

Quantum Mechanics

The important changes quantum mechanics has undergone in recent years are reflected in this new approach for students.

A strong narrative and over 300 worked problems lead the student from experiment, through general principles of the theory, to modern applications. Stepping through results allows students to gain a thorough understanding. Starting with basic quantum mechanics, the book moves on to more advanced theory, followed by applications, perturbation methods and special fields, and ending with new developments in the field. Historical, mathematical, and philosophical boxes guide the student through the theory. Unique to this textbook are chapters on measurement and quantum optics, both at the forefront of current research. Advanced undergraduate and graduate students will benefit from this new perspective on the fundamental physical paradigm and its applications.

Online resources including solutions to selected problems and 200 figures, with color versions of some figures, are available at www.cambridge.org/Auletta.

Gennaro Auletta is Scientific Director of Science and Philosophy at the Pontifical Gregorian University, Rome. His main areas of research are quantum mechanics, logic, cognitive sciences, information theory, and applications to biological systems.

Mauro Fortunato is a Structurer at Cassa depositi e prestiti S.p.A., Rome. He is involved in financial engineering, applying mathematical methods of quantum physics to the pricing of complex financial derivatives and the construction of structured products.

Giorgio Parisi is Professor of Quantum Theories at the University of Rome “La Sapienza.” He has won several prizes, notably the Boltzmann Medal, the Dirac Medal and Prize, and the Daniel Heineman prize. His main research activity deals with elementary particles, theory of phase transitions and statistical mechanics, disordered systems, computers and very large scale simulations, non-equilibrium statistical physics, optimization, and animal behavior.

Quantum Mechanics

GENNARO AULETTA

Pontifical Gregorian University, Rome

MAURO FORTUNATO

Cassa Depositi e Prestiti S.p.A., Rome

GIORGIO PARISI

“La Sapienza” University, Rome



CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE
 UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom
 One Liberty Plaza, 20th Floor, New York, NY 10006, USA
 477 Williamstown Road, Port Melbourne, VIC 3207, Australia
 314-321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi - 110025, India
 103 Penang Road, #05-06/07, Visioncrest Commercial, Singapore 238467

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning and research at the highest international levels of excellence.

www.cambridge.org

Information on this title: www.cambridge.org/9781107665897

© G. Auletta, M. Fortunato and G. Parisi 2009

This publication is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press.

First published 2009

First paperback edition 2013

A catalogue record for this publication is available from the British Library

Library of Congress Cataloging in Publication data

Auletta, Gennaro, 1957–

Quantum mechanics : into a modern perspective / Gennaro Auletta,
 Mauro Fortunato, Giorgio Parisi.

p. cm.

Includes bibliographical references and index.

ISBN 978-0-521-86963-8

1. Quantum theory. I. Fortunato, Mauro. II. Parisi, Giorgio. III. Title.
 QC174.12.A854 2009
 530.12–dc22
 2009004303

ISBN 978-0-521-86963-8 Hardback

ISBN 978-1-107-66589-7 Paperback

Cambridge University Press has no responsibility for the persistence or accuracy of URLs for external or third-party internet websites referred to in this publication, and does not guarantee that any content on such websites is, or will remain, accurate or appropriate.

Contents

<i>List of figures</i>	<i>page xi</i>
<i>List of tables</i>	xvii
<i>List of definitions, principles, etc.</i>	xviii
<i>List of boxes</i>	xx
<i>List of symbols</i>	xxi
<i>List of abbreviations</i>	xxxii
<i>Acknowledgements for the revised edition</i>	xxxiii
Introduction	1
Part I Basic features of quantum mechanics	
1 From classical mechanics to quantum mechanics	7
1.1 Review of the foundations of classical mechanics	7
1.2 An interferometry experiment and its consequences	12
1.3 State as vector	20
1.4 Quantum probability	28
1.5 The historical need of a new mechanics	31
Summary	40
Problems	41
Further reading	42
2 Quantum observables and states	43
2.1 Basic features of quantum observables	43
2.2 Wave function and basic observables	68
2.3 Uncertainty relation	82
2.4 Quantum algebra and quantum logic	92
Summary	96
Problems	97
Further reading	99
3 Quantum dynamics	100
3.1 The Schrödinger equation	101
3.2 Properties of the Schrödinger equation	107
3.3 Schrödinger equation and Galilei transformations	111
3.4 One-dimensional free particle in a box	113
3.5 Unitary transformations	117

