humidity (-)}
RR \hspace{5cm} \text{radius ratio of an annular duct (-)}
s \hspace{5cm} \text{Laplace transformation variable (1/s)}
S \hspace{5cm} \text{shape factor (m)}
S \hspace{5cm} \text{channel spacing (m)}
Sc \hspace{5cm} \text{Schmidt number (-)}
Sh \hspace{5cm} \text{Sherwood number (-)}
\overline{Sh} \hspace{5cm} \text{average Sherwood number (-)}
St \hspace{5cm} \text{Stanton number (-)}
t \hspace{5cm} \text{time (s)}
t_{sim} \hspace{5cm} \text{simulated time (s)}
th \hspace{5cm} \text{thickness (m)}
\text{tol} \hspace{5cm} \text{convergence tolerance}
T \hspace{5cm} \text{temperature (K)}
T_b \hspace{5cm} \text{base temperature of fin (K)}
T_{film} \hspace{5cm} \text{film temperature (K)}
T_m \hspace{5cm} \text{mean or bulk temperature (K)}
T_s \hspace{5cm} \text{surface temperature (K)}
T_{sat} \hspace{5cm} \text{saturation temperature (K)}
T_{\infty} \hspace{5cm} \text{free-stream or fluid temperature (K)}
T^* \hspace{5cm} \text{eddy temperature fluctuation (K)}
T' \hspace{5cm} \text{fluctuating component of temperature (K)}
\overline{T} \hspace{5cm} \text{average temperature (K)}
TR \hspace{5cm} \text{temperature solution that is a function of } r, \text{ for separation of variables}
T_t \hspace{5cm} \text{temperature solution that is a function of } t, \text{ for separation of variables}
TX \hspace{5cm} \text{temperature solution that is a function of } x, \text{ for separation of variables}
TY \hspace{5cm} \text{temperature solution that is a function of } y, \text{ for separation of variables}
Th \hspace{5cm} \text{thickness (m)}
U \hspace{5cm} \text{internal energy (J)}
\text{utilization (-)}
\text{u} \hspace{5cm} \text{specific internal energy (J/kg)}
\text{velocity in the x-direction (m/s)}
\text{u_{char}} \hspace{5cm} \text{characteristic velocity (m/s)}
\text{u_f} \hspace{5cm} \text{frontal or upstream velocity (m/s)}
\text{u_m} \hspace{5cm} \text{mean or bulk velocity (m/s)}
\text{u_{\infty}} \hspace{5cm} \text{free-stream velocity (m/s)}
\text{u^*} \hspace{5cm} \text{eddy velocity (m/s)}
\text{u^+} \hspace{5cm} \text{inner velocity (-)}
\tilde{u} \hspace{5cm} \text{dimensionless x-velocity (-)}
\text{u'} \hspace{5cm} \text{fluctuating component of x-velocity (m/s)}
\overline{\pi} \hspace{5cm} \text{average x-velocity (m/s)}
UA \hspace{5cm} \text{conductance (W/K)}
v \hspace{5cm} \text{velocity in the y- or r-directions (m/s)}
\text{v_3} \hspace{5cm} \text{y-velocity at the outer edge of the boundary layer, approximate scale of y-velocity in a boundary layer (m/s)}
\text{v_{ms}} \hspace{5cm} \text{mean velocity of micro-scale energy or momentum carriers (m/s)}
Nomenclature

\( \tilde{v} \)  dimensionless \( y \)-velocity (-)
\( v' \)  fluctuating component of \( y \)-velocity (m/s)
\( \overline{v} \)  average \( y \)-velocity (m/s)
\( V \)  volume (m\(^3\))
\( V \)  volume flow rate (m\(^3\)/s)
\( v_f \)  void fraction (-)
\( w \)  velocity in the \( z \)-direction (m/s)
\( \dot{w} \)  rate of work transfer (W)
\( W \)  width (m)
\( x \)  \( x \)-coordinate (m)
\( x_f \)  total amount of work transferred (J)
\( \tilde{x} \)  dimensionless \( x \)-coordinate (-)
\( X \)  particular solution that is only a function of \( x \)
\( x_{fd,h} \)  hydrodynamic entry length (m)
\( x_{fd,t} \)  thermal entry length (m)
\( X_{tt} \)  Lockhart Martinelli parameter (-)
\( y \)  \( y \)-coordinate (m)
\( y^+ \)  inner position (-)
\( \tilde{y} \)  dimensionless \( y \)-coordinate (-)
\( Y \)  particular solution that is only a function of \( y \)
\( z \)  \( z \)-coordinate (m)

