PRINCIPLES OF QUANTUM MECHANICS

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by

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PREFACE

It is the aim of this book to develop the principles of quantum mechanics on the basis of a few standard observations. In this way we hope to succeed in eliminating from the customary interpretation of the theory some unphysical ideas that have no counterpart in empirical facts. Such a task would be quite trivial in the case of classical mechanics whose path, from the eighteenth century down to Einstein's relativization of the absolute time, was marked by a gradual elimination of anthropomorphic concepts. In quantum mechanics, however, only twelve years have passed since this theory was introduced as a cryptic technique of mathematical operations with non-commutative quantities, corresponding to a still more mysterious behaviour of matter whose particles seemed to disregard the laws of mechanics in favour of wave rules. But in spite of the Heisenberg principle of uncertainty which clarified so much of the physical content of quantum mechanics, and in spite of the mathematical perfection of the theory, there seems still some work to be done before the interpretation of the formulae satisfy all requirements of consistency. In this respect we can learn a great deal from the general theory of relativity, as may be seen from the following list of analogies between relativity and quantum mechanics.

(*a*) In Einstein's theory one describes a phenomenon, for instance, the motion of a falling stone, in two equivalent ways: one either derives the curved path of the stone from the forces of gravity, or ascribes it to the inertia with respect to an accelerated frame of co-ordinates. There is then a unique mathematical relation between the two descriptions.

(*b*) It would contradict however the very idea of relativity if one should apply the concepts of both descriptions *simultaneously* —that is, if one should ask for the distribution and magnitude of gravitational forces *within* the accelerated frame.

(*c*) On the other hand, the two explanations of the curved path are *equivalent*—that is, there is a unique mathematical connection between them. The same coefficients $g_{\mu\nu}$ that play the role of coefficients of force in the one interpretation, appear to be metric coefficients in the second interpretation. Since, however, these coefficients g, by virtue of their metric character, have to obey certain inherent differential equations, the same differential equations hold then for the g's in their first role as coefficients of force. In this way Einstein found his fundamental equations of the gravitational field.

Similar considerations apply now to quantum mechanics also:

(*a*) In quantum theory we can explain one and the same phenomenon, for instance the diffraction of light at a grating, in two *equivalent* ways: either we ascribe the diffraction pattern to a periodic distribution of matter in the diffracting instrument, which serves as a Huygens source of secondary interfering light *waves*; or we explain the same diffraction with the help of directed impulses imparted to the incident *photons* by corpuscles of matter supposed to be present in the diffracting apparatus.

(*b*) It would contradict however the basic idea of quantum mechanics if we should apply both ideas *simultaneously*, e.g. if we should inquire for the location of the impulse giving particles of matter and attempt to place them mainly in the beat maxima of the periodic waves of matter (since the latter were introduced hypothetically only for the purpose of explaining the diffraction with the help of light waves). Forgetting that equivalence precludes simultaneity has often led to paradoxical situations in the theory of quanta, for instance, to the question of how a photon is able to know which way it has to go, after having passed through a periodic grating. The answer given first by P. Duane is that from the standpoint of photons the diffracting instrument is not a periodic grating but is an arrangement of matter that gives off momentum only by amounts which are multiples of a certain basic amount.

(*c*) On the other hand, the two explanations of the diffraction pattern with the help of waves and corpuscles are *equivalent*—that is, there is a mathematical relation between the wave data and the corresponding corpuscular data. The most familiar of these relations are the Planck formula $\epsilon = h\nu$ and the de Broglie formula $p = h/\lambda$. It is the aim of the methods of Schrödinger and of Heisenberg-Born-Jordan to connect the wave description with the corpuscular description of the same phenomena in a general way. Now since the waves have certain inherent features—for instance, since a standing wave can have only an integral number of nodes, the same feature will have a bearing on the corresponding feature of corpuscles; their energies and momenta appear to be "quantized". It would, however, be misleading to ask, *within* the frame of the wave picture, for the location of particles. The concepts "waves" and "corpuscles" are complementary, but they cannot be applied simultaneously. If one does it, one commits the same error as a certain book of military instruction which tells us that a bullet falls down for two reasons, firstly because of the attractive force of the earth, secondly because of its own heaviness (meaning probably its inertia in an upwards accelerated system).

In view of the complementarity of the two classical theories and in order to avoid a confusion of their respective ideas, it is a good policy to compare the tentative theoretical explanation of an observed process with a complementary interpretation in which the roles of particles and waves have been interchanged. If the latter interpretation turns out to be objectionable, one is prepared to criticize the former interpretation also, even though it may represent a generally accepted opinion. A similar programme of exploiting the perfect complementarity of the ideas of corpuscles and waves in interpreting the observed facts has been carried through in Heisenberg's University of Chicago lectures on "The Physical Principles of Quantum Theory" (1930), a standard work to which the author is very indebted. The aim of the present book is different from Heisenberg's in that more stress is laid on the mutual dependence of the various principles of quantum theory, and on developing them from the simple theory of observation which leads eventually to the transformation theorems of P. Jordan.

Considerations of relativistic invariance together with Dirac's theory of spinning electron have been omitted from this book. Nor was it convenient to discuss the applications of Pauli's exclusion principle, which is quite alien to the interwoven system of the other principles of quantum mechanics.

It is a pleasure to express my gratitude to my colleagues Jerome B. Green and George H. Shortley for their great help in revising the manuscript, and to the Cambridge University Press for the beautiful typographical work as exemplified in the following pages.

A. L.

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