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This Second Edition includes new material, including a section on Langmuir circulations and the Craik–Leibovich instability. The author has also added exercises to aid students' learning.

OLIVER BÜHLER is a Professor of Mathematics and Atmosphere Ocean Science at the Courant Institute of Mathematical Sciences, New York University.

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Waves and Mean Flows

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Courant Institute of Mathematical Sciences
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I will arise and go now, for always night and day
I hear lake water lapping with low sounds by the shore;
While I stand on the roadway, or on the pavements grey,
I hear it in the deep heart's core.
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W. B. Yeats 'The Lake Isle Of Innisfree'

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Preface

For the revised edition

The happy occasion of the revised paperback printing made it possible to add a section on Langmuir circulations and the Craik–Leibovich instability to chapter 11. These are important and fundamental topics that ought to have been included already in the first edition. This new material also prompted significant changes in section 13.4 on the vorticity generated by breaking surface-gravity waves, which hopefully make this crucial topic more transparent. In addition, there are smaller changes such as high-lighting the amazing curl–curvature formula for wave ray tracing in a weak vortical mean flow in §4.4.3, as well as numerous small fixes and some additional references. A small number of exercises has also been added to various chapters, which hopefully will aid the educational aspects of this book.

I am exceptionally grateful to Michael McIntyre for his very detailed reading of the first edition and for his support in preparing this revised edition. Thanks are also due to Rick Salmon and William Young for their insightful suggestions and to David Tranah for his continued support at Cambridge University Press.

Finally, I would like to dedicate this edition to the memory of my father by the last words of Mahler's *Lied der Erde: "Ewig, ewig"*.

New York, March 2013.

The aim of this book

This book is on waves and on their interactions with mean flows such as shear flows or vortices. Such interactions are generally a two-way street, with the waves being affected by the mean flow whilst the mean flow itself responds to the presence of the waves. For instance, readily observed examples of waves affected by mean flows are surface waves propagating on a sheared river current, or ripples that are refracted by a bath-tub vortex. Mean flows that are responding to waves are slightly less easily observed, here examples are given by the classic phenomenon of acoustic streaming, by longshore currents driven by breaking waves on beaches, and, as it turns out, also by many other flows in the atmosphere and ocean. Not surprisingly, *wave–mean interaction theory* is a very important topic in geophysical fluid dynamics (GFD).

For instance, the wave-induced transport of mass, momentum, and angular momentum plays a crucial role in the global-scale circulation of the atmosphere and the ocean, and in such complex multi-scale phenomena as the stratospheric ozone hole. However, many waves that contribute significantly to this transport are much too small in scale to be resolvable by even the most powerful present-day supercomputers, which implies that these small-scale waves, and their interactions with the large-scale mean flow, must be "parametrized" in numerical models, i.e., they must be put-in by hand based on a combination of theory and observational data. Our ability to predict the climate by large-scale numerical simulation therefore hinges delicately on our ability to model unresolvable wave processes in a consistent and accurate manner.

Arguably, the theoretical advances in wave–mean interaction theory that have been achieved in GFD in order to answer this need have far outpaced those in other fluid-dynamical fields, such as plasma physics, or condensed-matter physics including superfluids. For instance, the intricate interplay between the so-called *pseudomomentum* of the waves and the momentum and the vorticity of the mean flow has been fully worked out in GFD, but not yet in these other fields. Hopefully, by providing a comprehensive account of the tools and key results of wave–mean interaction theory in GFD, the present book will be equally useful for readers interested in GFD and also for readers who might want to apply such a theory in other fields.

The types of waves that are studied in this book include sound waves, surface and internal gravity waves, waves modified by Coriolis forces, and also Rossby waves, which are a peculiar form of large-scale vorticity waves that owe their principal restoring mechanism to the spherical curvature of the rapidly rotating Earth. To the best of my ability, I have tried to focus as closely as possible on the *fundamental principles of fluid dynamics* in the hope that this approach will best elucidate the essential workings of the diverse interaction effects that can occur.