Greek Symbols

\( \alpha \)  thermal diffusivity (m\(^2\)/s)
\( \alpha \)  absorption coefficient (1/m)
\( \alpha \)  absorptivity or absorptance (-), total hemispherical absorptivity (-)
\( \alpha_{surf} \)  surface area per unit volume (1/m)
\( \alpha_{eff} \)  effective thermal diffusivity of a composite (m\(^2\)/s)
\( \alpha_{\lambda} \)  hemispherical absorptivity (-)
\( \alpha_{\lambda,\phi} \)  spectral directional absorptivity (-)
\( \beta \)  volumetric thermal expansion coefficient (1/K)
\( \delta \)  film thickness for condensation (m)
\( \delta_f \)  boundary layer thickness (m)
\( \delta_m \)  momentum diffusion penetration depth (m)
\( \delta_m \)  momentum boundary layer thickness (m)
\( \delta_{visc} \)  viscous sublayer thickness (m)
\( \delta_t \)  energy diffusion penetration depth (m)
\( \delta_t \)  thermal boundary layer thickness (m)
\( \Delta i_{fus} \)  latent heat of fusion (J/kg)
\( \Delta i_{vap} \)  latent heat of vaporization (J/kg)
\( \Delta p \)  pressure drop (N/m\(^2\))
\( \Delta r \)  distance in \( r \)-direction between adjacent nodes (m)
Nomenclature

\( \Delta T \) temperature difference (K)
\( \Delta T_e \) excess temperature (K)
\( \Delta T_{lm} \) log-mean temperature difference (K)
\( \Delta t \) time step (s)
\( \Delta t_{crit} \) critical time step (s)
\( \Delta x \) distance in x-direction between adjacent nodes (m)
\( \Delta y \) distance in y-direction between adjacent nodes (m)
\( \epsilon \) heat exchanger effectiveness (-)
\( \overline{\epsilon} \) emissivity or emittance (-), total hemispherical emissivity (-)
\( \overline{\epsilon}_{fin} \) fin effectiveness (-)
\( \overline{\epsilon}_{H} \) eddy diffusivity for heat transfer (m\(^2\)/s)
\( \overline{\epsilon}_{\lambda, \theta, \phi} \) spectral, directional emissivity (-)
\( \overline{\epsilon}_M \) eddy diffusivity of momentum (m\(^2\)/s)
\( \epsilon_1 \) Lennard-Jones 12-6 potential characteristic energy for species 1 (J)
\( \epsilon_{1,2} \) characteristic energy parameter for a mixture of species 1 and species 2 (J)
\( \phi \) porosity (-)
\( \phi_0 \) overall efficiency of a finned surface (-)
\( \kappa \) von Kármán constant
\( \lambda \) dimensionless axial conduction parameter (-)
\( \lambda_i \) \( i^{th} \) eigenvalue of a solution (1/m)
\( \mu \) viscosity (N-s/m\(^2\))
\( \nu \) frequency of radiation (1/s)
\( \theta \) temperature difference (K)
\( \bar{\theta} \) dimensionless temperature difference (-)
\( \theta^+ \) inner temperature difference (-)
\( \theta_R \) temperature difference solution that is only a function of \( r \), for separation of variables
\( \theta_t \) temperature difference solution that is only a function of \( t \), for separation of variables
\( \theta_X \) temperature difference solution that is only a function of \( x \), for separation of variables
\( \theta_{Xt} \) temperature difference solution that is only a function of \( x \) and \( t \), for reduction of multi-dimensional transient problems
\( \theta_Y \) temperature difference solution that is only a function of \( y \), for separation of variables
\( \theta_{Yt} \) temperature difference solution that is only a function of \( y \) and \( t \), for reduction of multi-dimensional transient problems
\( \theta_{Zt} \) temperature difference solution that is only a function of \( z \) and \( t \), for reduction of multi-dimensional transient problems
Nomenclature

\( \rho \)  
- density (kg/m\(^3\))
- reflectivity or reflectance (-), total hemispherical reflectivity (-)

\( \rho_e \)  
electrical resistivity (ohm-m)

