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Greg Byrd Lynn Byrd and Chris Pearce
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1 Integers, powers and roots

Mathematics is about finding patterns.

How did you first learn to add and multiply negative integers? Perhaps you started with an addition table or a multiplication table for positive integers and then extended it. The patterns in the tables help you to do this.

Key words

Make sure you learn and understand these key words:

power
index (indices)

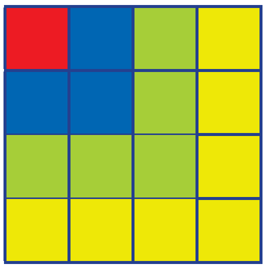
+	3	2	1	0	−1	−2	−3
3	6	5	4	3	2	1	0
2	5	4	3	2	1	0	−1
1	4	3	2	1	0	−1	−2
0	3	2	1	0	−1	−2	−3
−1	2	1	0	−1	−2	−3	−4
−2	1	0	−1	−2	−3	−4	−5
−3	0	−1	−2	−3	−4	−5	−6

This shows
$1 + -3 = -2$.
You can also subtract.
$-2 - 1 = -3$ and
$-2 - -3 = 1$.

×	3	2	1	0	−1	−2	−3
3	9	6	3	0	−3	−6	−9
2	6	4	2	0	−2	−4	−6
1	3	2	1	0	−1	−2	−3
0	0	0	0	0	0	0	0
−1	−3	−2	−1	0	1	2	3
−2	−6	−4	−2	0	2	4	6
−3	−9	−6	−3	0	3	6	9

This shows
$2 \times -3 = -6$.
You can also divide.
$-6 \div 2 = -3$ and
$-6 \div -3 = 2$.

Square numbers show a visual pattern.
$1 + 3 = 4 = 2^2$
$1 + 3 + 5 = 9 = 3^2$
$1 + 3 + 5 + 7 = 16 = 4^2$
Can you continue this pattern?



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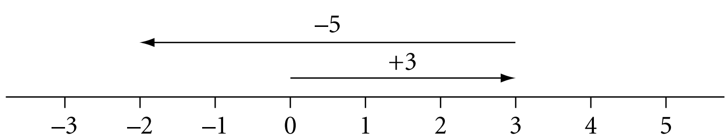
1.1 Directed numbers

Directed numbers have direction; they can be positive or negative. Directed numbers can be integers (whole numbers) or they can be decimal numbers.

Here is a quick reminder of some important things to remember when you add, subtract, multiply and divide integers. These methods can also be used with any directed numbers.

What is 3 + −5?



Think of a number line. Start at 0. Moving 3 to the right, then 5 to the left is the same as moving 2 to the left.

Or you can change it to a subtraction: 3 + −5 = 3 − 5.

Either way, the answer is −2.

add negative → subtract positive
subtract negative → add positive

What about 3 − −5?

Perhaps the easiest way is to add the inverse.

3 − −5 = 3 + 5 = 8

What about multiplication?

3 × 5 = 15   3 × −5 = −15   −3 × 5 = −15   −3 × −5 = 15

Multiply the corresponding positive numbers and decide whether the answer is positive or negative.

Division is similar.

15 ÷ 3 = 5   −15 ÷ 3 = −5   −15 ÷ −3 = 5   15 ÷ −3 = −5

Remember for multiplication and division:
same signs → positive answer
different signs → negative answer

These are the methods for integers.

You can use exactly the same methods for any directed numbers, even if they are not integers.

Worked example 1.1

Complete these calculations. **a** 3.5 + −4.1 **b** 3.5 − −2.8 **c** 6.3 × −3 **d** −7.5 ÷ −2.5

a	3.5 − 4.1 = −0.6	You could draw a number line but it is easier to subtract the inverse (which is 4.1).
b	3.5 + 2.8 = 6.3	Change the subtraction to an addition. Add the inverse of −2.8 which is 2.8.
c	6.3 × −3 = −18.9	First multiply 6.3 by 3. The answer must be negative because 6.3 and −3 have opposite signs.
d	−7.5 ÷ −2.5 = 3	7.5 ÷ 2.5 = 3. The answer is positive because −7.5 and −2.5 have the same sign.

Exercise 1.1

Do not use a calculator in this exercise.

