CAMBRIDGE

Cambridge University Press
978-1-107-66947-5 - Experimental Building Science: Volume Two: Being an Introduction to Mechanics and its Application in the Design and Erection of Buildings
J. Leask Manson and Francis E. Drury
Excerpt
More information

SECTION I

THE CONDITIONS OF STATICAL EQUILIBRIUM, WITH APPLICATIONS TO BUILDING STRUCTURES

CHAPTER I

INTRODUCTION. THE CONDITIONS OF EQUILIBRIUM FOR SYSTEMS OF CONCURRENT COPLANAR FORCES

1. Introduction to Section I. Mechanics is one of the oldest of the sciences. Its origins are of peculiar interest to students of Building, since they are to be found in man's early attempts to devise contrivances for the raising and moving of heavy weights, tasks which had to be faced, for example, when he undertook to erect the monumental buildings of early historical times. The term **Mechanics** (or Applied Mechanics) is now generally applied to the more practical aspects of the subject of **Dynamics**, the mathematical science which treats of the *motion* and *equilibrium* of material bodies under the action of forces.

The subject of **Mechanics** is usually divided into two main sections:

(1) **Kinetics** (sometimes also called Dynamics), which deals with systems of forces which produce *motion*; and

(2) **Statics**, which deals with systems of forces which produce *equilibrium, or a state of rest.*

In this volume we propose to investigate all the more important principles of Statics, since in the design and erection of buildings we are almost exclusively concerned with bodies which are at rest, under the action of systems of forces arising from the weights of structures and from the external loads which they have to carry.

The work will be divided into three sections, as follows:

Section I will treat almost exclusively of the equilibrium of structures and of parts of structures under the action of systems of external forces.

Section II will deal with the effects of these external forces upon the materials of which the structures are composed, giving particular attention to the important case of loaded beams. In this section are introduced the subjects of **Elasticity**, which deals with the equilibrium of strained bodies, and the **Strength of Materials**,

© in this web service Cambridge University Press www.cambridge.org

Cambridge University Press
978-1-107-66947-5 - Experimental Building Science: Volume Two: Being an Introduction to Mechanics and its Application in the Design and Erection of Buildings
J. Leask Manson and Francis E. Drury
Excerpt
More information

which treats of the power of structural materials to support the forces applied to them and to maintain the equilibrium of the structure as a whole.

Section III will carry the whole of the previous work a stage further by considering the application of statical knowledge to the main types of building construction.

2. Units for the measurement of statical forces. In the more general treatment given in Dynamics, **Force** is defined as *that which tends to change the state of rest or of uniform motion of a material body*, a definition which is derived from Newton's First Law of Motion. As explained in Vol. I, Chap. VI, however, it is possible to derive a satisfactory unit for the measurement of statical forces, which only involves the idea of motion by implication, from the definition of **Equal Forces.** *Two forces are said to be equal if, when applied to the same small body or particle but in exactly opposite directions, the particle remains at rest.*

Thus the force which will just support vertically the weight of one pound may be said to be "a force equal to one pound weight", or, more simply, "a force of one pound". Since the forces which occur most frequently in Statics are those due to weight, *we shall adopt the force of one pound as our statical unit of force.* (Any other weight or "gravitational" unit, such as the ton, the gram or the kilogram, may of course be used if more convenient.)

3. The Resolution of Forces. On the assumption that the reader is already acquainted with the elementary work in Statics which was included in Vol. I, it will be unnecessary to repeat that work in detail. As a convenient first step in our wider and more general consideration of the laws of statical equilibrium we may, however, reconsider the question of the resolution of forces, indicating at the same time the way in which the graphical methods described in Vol. I may be usefully supplemented by arithmetical and trigonometrical methods.

Definitions. In this chapter we will limit our treatment to systems of concurrent forces. Forces are **Concurrent** when their directions meet at a point, and **Non-concurrent** when they do not meet.

We have already seen (Vol. I, Chap. VI) that, where two (or more) forces are applied to the same particle—or have the same point of application—it is possible to obtain a single force, or **Resultant**, which would have the same effect upon the particle as the forces which it replaces.

Conversely, given a single force it is possible to "resolve" that force into two (or more) component forces, or **Components**, acting along any two (or more) lines drawn through the point of application of the single force.