\( \rho_{\text{eff}} \)  
effective density of a composite (kg/m\(^3\))

\( \rho_{\lambda} \)  
hemispherical reflectivity (-)

\( \rho_{\lambda,\theta,\phi} \)  
spectral, directional reflectivity (-)

\( \sigma \)  
surface tension (N/m), molecular radius (m)
- ratio of free-flow to frontal area (-)
- Stefan-Boltzmann constant (5.67 \times 10^{-8} \text{ W/m}^2\cdot\text{K}^4)

\( \tau \)  
time constant (s)
- shear stress (Pa)
- transmittivity or transmittance (-), total hemispherical transmittivity (-)

\( \tau_{\text{diff}} \)  
diffusive time constant (s)

\( \tau_{\text{lumped}} \)  
lumped capacitance time constant (s)

\( \tau_{\lambda} \)  
hemispherical transmittivity (-)

\( \tau_{\lambda,\theta,\phi} \)  
spectral, directional transmittivity (-)

\( \tau_v \)  
shear stress at surface (N/m\(^2\))

\( \nu \)  
kinematic viscosity (m\(^2\)/s)

\( \omega \)  
angular velocity (rad/s)

\( \Omega_D \)  
dimensionless collision integral for diffusion (-)

\( \Psi \)  
stream function (m\(^2\)/s)

\( \zeta \)  
tilt angle (rad)

\( \zeta_i \)  
the \( i \)th dimensionless eigenvalue (-)

Superscripts

\( o \)  
at infinite dilution

Subscripts

\( a \)  
- air
\( abs \)  
- absorbed
\( ac \)  
- axial conduction (in heat exchangers)
\( an \)  
- analytical
\( app \)  
- apparent
- approximate
\( b \)  
- blackbody
\( bl \)  
- boundary layer
\( bottom \)  
- bottom
\( c \)  
- condensate film
- corrected
\( C \)  
- cold
- cold-side of a heat exchanger
\( cc \)  
- complex conjugate, for complex combination problems
\( char \)  
- characteristic
\( cf \)  
- counter-flow heat exchanger
<table>
<thead>
<tr>
<th><strong>Nomenclature</strong></th>
<th>xxxv</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>cond</strong></td>
<td>conduction, conductive</td>
</tr>
<tr>
<td><strong>conv</strong></td>
<td>convection, convective</td>
</tr>
<tr>
<td><strong>crit</strong></td>
<td>critical</td>
</tr>
<tr>
<td><strong>CTHB</strong></td>
<td>cold-to-hot blow process</td>
</tr>
<tr>
<td><strong>dc</strong></td>
<td>dry coil</td>
</tr>
<tr>
<td><strong>df</strong></td>
<td>downward facing</td>
</tr>
<tr>
<td><strong>diff</strong></td>
<td>diffusive transfer</td>
</tr>
<tr>
<td><strong>eff</strong></td>
<td>effective</td>
</tr>
<tr>
<td><strong>emit</strong></td>
<td>emitted</td>
</tr>
<tr>
<td><strong>evap</strong></td>
<td>evaporative</td>
</tr>
<tr>
<td><strong>ext</strong></td>
<td>external</td>
</tr>
<tr>
<td><strong>f</strong></td>
<td>fluid</td>
</tr>
<tr>
<td><strong>fc</strong></td>
<td>forced convection</td>
</tr>
<tr>
<td><strong>fd,h</strong></td>
<td>hydrodynamically fully developed</td>
</tr>
<tr>
<td><strong>fd,t</strong></td>
<td>thermally fully developed</td>
</tr>
<tr>
<td><strong>fin</strong></td>
<td>fin, finned</td>
</tr>
<tr>
<td><strong>h</strong></td>
<td>homogeneous solution</td>
</tr>
<tr>
<td><strong>H</strong></td>
<td>hot</td>
</tr>
<tr>
<td><strong>hs</strong></td>
<td>hot-side of a heat exchanger</td>
</tr>
<tr>
<td><strong>HTCB</strong></td>
<td>hot-to-cold blow process</td>
</tr>
<tr>
<td><strong>i</strong></td>
<td>node $i$</td>
</tr>
<tr>
<td><strong>in</strong></td>
<td>inner</td>
</tr>
<tr>
<td><strong>ini</strong></td>
<td>initial</td>
</tr>
<tr>
<td><strong>int</strong></td>
<td>internal</td>
</tr>
<tr>
<td><strong>j</strong></td>
<td>node $j$</td>
</tr>
<tr>
<td><strong>l</strong></td>
<td>liquid</td>
</tr>
<tr>
<td><strong>lam</strong></td>
<td>laminar</td>
</tr>
<tr>
<td><strong>LHS</strong></td>
<td>left-hand side</td>
</tr>
<tr>
<td><strong>lumped</strong></td>
<td>lumped-capacitance</td>
</tr>
<tr>
<td><strong>m</strong></td>
<td>mean or bulk</td>
</tr>
<tr>
<td><strong>max</strong></td>
<td>maximum or maximum possible</td>
</tr>
<tr>
<td><strong>min</strong></td>
<td>minimum or minimum possible</td>
</tr>
<tr>
<td><strong>mod</strong></td>
<td>modified</td>
</tr>
<tr>
<td><strong>ms</strong></td>
<td>micro-scale carrier</td>
</tr>
<tr>
<td><strong>n</strong></td>
<td>normal</td>
</tr>
<tr>
<td><strong>nac</strong></td>
<td>without axial conduction (in heat exchangers)</td>
</tr>
<tr>
<td><strong>nb</strong></td>
<td>nucleate boiling</td>
</tr>
<tr>
<td><strong>nc</strong></td>
<td>natural convection</td>
</tr>
<tr>
<td><strong>no-fin</strong></td>
<td>without a fin</td>
</tr>
<tr>
<td><strong>out</strong></td>
<td>outer</td>
</tr>
<tr>
<td><strong>p</strong></td>
<td>particular (or non-homogeneous) solution</td>
</tr>
</tbody>
</table>
Nomenclature