1 Work these out.
a 5 + −3 **b** 5 + −0.3 **c** −5 + −0.3 **d** −0.5 + 0.3 **e** 0.5 + −3

2 Work these out.
a 2.8 + −1.3 **b** 0.6 + −4.1 **c** −5.8 + 0.3 **d** −0.7 + 6.2 **e** −2.25 + −0.12

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3 Work these out.
a $7 - -4$ **b** $-7 - 0.4$ **c** $-0.4 - -7$ **d** $-0.4 - 0.7$ **e** $-4 - -0.7$

4 Work these out.
a $2.8 - -1.3$ **b** $0.6 - -4.1$ **c** $-5.8 - 0.3$ **d** $-0.7 - 6.2$ **e** $-2.25 - -0.12$

5 The midday temperature, in Celsius degrees (°C), on four successive days is 1.5, −2.6, −3.4 and 0.5. Calculate the mean temperature.

6 Find the missing numbers.
a $\square + 4 = 1.5$ **b** $\square + -6.3 = -5.9$ **c** $4.3 + \square = -2.1$ **d** $12.5 + \square = 3.5$

7 Find the missing numbers.
a $\square - 3.5 = -11.6$ **b** $\square - -2.1 = 4.1$ **c** $\square - 8.2 = 7.2$ **d** $\square - -8.2 = 7.2$

8 Copy and complete this addition table.

+	−3.4	−1.2
5.1		
	−4.7	

9 Use the information in the box to work these out.

$2.3 \times 9.6 = 22.08$

a -2.3×-9.6 **b** $-22.08 \div 2.3$ **c** $22.08 \div -9.6$
d -4.6×-9.6 **e** $-11.04 \div -2.3$

10 Work these out.
a 2.7×-3 **b** $2.7 \div -3$ **c** -1.2×-1.2 **d** -3.25×-4 **e** $17.5 \div -2.5$

11 Copy and complete this multiplication table.

×	3.2	−0.6
−1.5		
		1.5

12 Complete these calculations.
a -2×-3 **b** $(-2 \times -3) \times -4$ **c** $(-3 \times 4) \div -8$

13 Use the values given in the box to work out the value of each expression.

$p = -4.5 \quad q = 5.5 \quad r = -7.5$

a $p - q$ **b** $(p + q) \times r$
c $(q + r) \times p$ **d** $(r - q) \div (q - p)$

14 Here is a multiplication table.
Use the table to calculate these.
a $(-2.4)^2$ **b** $13.44 \div -4.6$
c $-16.1 \div -3.5$ **d** $-84 \div 2.4$

×	2.4	3.5	4.6
2.4	5.76	8.4	13.44
3.5	8.4	12.25	16.1
4.6	13.44	16.1	21.16

15 p and q are numbers, $p + q = 1$ and $pq = -20$. What are the values of p and q?

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1.2 Square roots and cube roots

You should be able to recognise:

- the squares of whole numbers up to 20×20 and their corresponding square roots
- the cubes of whole numbers up to $5 \times 5 \times 5$ and their corresponding cube roots.

Only squares or cubes of integers have integer square roots or cube roots.

You can use a calculator to find square roots and cube roots, but you can estimate them without one.

Worked example 1.1

Estimate each root, to the nearest whole number. **a** $\sqrt{295}$ **b** $\sqrt[3]{60}$

a $17^2 = 289$ and $18^2 = 324$

295 is between 289 and 324 so $\sqrt{295}$ is between 17 and 18.

$\sqrt{295}$ is 17 to the nearest whole number.

It will be a bit larger than 17.

b $3^3 = 27$ and $4^3 = 64$

60 is between 27 and 64 so $\sqrt[3]{60}$ is between 3 and 4.

$\sqrt[3]{60}$ is 4, to the nearest whole number.

It will be a bit less than 4. A calculator gives 3.91 to 2 d.p.

Exercise 1.2

Do not use a calculator in this exercise, unless you are told to.