If in the first case a force is applied to the particle which is equal in magnitude but acts in the opposite direction to the resultant, then this force is known as the **Equilibrant.** When acting with the original forces the equilibrant produces a state of rest or equilibrium of the particle.

© in this web service Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-1-107-66947-5 - Experimental Building Science: Volume Two: Being an Introduction to Mechanics and its Application in the Design and Erection of Buildings
J. Leask Manson and Francis E. Drury
Excerpt
More information

I. The Parallelogram of Forces. If two forces having the same point of application be represented in magnitude and direction by the adjacent sides of a parallelogram drawn from (or to) their point of application, then their resultant shall be represented by the diagonal of the parallelogram drawn from (or to) that point.

When the lines along which the components of a given force act are at right angles to each other, then the components are known as the **Rectangular Components** of the force; see Fig. 1. This is an important case and of frequent occurrence; the following examples deal with rectangular components.

Example 1. *A force of* 10 *lbs. acts at* O *along the line* OM *and away from* O, *as shown in Fig.* 1. *It is required to find its components along the lines* ON *and* OP, *which are at right angles to each other.*

(*a*) **Graphical solution.** Mark off OM to a suitable scale to represent 10 lbs. Draw MN parallel to OP and MP parallel to NO. Then by the converse of the Parallelogram of Forces the components of the force OM are the force ON, acting vertically along the line ON, and the force OP, acting horizontally along the line OP. The magnitude of each of these forces is obtained by measuring the lengths of the lines ON and OP respectively on the force scale used in marking off the force OM.

By actual measurement we get:

$$\text{Component force } OP = 8 \text{ lbs.}$$
$$\text{Component force } ON = 6 \text{ lbs.}$$

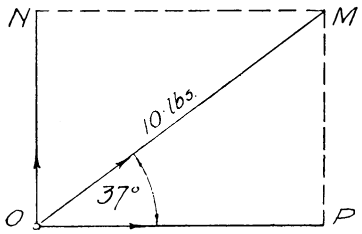

Fig. 1.

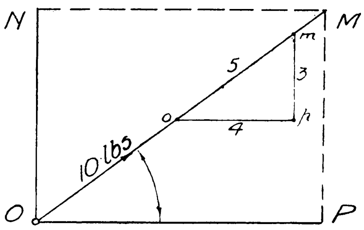

Fig. 2.

(*b*) **Trigonometrical solution.** See Fig. 1. Since the triangle MOP is a right-angled triangle, we have that

$$OP = OM \times \text{cosine } 37°$$
$$= 10 \times 0{\cdot}7986$$
$$= 7{\cdot}986 \text{ lbs.}$$

Similarly $$ON = PM = OM \times \text{sine } 37°$$
$$= 10 \times 0{\cdot}6018$$
$$= 6{\cdot}018 \text{ lbs.}$$

(*c*) **Arithmetical solution.** See Fig. 2. As will be found at a later stage, this is a rapid and powerful method for dealing with framed structures having rectangular outlines, the relations between the forces acting in the various members being obtained from a consideration of their relative lengths. For the sake of simplicity the ratios of the sides of the

© in this web service Cambridge University Press www.cambridge.org

Cambridge University Press
978-1-107-66947-5 - Experimental Building Science: Volume Two: Being an Introduction to Mechanics and its Application in the Design and Erection of Buildings
J. Leask Manson and Francis E. Drury
Excerpt
More information

small triangle *mop*, which gives the inclination of the force OM, have, in the present example, been expressed in whole figures which give an inclination only approximately equal to 37°.

Since the figure $OPMN$ is a rectangle, $MP = ON$.

Also, since triangles MOP and *mop* are similar triangles, evidently

$$\frac{OP}{OM} = \frac{op}{om} = \frac{4}{5}.$$

Now OM represents 10 lbs., therefore

$$OP = \frac{4 \times OM}{5} = \frac{40}{5} = 8 \text{ lbs.}$$

Similarly $$ON = MP = \frac{mp}{om} \times OM = \frac{3 \times 10}{5} = 6 \text{ lbs.}$$

Example 2. *To find a force from its rectangular components. In Fig. 2 let* ON *and* OP *be the known rectangular components of the force* OM, *of which we know only the direction and require to know the magnitude.*

Since the figure $OPMN$ is a rectangle, $MP = ON$. Also ON is the vertical component of the force OM.