\( pf \)  parallel-flow heat exchanger
\( pp \)  pinch-point
\( r \)  regenerator matrix
  at position \( r \)
\( rad \)  radiation, radiative
\( ref \)  reference
\( RHS \)  right-hand side
\( s \)  at the surface
\( sat \)  saturated
  saturated section of a heat exchanger
\( sat,l \)  saturated liquid
\( sat,v \)  saturated vapor
\( sc \)  sub-cooled section of a heat exchanger
\( semi-\infty \)  semi-infinite
\( sh \)  super-heated section of a heat exchanger
\( sph \)  sphere
\( sur \)  surroundings
\( sus \)  sustained solution
\( T \)  constant temperature boundary condition
  at temperature \( T \)
\( top \)  top
\( tot \)  total
\( turb \)  turbulent
\( uf \)  upward-facing
\( unfin \)  not finned
\( v \)  vapor
  vertical
  viscous dissipation
\( w \)  water
\( wb \)  wet-bulb
\( wc \)  wet coil
\( x \)  at position \( x \)
  in the \( x \)-direction
\( x^- \)  in the negative \( x \)-direction
\( x^+ \)  in the positive \( x \)-direction
\( y \)  at position \( y \)
  in the \( y \)-direction
\( \infty \)  free-stream, fluid
\( 90^\circ \) solution that is \( 90^\circ \) out of phase, for complex combination problems

Other notes

\( A \)  arbitrary variable
\( A' \)  fluctuating component of variable \( A \)
  value of variable \( A \) on a unit length basis
\( A'' \)  value of variable \( A \) on a unit area basis
\( A''' \)  value of variable \( A \) on a unit volume basis
\( \tilde{A} \)  dimensionless form of variable \( A \)
\( \hat{A} \)  a guess value or approximate value for variable \( A \)
\( \mathcal{A} \)  Laplace transform of the function \( A \)
# Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\overline{A}$</td>
<td>average of variable A</td>
</tr>
<tr>
<td>$\mathbf{A}$</td>
<td>denotes that variable A is a vector</td>
</tr>
<tr>
<td>$\mathbf{A}$</td>
<td>denotes that variable A is a matrix</td>
</tr>
<tr>
<td>$\Delta A$</td>
<td>differential change in the variable A</td>
</tr>
<tr>
<td>$\delta A$</td>
<td>uncertainty in the variable A</td>
</tr>
<tr>
<td>$\Delta A$</td>
<td>finite change in the variable A</td>
</tr>
<tr>
<td>$O(A)$</td>
<td>order of magnitude of the variable A</td>
</tr>
</tbody>
</table>