1 Read the statement on the right. Write a similar statement for each root.

a $\sqrt{20}$ **b** $\sqrt{248}$ **c** $\sqrt{314}$ **d** $\sqrt{83.5}$ **e** $\sqrt{157}$

$2 < \sqrt{8} < 3$

2 Explain why $\sqrt[3]{305}$ is between 6 and 7.

3 Estimate each root, to the nearest whole number.

a $\sqrt{171}$ **b** $\sqrt{35}$ **c** $\sqrt{407}$ **d** $\sqrt{26.3}$ **e** $\sqrt{292}$

4 Read the statement on the right. Write a similar statement for each root.

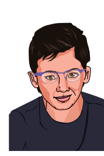
a $\sqrt[3]{100}$ **b** $\sqrt[3]{222}$ **c** $\sqrt[3]{825}$ **d** $\sqrt[3]{326}$ **e** $\sqrt[3]{58.8}$

$10 < \sqrt[3]{1200} < 11$

5 What Ahmad says is not correct.

a Show that $\sqrt{160}$ is between 12 and 13.

b Write down the number of which 40 is square root.



6 a Find $\sqrt{1225}$. **b** Estimate $\sqrt[3]{1225}$ to the nearest whole number.

$35^2 = 1225$

7 Show that $\sqrt[3]{125}$ is less than half of $\sqrt{125}$.

8 Use a calculator to find these square roots and cube roots.

a $\sqrt{625}$ **b** $\sqrt{20.25}$ **c** $\sqrt{46.24}$ **d** $\sqrt[3]{1728}$ **e** $\sqrt[3]{6.859}$

9 Use a calculator to find these square roots and cube roots. Round your answers to 2 d.p.

a $\sqrt{55}$ **b** $\sqrt{108}$ **c** $\sqrt[3]{200}$ **d** $\sqrt[3]{629}$ **e** $\sqrt[3]{10\,000}$

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1.3 Indices

This table shows powers of 3. Look at the patterns in the table.

Power	3^{-4}	3^{-3}	3^{-2}	3^{-1}	3^0	3^1	3^2	3^3	3^4	3^5
Value	$\frac{1}{81}$	$\frac{1}{27}$	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9	27	81	243

> 3^4 is 3 to the **power** 4.
> 4 is called the **index**.
> The plural of index is **indices**.

Negative powers of any positive integer are fractions. Here are some more examples.

$2^4 = 2 \times 2 \times 2 \times 2 = 16$ $\quad 2^{-4} = \frac{1}{16}$ $\quad 7^3 = 7 \times 7 \times 7 = 353$ $\quad 7^{-3} = \frac{1}{343}$

Any positive integer to the power 0 is 1. $\quad 2^0 = 1 \quad 7^0 = 1 \quad 12^0 = 1$

Worked example 1.3

Write these as fractions. **a** 2^{-6} **b** 6^{-2}

a $2^{-6} = \frac{1}{2^6} = \frac{1}{64}$ $\quad 2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$

b $6^{-2} = \frac{1}{6^2} = \frac{1}{36}$ $\quad 6^2 = 36$

Exercise 1.3

1 Write each number as a fraction. **a** 5^{-1} **b** 5^{-2} **c** 5^{-3} **d** 5^{-4}

2 Write each number as a fraction or as an integer.
a 7^2 **b** 7^{-2} **c** 7^{-1} **d** 7^0 **e** 7^3

3 Write each number as a fraction.
a 4^{-1} **b** 10^{-2} **c** 2^{-3} **d** 12^{-1} **e** 15^{-2} **f** 20^{-2}

4 **a** Simplify each number. **i** 2^0 **ii** 5^0 **iii** 10^0 **iv** 20^0
b Write the results in part **a** as a generalised rule.

5 Write each expression as a single number.
a $2^0 + 2^{-1} + 2^{-2}$ **b** $3^2 + 3 + 3^0 + 3^{-1}$ **c** $5 - 5^0 - 5^{-1}$

6 Write each number as a decimal.
a 5^{-1} **b** 5^{-2} **c** 10^{-1} **d** 10^{-2} **e** 10^{-3}

7 Write each number as a power of 2.
a 8 **b** $\frac{1}{2}$ **c** $\frac{1}{4}$ **d** $\frac{1}{16}$ **e** 1

8 $2^{10} = 1024$. In computing this is called 1K. Write each of these as a power of 2.
a 2K **b** 0.5K **c** $\frac{1}{1\text{K}}$

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1.4 Working with indices

You can write the numbers in the boxes as powers.
Look at the indices. 2 + 3 = 5 and 5 + 3 = 8.

9 × 27 = 243	32 × 8 = 256
$3^2 \times 3^3 = 3^5$	$2^5 \times 2^3 = 2^8$

This is an example of a general result.