Since the triangle MOP is a right-angled triangle, we know that

$$OM^2 = OP^2 + PM^2$$
$$= OP^2 + ON^2;$$

hence we have that:

$$(\text{force } OM)^2 = (\text{component } OP)^2 + (\text{component } ON)^2,$$

or $$\textbf{Force OM} = \sqrt{\textbf{OP}^2 + \textbf{ON}^2}. \qquad \text{......(i)}$$

Example 3. *Rectangular component of a force along a line at right angles to its own length.*

Let the force OM in Fig. 3 act at O in the line OP, OM being at right angles to OP.

Using Fig. 1, the vertical component ON in this case coincides with and is equal to the force OM, hence the horizontal component along OP must be equal to zero, or, *the rectangular component of a force along a line at right angles to its own direction is zero.*

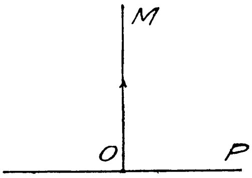

Fig. 3.

4. The rectangular components of two forces and their resultant. See Fig. 4 (A). Let F_1 and F_2 be any two forces and R be their resultant, obtained from the parallelogram $ONMP$, and let OX and OY be *any* two axes at right angles to each other.

Draw Nn, Pp and Mm at right angles to OX, and draw Pp' at right angles to OY. Then On, Op and Om are the rectangular components of F_1, F_2 and R along the axis OX.

Now since in the triangles ONn and PMp' the sides ON and PM are equal and the corresponding sides of each triangle are parallel, therefore the triangles are equal and $On = Pp' = pm$. Therefore

Om, the rectangular component of R, $= Op + pm = Op + On$, or
$=$ the sum of the rectangular components of F_1 and F_2. ...(a)

© in this web service Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-1-107-66947-5 - Experimental Building Science: Volume Two: Being an Introduction to Mechanics and its Application in the Design and Erection of Buildings
J. Leask Manson and Francis E. Drury
Excerpt
More information

In the case illustrated in Fig. 4 (B), where OY falls within the angle made by the two forces F_1 and F_2, if components acting towards the left be called negative, we have, since $On = Pp' = pm$,

$$Om = Op - pm = Op - On; \qquad \text{......(b)}$$

or, the rectangular component of R is equal to the *algebraic* sum of the rectangular components of the forces F_1 and F_2.

Again, the above relation must hold even when there are more than two concurrent forces, since in such a case we may first find the resultant of one pair of forces and then add this resultant to

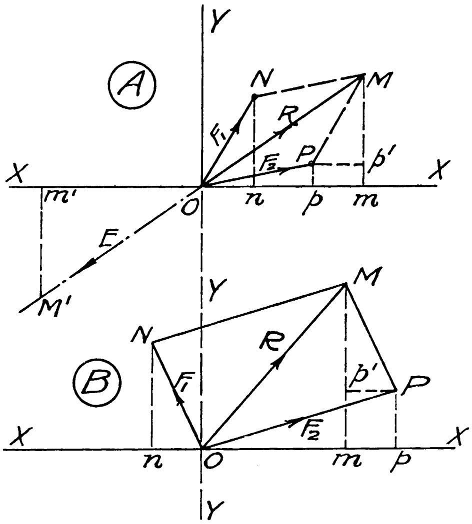

Fig. 4.

the next force, and so on until we are finally left with two forces and their resultant. Hence we may make the following general statement:

II. The algebraic sum of the rectangular components of two (or more) concurrent forces in any direction is equal to the rectangular component of their resultant in the same direction.

5. The rectangular components of a system of forces in equilibrium. In the case of a system of forces which is in equilibrium, since the resultant is zero the algebraic sum of the rectangular components in *any* direction must of necessity be also zero. We may, however, consider the simple case shown in Fig. 4 (A), when we replace R by the equilibrant E; then, since E is equal and opposite to R, Om', the rectangular component of E, must be

© in this web service Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-1-107-66947-5 - Experimental Building Science: Volume Two: Being an Introduction to Mechanics and its Application in the Design and Erection of Buildings
J. Leask Manson and Francis E. Drury
Excerpt
More information

equal and opposite to Om, that is to $(Op + On)$, see (a) in para. 4. Hence in this case the algebraic sum of the rectangular components is zero. The same may be similarly shown to hold for larger systems. Hence:

III. If a system of concurrent forces be in equilibrium, the algebraic sum of the rectangular components of the forces in any direction is zero.