3.6	Different pictures	125
3.7	Time derivatives and the Ehrenfest theorem	129
3.8	Energy–time uncertainty relation	131
3.9	Towards a time operator	135
	Summary	138
	Problems	139
	Further reading	140
4	Examples of quantum dynamics	141
4.1	Finite potential wells	141
4.2	Potential barrier	145
4.3	Tunneling	150
4.4	Harmonic oscillator	154
4.5	Quantum particles in simple fields	165
	Summary	169
	Problems	170
5	Density matrix	174
5.1	Basic formalism	174
5.2	Expectation values and measurement outcomes	177
5.3	Time evolution and density matrix	179
5.4	Statistical properties of quantum mechanics	180
5.5	Compound systems	181
5.6	Pure- and mixed-state representation	187
	Summary	188
	Problems	189
	Further reading	190

Part II More advanced topics

6	Angular momentum and spin	193
6.1	Orbital angular momentum	193
6.2	Special examples	207
6.3	Spin	217
6.4	Composition of angular momenta and total angular momentum	226
6.5	Angular momentum and angle	239
	Summary	241
	Problems	242
	Further reading	244
7	Identical particles	245
7.1	Statistics and quantum mechanics	245
7.2	Wave function and symmetry	247
7.3	Spin and statistics	249

7.4	Exchange interaction	254
7.5	Two recent applications	255
	Summary	257
	Problems	257
	Further reading	258
8	Symmetries and conservation laws	259
8.1	Quantum transformations and symmetries	259
8.2	Continuous symmetries	264
8.3	Discrete symmetries	266
8.4	A brief introduction to group theory	267
	Summary	275
	Problems	275
	Further reading	276
9	The measurement problem in quantum mechanics	277
9.1	Statement of the problem	278
9.2	A brief history of the problem	284
9.3	Schrödinger cats	291
9.4	Decoherence	297
9.5	Reversibility/irreversibility	308
9.6	Interaction-free measurement	315
9.7	Delayed-choice experiments	320
9.8	Quantum Zeno effect	322
9.9	Conditional measurements or postselection	325
9.10	Positive operator valued measure	327
9.11	Quantum non-demolition measurements	335
9.12	Decision and estimation theory	341
	Summary	349
	Problems	351
	Further reading	353

Part III Matter and light

10	Perturbations and approximation methods	357
10.1	Stationary perturbation theory	357
10.2	Time-dependent perturbation theory	366
10.3	Adiabatic theorem	369
10.4	The variational method	371
10.5	Classical limit	372
10.6	Semiclassical limit and WKB approximation	378
10.7	Scattering theory	384
10.8	Path integrals	389
	Summary	398

viii	Contents
	Problems 399
	Further reading 399
11	Hydrogen and helium atoms 401
11.1	Introduction 401
11.2	Quantum theory of the hydrogen atom 403
11.3	Atom and magnetic field 413
11.4	Relativistic corrections 423
11.5	Helium atom 426
11.6	Many-electron effects 431
	Summary 436
	Problems 437
	Further reading 438
12	Hydrogen molecular ion 439
12.1	The molecular problem 439
12.2	Born–Oppenheimer approximation 440
12.3	Vibrational and rotational degrees of freedom 443
12.4	The Morse potential 447
12.5	Chemical bonds and further approximations 449
	Summary 453
	Problems 453
	Further reading 454
13	Quantum optics 455
13.1	Quantization of the electromagnetic field 457
13.2	Thermodynamic equilibrium of the radiation field 462
13.3	Phase–number uncertainty relation 463
13.4	Special states of the electromagnetic field 465
13.5	Quasi-probability distributions 474
13.6	Quantum-optical coherence 481
13.7	Atom–field interaction 484
13.8	Geometric phase 497
13.9	The Casimir effect 501
	Summary 506
	Problems 507
	Further reading 509
Part IV Quantum information: state and correlations	
14	Quantum theory of open systems 513
14.1	General considerations 514
14.2	The master equation 516

ix	Contents
	14.3 A formal generalization 523
	14.4 Quantum jumps and quantum trajectories 528
	14.5 Quantum optics and Schrödinger cats 533
	Summary 540
	Problems 541
	Further reading 542
	15 State measurement in quantum mechanics 544
	15.1 Protective measurement of the state 544
	15.2 Quantum cloning and unitarity violation 548
	15.3 Measurement and reversibility 550
	15.4 Quantum state reconstruction 554
	15.5 The nature of quantum states 564
	Summary 565
	Problems 565
	Further reading 566
	16 Entanglement: non-separability 567
	16.1 EPR 568
	16.2 Bohm’s version of the EPR state 573
	16.3 HV theories 577
	16.4 Bell’s contribution 582
	16.5 Experimental tests 595
	16.6 Bell inequalities with homodyne detection 605
	16.7 Bell theorem without inequalities 613
	16.8 What is quantum non-locality? 619
	16.9 Further developments about inequalities 623
	16.10 Conclusion 625
	Summary 625
	Problems 626
	Further reading 627
	17 Entanglement: quantum information and computation 628
	17.1 Information and entropy 628
	17.2 Entanglement and information 631
	17.3 Measurement and information 639
	17.4 Qubits 642
	17.5 Teleportation 643
	17.6 Quantum cryptography 646
	17.7 Elements of quantum computation 650
	17.8 Quantum algorithms and error correction 659
	Summary 671