Specifically, at its core and throughout all its chapters, this book aims to make obvious and to exploit the intimate connections between three key concepts in fluid dynamics:

1. *Kelvin's circulation theorem*,
2. the material invariance of *potential vorticity* in ideal fluid flow, and
3. the dynamics of the waves' *pseudomomentum vector* field.

As we shall see, putting the focus on the fundamental fluid dynamics becomes particularly important if one considers interactions outside the setting of spatially symmetric mean flows, which is the classical setting that is considered in detail in the current textbooks on GFD. The present book gives a detailed discussion of this classical setting, but it also moves significantly beyond it, with nontrivial implications for the "parametrization" problem as well as, for instance, the ocean-beach problem.

With regard to the mathematical methods of wave–mean interaction theory, I have tried to combine freely the standard fixed-position, Eulerian methods with the less standard particle-following, Lagrangian methods. Each kind of method has its merits and drawbacks in different situations, and I think it is very advantageous to know how to use both kinds. After all, the interplay between Eulerian and Lagrangian viewpoints lies at the very heart of understanding in fluid dynamics.

Before moving on to the specifics of the book content, I would like to make just one comment on terminology, namely on the use of the phrase "wave–mean interactions". It is far from obvious that this is the most logical term. Indeed, from a physical point of view it might be more logical to speak of interactions between "waves" and "vortices", so we should prefer the term "wave–vortex interactions". On the other hand, from a mathematical point of view one might prefer to juxtapose "disturbance fields" and "mean fields", as these can be defined rigorously in terms of a suitable averaging operator. After all, mean fields can be wave-like and some disturbance fields (such as Rossby waves) can be vortex-like. Thus perhaps one should prefer the term "disturbance–mean interactions". In the light of these valid objections, we will use the cross-bred term "wave–mean interactions" precisely because it reminds us that our physical and mathematical categories never fully catch all the facets of the slippery reality we seek to understand—but we try.

Contents outline

The book is structured such that it can be read cover-to-cover by a general reader with little previous knowledge of the subject matter, although one might have to assume a considerable enthusiasm for fluid dynamics in this case. Alternatively, I have tried to keep the parts and the chapters within the parts as self-contained as possible, with clear references to results established elsewhere, so for the specialist it should be easy to pull out particular sections of interest. Some of the chapter materials were first tested in several graduate classes at the Courant Institute and at various summer schools, but the book as it stands is all new.

Each chapter ends with a very brief discussion of some references, which are very far from being comprehensive. Basically, I have only endeavoured to refer to some original sources and to particular review articles or textbooks that I have read myself. Today, it is very easy to find and to obtain a multitude of references from the web once one knows what to look for.

Part I covers the linear theory of dispersive waves in moving media, which is a necessary basis for the study of interaction theory. This also includes a brief summary of fluid dynamics in order to keep the book self-contained. Particular attention is given to group-velocity concepts, geometric wave theory, WKB techniques, and ray tracing, because these are the essential theoretical tools for the study of small-scale waves propagating on a large-scale flow. The conservation laws for wave activity measures such as *wave action* and *pseudomomentum* are derived, and it is explained how these conservation laws are related to the continuous symmetries of the basic state on which the waves are propagating. Caustics, where ray tracing fails and predicts infinite wave amplitudes, are ubiquitous in the atmosphere and ocean, and their treatment within linear theory is discussed in some detail.

Part II presents the classic wave–mean interaction theory that has been developed mostly in atmospheric science. This theory is based on *simple geometry*, which assumes spatially periodic flows and uses zonal averaging, i.e., averaging over the periodic coordinate such as longitude in the atmosphere. Moreover, the basic state on which the waves are propagating is assumed to be zonally symmetric, which greatly simplifies the situation.

In simple geometry the focus is on the zonal mean flow and on the corresponding zonal component of the pseudomomentum of the waves. The central result in this area is the *dissipative pseudomomentum rule*, which relates the dissipation rate of pseudomomentum due to viscous effects or wave breaking to an effective mean force felt by the mean flow. The most fundamental derivation of this result is based on Kelvin's circulation theorem and on the potential vorticity of the mean flow.