Example 1. See Fig 5. *Let the three forces* MO, NO *and* PO *acting at* O *be in equilibrium. If* MO *be a force of* 1000 *lbs. of the given inclination and the directions of* NO *and* PO *are as indicated, find the magnitude of the force* NO *and of the force* PO.

Produce PO and NO to X and Y respectively.

Mark off MO to represent force MO to some suitable scale.

Draw MR and MS parallel to OX and OY respectively.

By the methods previously adopted it is easy to show that RO, which is the vertical component of MO, equals 600 lbs.

Similarly SO, which is the horizontal component of MO, equals 800 lbs.

Then, for equilibrium along the vertical line NOY, force NO must be equal to force RO (600 lbs.) and act upwards.

Similarly, for equilibrium along the horizontal axis XOP, force PO must be equal to force SO (800 lbs.) and act towards the right.

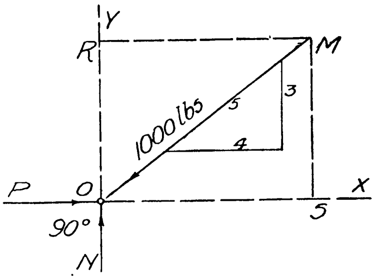

Fig. 5.

Example 2. See Fig. 6 (A). *Let the three forces* OM, NO *and* OP *acting at the point* O *be in equilibrium. The magnitude of the force* OM *being given, find the magnitudes of the other two forces.*

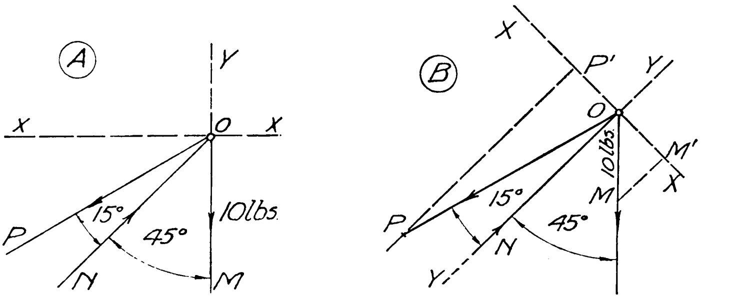

Fig. 6.

It will be evident from an inspection of the diagram that the vertical components of the inclined forces NO and OP acting along the line MOY must together balance the vertical force OM, but since it is impossible at

© in this web service Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-1-107-66947-5 - Experimental Building Science: Volume Two: Being an Introduction to Mechanics and its Application in the Design and Erection of Buildings
J. Leask Manson and Francis E. Drury
Excerpt
More information

this stage to say what proportion of the total force would be contributed by NO and what by OP we must proceed differently. Instead of using vertical and horizontal axes we may use one of the inclined forces (NO) as the OY axis and draw the OX axis at right angles to it; see Fig. 6 (B).

If OM and OP represent these two forces to some scale and OM' and OP' are their components along the OX axis, then it is clear that, for equilibrium along this axis, these two components must be equal.

Mark off OM to some suitable scale to represent force OM.

Draw MM' parallel to YOY.

By construction angle MOM' is an angle of 45°, therefore

$$\begin{aligned} OM' &= OM \times \text{cosine } 45° = 10 \times 0{\cdot}707 \\ &= 7{\cdot}07 \text{ lbs.} \qquad \ldots\ldots\text{(a)} \end{aligned}$$

Hence OP' must also represent a force of 7·07 lbs.

By construction angle POP' is an angle of 75°, therefore

$$\begin{aligned} P'P &= OP' \times \text{tangent } 75° \\ &= 7{\cdot}07 \times 3{\cdot}732 \\ &= 26{\cdot}4 \text{ lbs.} \qquad \ldots\ldots\text{(b)} \end{aligned}$$

The magnitude of force OP can now be obtained from (a) and (b); since

$$(OP)^2 = (OP')^2 + (P'P)^2,$$

therefore

$$\begin{aligned} OP &= \sqrt{7{\cdot}07^2 + 26{\cdot}4^2} \\ &= 27{\cdot}3 \text{ lbs.} \end{aligned}$$

Consider next the components of the forces which act along the line NOY; there are evidently three such components, viz. NO acting towards O, a force equal to $P'P$ and representing the component of OP acting away from O, and also a force equal to $M'M$ and representing the component of OM acting in the same direction as $P'P$.