> To multiply powers of a number, add the indices. $A^m \times A^n = A^{m+n}$

$9 \times 9 = 81 \quad \Rightarrow \quad 3^2 \times 3^2 = 3^4 \quad 2 + 2 = 4$

$4 \times 8 = 32 \quad \Rightarrow \quad 2^2 \times 2^3 = 2^5 \quad 2 + 3 = 5$

The multiplications above can be written as divisions.
You can write the numbers as powers.
Again, look at the indices. 5 − 3 = 2 and 8 − 3 = 5.
This shows that:

243 ÷ 27 = 9	256 ÷ 8 = 32
$3^5 \div 3^3 = 3^2$	$2^8 \div 2^3 = 2^5$

> To divide powers of a number, subtract the indices. $A^m \div A^n = A^{m-n}$

$27 \div 3 = 9 \quad \Rightarrow \quad 3^3 \div 3^1 = 3^2 \quad 3 - 1 = 2$

$4 \div 8 = \frac{1}{2} \quad \Rightarrow \quad 2^2 \div 2^3 = 2^{-1} \quad 2 - 3 = -1$

Worked example 1.4

a Write each expression as a power of 5. **i** $5^2 \times 5^3$ **ii** $5^2 \div 5^3$
b Check your answers by writing the numbers as decimals.

a **i** $5^2 \times 5^3 = 5^{2+3} = 5^5$ — $2 + 3 = 5$
ii $5^2 \div 5^3 = 5^{2-3} = 5^{-1} = \frac{1}{5}$ — $2 - 3 = -1$
b **i** $25 \times 125 = 3125$ — 3125 is 5^5
ii $25 \div 125 = \frac{1}{5} = 0.2$

Exercise 1.4

1 Simplify each expression. Write your answers in index form.
a $5^2 \times 5^3$ **b** $6^4 \times 6^3$ **c** $10^4 \times 10^2$ **d** $a^2 \times a^2 \times a^3$ **e** $4^5 \times 4$

2 Simplify each expression. Leave your answers in index form where appropriate.
a $2^5 \times 2^3$ **b** $8^2 \times 8^4$ **c** $a^3 \times a^2$ **d** $2^3 \times 2^3$ **e** $b^3 \times b^4$

3 Simplify each expression.
a $3^5 \div 3^2$ **b** $k^4 \div k^3$ **c** $10^6 \div 10^4$ **d** $5^2 \div 5^4$ **e** $7 \div 7^1$

4 Simplify each expression.
a $2^2 \div 2^2$ **b** $2^2 \div 2^3$ **c** $2^2 \div 2^4$ **d** $2^4 \div 2^2$ **e** $2^4 \div 2^6$

5 Write each expression as a power or fraction.
a $8^3 \times 8^4$ **b** $5^2 \times 5$ **c** $4^2 \times 4^4$ **d** $9^2 \div 9^3$ **e** $12^2 \div 12^4$

6 Find the value of N in each part.
a $10^2 \times 10^N = 10^4$ **b** $10^2 \div 10^N = 10$ **c** $10^2 \times 10^N = 10^7$ **d** $10^2 \div 10^N = 10^{-1}$

7 This table shows values of powers of 7.
Use the table to find the value of:
a 49×2401 **b** $16\,807 \div 343$ **c** 343^2.

7^1	7^2	7^3	7^4	7^5	7^6
7	49	343	2401	16 807	117 649

8 **a** Write the numbers in the box as powers of 4. Check that the division rule for indices is correct.
b Write the numbers as powers of 2 and check that the division rule for indices is correct.

$1024 \div 16 = 64$

9 **a** Write 9 and 243 as powers of 3.
b Use your answers to part **a** to find, as powers of 3: **i** 9×243 **ii** $9 \div 243$.

10 Simplify each fraction.
a $\frac{2^3 \times 2^4}{2^5}$ **b** $\frac{a^3 \times a^2}{a^2}$ **c** $\frac{d^3 \times d}{d^1}$ **d** $\frac{10^6 \times 10^4}{10^2 \times 10^3}$

11 **a** Write each of these as a power of 2.
i $(2^2)^2$ **ii** $(2^2)^3$ **iii** $(2^4)^2$ **iv** $(2^4)^3$ **v** $(2^2)^4$
b What can you say about $(2^m)^n$ if m and n are positive integers?