Examples discussed in this part include internal waves forced by flow over undulating topography and the interplay between large-scale Rossby waves and two-dimensional turbulence. The latter problem is relevant not only on Earth, but also on other planets such as Jupiter.

Part II also includes a detailed introduction to the so-called *generalized Lagrangian-mean theory*, which is based on particle-following averages and which formally allows extending all of the earlier results to finite wave amplitude. In particular, a detailed account of Kelvin's circulation theorem for the Lagrangian-mean flow can be given, which makes obvious the essential role of the pseudomomentum vector in this connection. Also, the previous discussion of conserved wave activities is extended here to apply to fully nonlinear, finite-amplitude waves and to the symmetries of the Lagrangian-mean flow.[1]

Part III discusses wave–mean interactions outside simple geometry, which is a profound change in the physical and mathematical situation. This makes the theory at once more complicated, but also much more relevant to the study of small-scale waves propagating generally on a slowly varying basic flow. Such a setting is natural in the ocean, for instance, because with rare exceptions it is not possible to average oceanic flows zonally, as the continents get in the way. It is also needed in the "parametrization" problem.

The crucial role of long-range mean pressure effects is then discussed in detail and with several examples. Such pressure effects are absent in simple geometry, and this marks the essential difference between the two situations. It is argued that at this stage wave–mean interaction theory must restrict itself to the dynamics of vorticity rather than of momentum, essentially in order to control the non-local effects due to the mean pressure field. With some further assumptions it is then possible to formulate a conservation law for a suitably defined *impulse* of the mean flow plus the pseudomomentum of the waves, and this law replaces the classic pseudomomentum rule of simple geometry.

A study of wave-driven nearshore circulations on beaches is then used to illustrate the new situation, which here involves mean-flow vortices driven by the waves. This is followed by a number of idealized scenarios that show how wavepackets can interact with isolated vortices at long range via refraction effects. These interactions are intrinsically non-local in nature. Finally, the peculiar kinship between wavepackets with low intrinsic group velocities and vortex couples will be discussed, which is relevant to packets of internal gravity waves undergoing a scale cascade to smaller wavelengths. This provides the most intriguing example of the interplay between the three fundamental concepts listed above.

Acknowledgements

I owe thanks to many people and institutions who, in different ways, enabled me to write this book. Foremost, these include Michael McIntyre and the

[1] Practical evaluations of finite-amplitude wave activity measures are usually computed differently, namely using the elegant methods of Hamiltonian fluid mechanics. Excellent accounts of these methods exist in the literature, but here we work with Lagrangian-mean methods because the Hamiltonian methods cannot be applied with equal ease to the topic of wave–mean interactions, where the Lagrangian-mean theory excels.

Dept of Applied Mathematics and Theoretical Physics in Cambridge (UK), and Andrew Majda and the Courant Institute of Mathematical Sciences at New York University. I would also like to acknowledge the kind hospitality of Rupert Klein and his group, as well as that of the Zuse Zentrum at the Freie Universität Berlin (Germany), during my 2007-08 sabbatical year, when the bulk of this book was written. David Tranah and the staff at Cambridge University Press kept the faith over long dry spells. My research has been supported financially by the National Science Foundation. Some of the illustrations were created using an author licence for Matlab.

I have been fortunate to have had many excellent teachers of fluid dynamics over the years: Heinz Schade, Alfons Michalke, Ken Powell, Phil Roe, Michael McIntyre, Peter Haynes, and David Crighton. The Woods Hole summer study in GFD has provided a home-away-from-home for many scientists, and I am thankful for its staff, especially Lou Howard, Joe Keller, and Ed Spiegel, for their inspiration and their teaching in GFD.

Once more I would like to thank Michael McIntyre, who taught me almost all I know about the topic of this book, and who kindly shared with me his draft of an earlier book project in the same direction; Michael also commented somewhat extensively on chapter 9.

Finally, as ever, all would have been for naught had it not been for SCK, now SCB, and our corollaries JBB and AKB.

New York, October 2008.