Since the force NO acts in the opposite direction to the components $P'P$ and $M'M$, NO will be equal in magnitude to the sum of $P'P$ and $M'M$.

Therefore $\qquad NO = 26{\cdot}4 + 7{\cdot}07 = 33{\cdot}47$ lbs.

6. The general conditions of equilibrium for systems of concurrent forces. In the two examples which were worked out in the preceding paragraph we saw that, if the algebraic sum of the components of three forces in equilibrium were taken along two axes at right angles to each other, then the sum in each of these directions was zero. This is an important result and one of quite general application. Before summarising it, however, in a more precise form, we will consider the case from another aspect.

Case I. *A system of three concurrent forces in equilibrium.*

IV. The following statement of the **Triangle of Forces** is taken from Vol. I.

If three forces having the same point of application are in equilibrium, then any triangle whose sides are parallel to the directions of the forces will have the lengths of those sides proportional to the magnitudes of the forces.

The Converse of the Triangle of Forces is equally true.

© in this web service Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-1-107-66947-5 - Experimental Building Science: Volume Two: Being an Introduction to Mechanics and its Application in the Design and Erection of Buildings
J. Leask Manson and Francis E. Drury
Excerpt
More information

In Fig. 7 let the three forces *OM*, *ON* and *OP* be in equilibrium. The forces being fully defined, let a force triangle *abc* be drawn in the usual way, the sense or direction of each force being indicated thereon.* Enclose the force triangle in *any* rectangle *a′bb′c′* as shown.

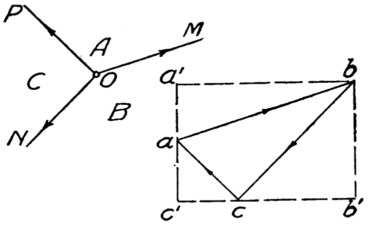

Fig. 7.

Then the horizontal component of force *ab* is represented by *a′b* and acts towards the right. Similarly the horizontal components of forces *bc* and *ca* are *b′c* and *cc′* respectively and act towards the left.

Since *a′bb′c′* is a rectangle, the lengths *b′c* and *cc′* are together equal to *a′b*; hence the algebraic sum of the "horizontal" components of the three forces is zero.

Similarly *c′a* and *aa′*, the components acting upwards, are together equal to *bb′*, the component acting downwards; hence the algebraic sum of the "vertical" components is also zero.

Case II. *A system having more than three forces.*

V. The following statement of the **Polygon of Forces** is taken from Vol. I.

If any number of forces having the same point of application are in equilibrium, then a closed polygon may be drawn whose sides shall represent these forces in magnitude and direction.

The Converse of the Polygon of Forces, though important, cannot be expressed in the same general terms, since the polygon cannot be drawn if the magnitudes of more than two of the forces are unknown, but we can say that, *for equilibrium, the Force Polygon must be a "closed" figure.*

Let the five forces given in Fig. 8 be in equilibrium and let a corresponding force polygon *abcde* be drawn to a suitable scale, the sense or direction of each of the forces being indicated as before.

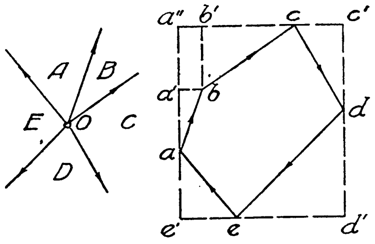

Fig. 8.

Enclose this figure in *any* rectangle *a″c′d′e′* and draw the lines *ba′* and *bb′* parallel to the sides of the rectangle.

Then, as before, the "horizontal" components of the forces *ab*, *bc* and *cd* are *a″b′*, *b′c* and *cc′* and act towards the right.

Similarly the "horizontal" components of the remaining forces *de* and *ea* are *d′e* and *ee′* and act towards the left.

* For the lettering of the forces and force triangle see Bow's Notation in Vol. I, Chap. VI. See also para. 17.