12 In computing, $1\text{K} = 2^{10} = 1024$. Write each of these in K.
a 2^{12} **b** 2^{15} **c** 2^{20} **d** 2^7

13 Find the value of n in each equation.
a $3^n \times 3^2 = 81$ **b** $5^n \times 25 = 625$ **c** $2^n \div 2 = 8$ **d** $n^2 \times n = 216$

Summary

You should now know that:

- ★ You can add, subtract, multiply or divide directed numbers in the same way as integers.
- ★ Using inverses can simplify calculations with directed numbers.
- ★ Only square numbers or cube numbers have square roots or cube roots that are integers.
- ★ $A^0 = 1$ if A is a positive integer.
- ★ $A^{-n} = \frac{1}{A^n}$ if A and n are positive integers.
- ★ $A^m \times A^n = A^{m+n}$
- ★ $A^m \div A^n = A^{m-n}$

You should be able to:

- ★ Add, subtract, multiply and divide directed numbers.
- ★ Estimate square roots and cube roots.
- ★ Use positive, negative and zero indices.
- ★ Use the index laws for multiplication and division of positive integer powers.
- ★ Use the rules of arithmetic and inverse operations to simplify calculations.
- ★ Calculate accurately, choosing operations and mental or written methods appropriate to the number and context.
- ★ Manipulate numbers and apply routine algorithms.

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End-of-unit review

1 Complete these additions.
a $-3 + 6$ **b** $12 + -14.5$ **c** $-3.5 + -5.7$ **d** $-3.6 + 2.8 + -1.3$

2 Subtract.
a $12 - -4$ **b** $-6.4 - 8.3$ **c** $3.7 - -8.3$ **d** $-5.1 - -5.2$

3 $2.5 \times 4.5 = 11.25$. Use this to find the value of each expression.
a -2.5×-4.5 **b** $-11.25 \div -4.5$ **c** -4.5×1.25

4 Solve these equations.
a $x + 17.8 = 14.2$ **b** $y - 3.4 = -9.7$ **c** $3y + -4.9 = 2.6$

5 Look at the statement in the box. Write a similar statement for each number.
a $\sqrt{111}$ **b** $\sqrt{333}$ **c** $\sqrt{111}$ **d** $\sqrt[3]{333}$

$4 < \sqrt{19} < 5$

6 a Estimate $\sqrt{200}$ to the nearest whole number.
b Estimate $\sqrt[3]{200}$ to the nearest whole number.

7 Choose the number that is closest to $\sqrt{250}$.
14.9 15.1 15.4 15.8 16.2

8 Choose the number that is closest to $\sqrt[3]{550}$.
7.6 7.8 8.2 8.5 8.8

9 Show that $\sqrt{1000}$ is more than three times $\sqrt[3]{1000}$.

10 Write each of these numbers as a decimal.
a 2^{-1} **b** 4^{-1} **c** 2^{-2} **d** 5^{-2}

11 Write each number as a fraction.
a 3^{-2} **b** 2^{-3} **c** 6^{-1} **d** 12^{-2}

12 Write each expression as a single number.
a $2^2 + 2^0 + 2^{-2}$ **b** $10^{-1} + 10^0 + 10^3$

13 Write each number as a power of 10.
a 100 **b** 1000 **c** 0.01 **d** 0.001 **e** 1

14 Write each expression as a single power.
a $9^2 \times 9^3$ **b** 8×8^2 **c** $7^5 \div 7^2$ **d** $a \div a^3$ **e** $n^1 \div n^2$

15 Simplify each expression.
a $2^4 \div 2^5$ **b** $15^0 \times 15^2$ **c** $20^5 \div 20^3$ **d** $5^2 \div (5^3 \times 5^1)$

16 Write each expression as a power of a.
a $a^2 \times a^4$ **b** $a^2 \div a^4$ **c** $a^2 \times a^0$ **d** $a^1 \times a^4$ **e** $a^2 \div a^4$

17 Simplify each expression.
a $\frac{4^2 \times 4^4}{4^3}$ **b** $\frac{a^2}{a^3 \times a}$ **c** $\frac{n^2 \times n^1}{n^2}$

18 Find the value of n in each of these equations.
a $4^n = 1$ **b** $5^n = 0.2$ **c** $n \times n^2 = 343$ **d** $2^4 \div 2^n = 4$