Cambridge University Press
978-1-107-66589-7 — Quantum Mechanics
Gennaro Auletta, Mauro Fortunato, Giorgio Parisi
Frontmatter
[More Information](#)

Problems	672
Further reading	673
<i>Bibliography</i>	674
<i>Author index</i>	710
<i>Subject index</i>	716

Figures

1.1	Graphical representation of the Liouville theorem	page 12
1.2	Photoelectric effect	13
1.3	Mach–Zehnder interferometer	15
1.4	The Michelson–Morley interferometer	16
1.5	Interferometer for detecting gravitational waves	17
1.6	Interference in the Mach–Zehnder interferometer	17
1.7	Results of the experiment performed by Grangier, Roger, and Aspect	18
1.8	Oscillation of electric and magnetic fields	20
1.9	Polarization of classical light	21
1.10	Decomposition of an arbitrary vector $ a\rangle$	22
1.11	Poincaré sphere representation of states	28
1.12	Mach–Zehnder interferometer with the lower path blocked by the screen S	30
1.13	Black-body radiation intensity corresponding to the formula of Rayleigh–Jeans (1), Planck (2), and Wien (3)	32
1.14	Planck’s radiation curves in logarithmic scale for the temperatures of liquid nitrogen, melting ice, boiling water, melting aluminium, and the solar surface	33
1.15	Compton effect	34
1.16	Dulong–Petit’s, Einstein’s, and Debye’s predictions for specific heat	36
1.17	Lyman series for ionized helium	37
1.18	The Stern–Gerlach Experiment	39
1.19	Momentum conservation in the Compton effect	41
2.1	Polarization beam splitter	44
2.2	Change of basis	52
2.3	Filters	62
2.4	Two sequences of two rotations of a book	65
2.5	Probability distributions of position and momentum for a momentum eigenfunction	85
2.6	Probability distributions of position and momentum for a position eigenfunction	86
2.7	Time evolution of a classical degree of freedom in phase space and graphical representation of the uncertainty relation	88
2.8	Inverse proportionality between momentum and position uncertainties	89
2.9	Smooth complementarity between wave and particle	90
2.10	Illustration of the distributive law	93
2.11	Proposed interferometry and resulting non-Boolean algebra and Boolean subalgebras	94

xii	Figures
2.12	Hasse diagrams of several Boolean and non-Boolean algebras 95
3.1	Positive potential vanishing at infinity 109
3.2	Potential function tending to finite values as $x \rightarrow \pm\infty$ 110
3.3	Potential well 111
3.4	Relation between two different inertial reference frames \mathcal{R} and \mathcal{R}' under Galilei transformations 112
3.5	Particle in a box of dimension a 113
3.6	Energy levels of a particle in a one-dimensional box 115
3.7	First three energy eigenfunctions for a one-dimensional particle confined in a box of dimension a 115
3.8	Beam Splitters as unitary operators 119
3.9	Projector as a residue of the closed contour in a complex plane 125
3.10	A graphical representation of the apparatus proposed by Bohr 133
4.1	Schematic and asymmetric one-dimensional potential wells 142
4.2	Solution of Eq. (4.8) 143
4.3	Wave functions and probability densities for the first three eigenfunctions for the symmetric finite-well potential 144
4.4	Stepwise continuity 145
4.5	Potential barrier 147
4.6	Closed surface used to compute the flux of \mathbf{J} 148
4.7	Delta potential barrier 149
4.8	Classical turning points and quantum tunneling 151
4.9	Tunneling of α -particles 152
4.10	Carbon atoms shown by scanning tunneling microscopy 153
4.11	Potential and energy levels of the harmonic oscillator 154
4.12	Eigenfunctions for the one-dimensional harmonic oscillator 162
4.13	Potential energy corresponding to a particle in a uniform field 165
4.14	Triangular well 167
4.15	A quantum particle with energy E encounters a potential step of height $V_0 < E$ 171
4.16	Rectangular potential barrier with finite width a 171
5.1	Representation of pure and mixed states on a sphere 187
6.1	Angular momentum of a classical particle 194
6.2	Levi-Civita tensor 195
6.3	Relationship between rectangular and spherical coordinates 200
6.4	s - and p -states 206
6.5	Rigid rotator 207
6.6	Energy levels and transition frequencies for a rigid rotator 209
6.7	Cylindrical coordinates 212
6.8	Energy levels of the three-dimensional harmonic oscillator 215
6.9	Levels in the spectrum of hydrogen atom 218
6.10	An electric dipole with charges $+e$ and $-e$ in an electric field gradient 218
6.11	Scheme of spin superposition in single-crystal neutron interferometry 223
6.12	Landé vectorial model for angular momentum 227