© in this web service Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-1-107-66947-5 - Experimental Building Science: Volume Two: Being an Introduction to Mechanics and its Application in the Design and Erection of Buildings
J. Leask Manson and Francis E. Drury
Excerpt
More information

Since the figure $a''c'd'e'$ is a rectangle, it is evident that $a''c'$ (that is, the sum of the "horizontal" components acting to the right) is equal to $d'e'$ (that is, the sum of the components acting to the left), from which it follows that the algebraic sum of the "horizontal" components must be zero.

In a similar manner, if we consider the "vertical" components acting upwards and those acting downwards, we may show that the algebraic sum of the "vertical" components is likewise zero.

The above proof is quite general, since it may be shown to be unaffected by the shape of the polygon or the direction of the sides of the enclosing rectangle.

Hence it would appear that what we may call the arithmetical or mathematical method of checking the equilibrium of a system of concurrent forces, by equating the rectangular components to zero, is *equivalent* to the graphical method, which requires that the force polygon must be "closed" to ensure that the system is in equilibrium.

In practice either method may be used which is most convenient, or one may be used as check upon the other.

The same results would of course be obtained if the two axes selected formed any other angle than a right angle, provided always that the rectangular components are taken in the two directions selected (see para. 5, III), and that the axes do not coincide. It is, however, usually more convenient to use two axes at right angles. In all cases we shall continue to refer to the "vertical" and "horizontal" components. No confusion need arise if these terms are taken to include the case of rectangular components in two directions not at right angles to each other.

VI. Conditions of equilibrium for systems of concurrent coplanar forces.

In order that a particle, acted on by a system of concurrent coplanar forces, may be in equilibrium it is necessary and sufficient that the sum of the components of these forces in each of two directions shall be separately zero.

Or, alternatively, **the Force Polygon must close.**

More briefly,

If V and H stand for the "vertical" and "horizontal" components of these forces respectively, and the sign Σ stands for "the algebraic sum of all such quantities as...",

then, **for equilibrium:**

The sum of the "vertical" components must be equal to zero, or $\Sigma V = 0$;

Similarly the sum of the "horizontal" components must also be equal to zero, or $\Sigma H = 0$.

© in this web service Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-1-107-66947-5 - Experimental Building Science: Volume Two: Being an Introduction to Mechanics and its Application in the Design and Erection of Buildings
J. Leask Manson and Francis E. Drury
Excerpt
More information

The principles enunciated above may be applied in the solution of the following problems, the sum of the components in each of two directions being equated to zero.

Problems I

(**Note.** As each of the following problems contains a special difficulty which may be met with in practice, they should be worked in the order given.)

1. Fig. *A* represents the three forces acting at the foot of a roof truss. The magnitude and sense of the force *RO* is given and the sense of the force *SO* is known. Find the magnitude of the force *SO* and the magnitude and sense of the force acting along the line *TO*.

2. Fig. *B* gives the sense and direction of the forces acting at the head of a jib crane. The magnitude of the load being supported by the chain is 10 tons. Find the magnitudes of the forces acting in the jib and the tie.

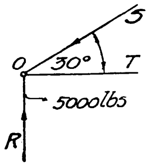

Fig. *A*.

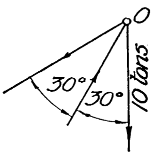

Fig. *B*.

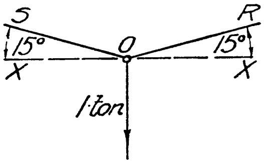

Fig. *C*.

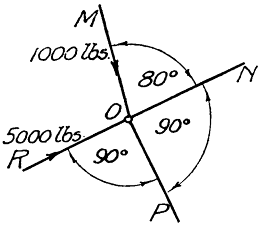

Fig. *D*.

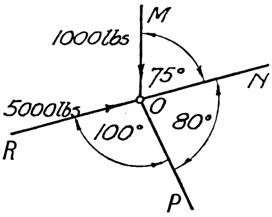

Fig. *E*.

3. Fig. *C* represents a load supported by two chains equally inclined to the horizontal. Find the forces acting in the two chains.

4. Fig. *D* represents the forces acting at a joint *O* in a roof truss. The sense and magnitude of each of two forces are given and the directions of the two remaining forces. Find the magnitude and sense of the forces in *NO* and *PO*.

5. Fig. *E* represents a similar case to that shown in Fig. *D* with the exception that the member *OP* is not at right angles to the line *RON*. Find the magnitude and sense of the force in *NO* and in *PO*.

© in this web service Cambridge University Press www.cambridge.org