6.13	Graphical representation of the distribution of eigenvalues of the z component of the angular momenta of two independent particles	228
6.14	Angle observable and step function	241
7.1	Interferometric example of indistinguishability	246
7.2	Example of counting the number of possible configurations of bosons	253
7.3	Potential wells of a natural atom and of a quantum dot	257
8.1	Passive and active transformations	260
9.1	Representation of a measurement on the sphere of density matrices	280
9.2	Two ways of tuning the coupling function	283
9.3	Decohering histories	290
9.4	Schrödinger cat	292
9.5	Experimental realization of a Schrödinger cat with a trapped ion	293
9.6	SQUID	294
9.7	Wigner function of an entangled state	296
9.8	Schematic representation of the experiment proposed by Scully and co-workers	301
9.9	Schematic representation of Mandel's experiment	309
9.10	Interference and visibility in Mandel's experiment	310
9.11	Scully, Englert, and Walther's proposed experiment	312
9.12	Interaction-free measurement	316
9.13	Repeated interaction-free measurements	317
9.14	Probability of success in repeated interaction-free measurements	318
9.15	Interaction-free measurement with two cavities	318
9.16	Schematic representation of Mandel's experiment on empty waves	319
9.17	Depiction of Wheeler's experiment	320
9.18	Interferometry experiment for testing delayed-choice	321
9.19	Optical version of the Zeno effect	324
9.20	Example of POVMs	334
9.21	Plot of the estimate of the wave function	349
9.22	Another example of POVM	352
10.1	Stark effect	364
10.2	WKB approximation: forbidden regions outside a potential well	381
10.3	WKB and potential well	382
10.4	WKB and potential barrier	383
10.5	The different paths	390
10.6	The analogy of path integral integration	391
10.7	The sum over paths	392
10.8	Two possible paths from i to f both passing through the same central point c	394
10.9	Path integrals and scattering	397
11.1	Electron coordinates in the atomic system	406
11.2	Resulting potential in the hydrogen atom	407
11.3	Grotrian scheme	411
11.4	Plot of the radial eigenfunctions of the hydrogenoid atom	412

xiv	Figures
11.5	Plot of the radial probability densities 413
11.6	<i>s</i> -, <i>p</i> -, and <i>d</i> -states versus energy levels 414
11.7	Landé vectorial model for the Paschen-Bach effect 417
11.8	<i>s</i> and <i>p</i> levels in presence of the Paschen-Bach effect 418
11.9	Paschen-Bach spectroscopical lines 419
11.10	Landé vectorial model for the Zeeman effect 420
11.11	Energy levels for the Zeeman effect 422
11.12	Spectroscopical lines for the Zeeman effect 422
12.1	Spheroidal coordinates for the H ₂ ⁺ ion 440
12.2	Molecular potential energy 446
12.3	Vibrational and rotational levels of two electronic states <i>I</i> and <i>II</i> in a diatomic molecule 447
12.4	Schematic diagram of the LCAO function <i>f</i> (<i>E</i>) 450
12.5	LCAO energy solutions for the H ₂ ⁺ molecular ion 451
12.6	Symmetric and antisymmetric states of the ground level of the H ₂ ⁺ molecule 452
13.1	The three directions of the electromagnetic field 460
13.2	Displacement operator for coherent states 469
13.3	Phase-number uncertainty properties of coherent states 471
13.4	Phase convention for squeezed states 472
13.5	Generation of a squeezed state 473
13.6	Phase-space of amplitude- and phase-squeezed states 473
13.7	Representation of the Q-function of coherent, number, and squeezed states 475
13.8	Representation of the W-function of coherent, number, and squeezed states 480
13.9	Homodyne detection 483
13.10	Jaynes–Cummings energy levels 490
13.11	Rabi oscillations 491
13.12	Collapse and revival 492
13.13	Spontaneous and stimulated emission for a two-level atom 495
13.14	Spontaneous and stimulated emission for a three-level atom 495
13.15	Schematic diagram of a laser 495
13.16	Parametric down conversion 496
13.17	Magnetic and electric AB effects 497
13.18	Parallel transport 501
14.1	Bloch-sphere representation of states 527
14.2	Techniques for integrating a function 531
14.3	BS model for dissipation 534
14.4	Interference fringes and their sensitivity to losses in the Yurke–Stoler model 536
14.5	Pictorial representation of a coherent state and separation between the two components 537
14.6	Haroche’s experiment 538
14.7	Interference fringes in Haroche’s experiment 540
15.1	Tomographic method for reconstructing the W-function 559
15.2	Tomographic measurements of the state 560
16.1	Overview of the EPR – Bohm experiment 574

xv	Figures
16.2	Preparation of a singlet state 576
16.3	Particle trajectories for two Gaussian slit systems after Bohm’s model, and the corresponding quantum potential 580
16.4	Trajectories for a potential barrier ($E = V/2$) after Bohm’s model, and the corresponding quantum potential 581
16.5	The three-dimensional Hilbert space proposed by Bell 585
16.6	Scheme of the experiment proposed for proving the second Bell theorem 589
16.7	Experiment proposed by CHSH 590
16.8	Optimal orientation for a , a' , b , and b' for testing the CHSH inequality 592
16.9	Typical dependence of $f(\theta)$ upon $n\theta$ for cases I-III 595
16.10	Partial Grotrian diagram of atomic calcium for Freedman and Clauser’s experiment 599
16.11	Schematic diagram of apparatus and associated electronics of the experiment by Freedman and Clauser 599
16.12	Freedman–Clauser experiment and Aspect and co-workers’ experiment 600
16.13	Alley-Shih and Ou-Mandel’s experiment 602
16.14	Measured coincidence counting rate as a function of the polarizer angle θ_1 , with θ_2 fixed at 45° 604
16.15	Experimental set-up in order to solve detection loopholes 605
16.16	“Entanglement” with vacuum 606
16.17	Yurke and Stoler’s experiment 608
16.18	Entanglement swapping 611
16.19	Variation of entanglement swapping 612
16.20	Orientations for the proof of Stapp’s theorem 614
16.21	The GHSZ proposed experiment 616
16.22	Conditional entanglement 619
16.23	Necessary criterion for separability 623
17.1	Information difference in bits versus angle θ for the information-theoretic Bell inequality 635
17.2	Informational distance by quadrilateral inequality 635
17.3	Schematic representation of quantum non-separability 636
17.4	Diagram for entangled and disentangled states 637
17.5	Representation of all density matrices 638
17.6	Decompression of information 639
17.7	Teleportation 644
17.8	Realization of teleportation with photons 646
17.9	The CNOT gate 652
17.10	Implementation of a CNOT gate by means of a polarization interferometer 653
17.11	The quantum computation device as an equivalent of a Mach–Zehnder interferometer 653
17.12	Generation of Bell states by means of a Hadamard gate followed by a CNOT gate 655
17.13	Preparation of a GHSZ state 655
17.14	Toffoli gate 657

17.15	Representation of computational complexity	660
17.16	Boolean transformation of an initial bit	660
17.17	Device for solving Deutsch’s problem	661
17.18	Device for solving Deutsch’s problem for $n + 1$ input states	662
17.19	Device for solving Deutsch’s problem for $n + m$ input states	662
17.20	Representation of Shor’s theorem	664
17.21	Implementation of Grover’s algorithm	666
17.22	Computational steps in Grover’s algorithm	666
17.23	Classical error correction	667
17.24	Quantum circuit for error correction	669
17.25	Environmental wave functions and their overlapping as a function of time in quantum computation	670

Tables

2.1	Different cases and ways of expressing the basic quantum formalism	<i>page</i> 71
6.1	Eigenstates and eigenvalues of \hat{l}_z	199
6.2	Eigenvalues, for the three-dimensional harmonic oscillator	215
6.3	Values of j and m and the corresponding number of possible states	233
6.4	Clebsch–Gordan coefficients for $j_1 = j_2 = 1/2$	238
6.5	Clebsch–Gordan coefficients for $j_1 = 1$ and $j_2 = 1/2$	238
6.6	Clebsch–Gordan coefficients for $j_1 = j_2 = 1$	238
7.1	Fermions and bosons	250
7.2	Fermionic distributions	252
7.3	Bosonic distributions	254
11.1	Ground-state energy of helioid atoms	429
13.1	Electromagnetic spectrum	456
17.1	Sequence transmission	648
17.2	Toffoli truth table	658
17.3	Fredkin truth table	658
17.4	Classical error correction	668