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AN ESSAY WHICH OBTAINED THE ADAMS PRIZE
IN THE UNIVERSITY OF CAMBRIDGE FOR THE YEAR 1907

BY

ERNEST W. BROWN Sc.D. F.R.S.

PROFESSOR OF MATHEMATICS IN YALE UNIVERSITY CONNECTICUT
SOMETIME FELLOW OF CHRIST'S COLLEGE CAMBRIDGE AND PROFESSOR OF
MATHEMATICS IN HAVERFORD COLLEGE PENNSYLVANIA

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TO

GEORGE HOWARD DARWIN

AT WHOSE SUGGESTION THE STUDY OF THE MOON'S MOTIONS
WAS UNDERTAKEN BY THE AUTHOR
AND WHOSE ADVICE AND SYMPATHY HAVE BEEN FREELY GIVEN
DURING THE PAST TWENTY YEARS
THIS ESSAY IS GRATEFULLY DEDICATED

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INTRODUCTION.

THIS Essay aims at a complete calculation of the effects produced by the action of a planet on the motion of the moon under the following limitations and conditions:

(1) The problem of the motion of the moon under the action of the sun (supposed to move round the centre of mass of the earth and moon in a fixed elliptic orbit) and the earth, is considered to have been completely solved.

(2) All the bodies are supposed to attract in the same manner as particles of masses equal to their actual masses and situated at the centres of mass.

(3) All the planets are supposed to move in fixed elliptic orbits, *i.e.*, the effect of the action of a planet transmitted either through the earth or through another planet is neglected.

(4) Perturbations of the first order with respect to the ratio of the mass of a planet to that of the sun are alone calculated.

(5) The exception to the above limitations occurs in the periods of revolution of the apse and node of the moon's orbit. These periods are not exactly those arising from (1) but they are the observed periods or, what amounts to the same thing owing to the close agreement between the observed and calculated periods, the periods after all known causes have been included. The point is only of importance in terms of very long period.

(6) All coefficients greater than $0''\cdot 01$ in longitude, latitude and parallax have been obtained. Many are also given which are less than $0''\cdot 01$ whenever they have been accurately calculated. There are, in addition, classes of terms of short period which run in series and which in the aggregate will add up at certain times to much more than $0''\cdot 01$: these have been found to be $0''\cdot 002$.

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(7) The maximum period considered is 3500 years, but as the sieve in Section IV. retained a few terms of longer period, these were also included in the general scheme.

The methods here adopted have been constructed mainly to overcome the difficulties which have in the past prevented an accurate computation of the long period terms. They were, however, found to be equally useful for finding the terms which have periods of a year or less. These difficulties include the development of the parts of the disturbing function which depend on the coordinates of the earth and planet; the accurate calculation of the derivatives of the moon's coordinates with respect to n, the moon's mean motion; the uncertainty arising from the possible omission of terms of long period; and the frequent appearance of small coefficients as the difference of two large numbers.

In Section I., the equations of variations for the lunar elements have been recomputed with the use of a semi-canonical system of elements. The equations for the ordinary system of elements were first given by G. W. Hill and independently, though at a later date, by S. Newcomb; they were recalculated in Hill's form by R. Radau. In the present system, errors arising from the slow convergence of Delaunay's series have been avoided; in fact his literal expressions have only been used in small terms where derivatives with respect to n were required but where the maximum possible error could make no difference in the final results.

In Section II. Hill's method of dividing the disturbing function into a sum of products in which the first factor of each product is independent of the lunar coordinates and the other of the planet's coordinates, is exhibited as part of a general theorem.

By referring these coordinates to the *true* place of the sun's radius vector, I have obtained the first factors directly from the expansion of the inverse first power of the distance between the planet and the earth $(1/\Delta)$ and of its derivatives with respect to certain of the elements of the earth and planet. Only one expansion is therefore required for all these factors, namely, that of $1/\Delta$, and this has been given by Leverrier in a literal form in powers of the eccentricities and mutual inclination. The expansion also contains the coefficients in the expansion of $1/\Delta_0$ (the value of a'/Δ when the eccentricities and inclination are zero) and its derivatives with respect to the planet's mean distance: the formulae for finding the coefficients and their derivatives are here put into forms which admit of rapid and simple computation.

The factors containing the moon's coordinates, together with their derivatives with respect to all the lunar elements except n, are found from the results of my lunar theory; a special method which I gave five years ago and which does not require the use of literal series in powers of n'/n has been

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used to find the derivatives with respect to n. These methods for finding the planetary and lunar factors are set forth in Section III.

The only one of the difficulties previously mentioned which has not been considered up to this point is the danger of omitting long-period terms with sensible coefficients. In Section IV. formulae are constructed which permit one to find rapidly an upper limit to the magnitude of any coefficient. By means of them, all terms having coefficients greater than 0″·01 and periods less than 3500 years have been sifted out; there are about 100 of such terms, excluding the terms of short period for which no sieve was required.

Sections V., VI. consist of numerical results. It may be noted that the values of A_p, B_p, ... are also required for finding the perturbations of the earth by the planets and of the planets by the earth, while those of M_i and of their derivatives (as well as the equations of variations contained in Section I.) are available for the computation of lunar perturbations other than those due to the direct action of the planets.

No new inequalities sufficiently great to account for the observed discrepancy between theory and observation appear from the direct action of the planets, as shown in the tables of Section VI. Radau's well-known list of terms in longitude has required considerable extension as far as the short period terms are concerned, and a few new long period inequalities with small coefficients have been computed. The more extensive developments of this essay have shown that some of his coefficients require alteration, but there is a general agreement for all those portions which he has taken into account.

Only a few slight verbal changes and corrections of errors in copying have been made to the first five sections, with two exceptions mentioned below, since the award of the examiners. But I have gone over all the computations for finding the short period terms and the larger long period terms during the year that has elapsed and have made the following corrections to the results in Section VI.:

Argument	Former coefficient	Corrected coefficient
$l+3T-10V+33°$	− 0″·35	+ 0″·35
$-l-16T+18V-151°$	−15″·22	−14″·55
$l+29T-26V+112°$	+ 0″·117	+ 0″·108
$2D-l+21T-20V-87°$	+ 0″·111	+ 0″·126
$2l-2D+6M-5T+211°-l$	+ 0″·040	− 0″·038

together with their accompanying short period secondaries.

The signs of those coefficients containing h on p. 86 have been changed.

The annual mean motion of the perigee has been altered from 2″·66* to 2″·69.

* I gave this value in a paper referred to on page 3 below.

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The values of M_2 and of its derivatives with respect to n, e, k under the argument 0 on page 61 have required a factor 2. This error necessitated slight changes in some of the coefficients whose primaries were independent of the lunar angles; the largest correction was one of $-0''\cdot019$ in the coefficient of $\sin(T-V)$.

A wrong sign in the computation of the equations of variations (see Errata at the end of the volume) gave rise to a few almost insensible changes in certain coefficients.

The additions are:

Argument	Coefficient
$4M-2T+63°$	$-0''\cdot012$
$2l-2D+4(M-T)-l$	$+0''\cdot017$
$2l-2D+8M-6T+63°$	$-0''\cdot019$
$2l-2D+8M-6T+63°-l$	$-0''\cdot031$

No other change or additional coefficient has been greater than $0''\cdot010$.

The Addendum containing the results obtained by adding together terms of the same argument in Section VI. is also new.

During the summer of last year Professor Newcomb's new work* on this subject was published. His methods differ so completely from those given here that no comparison is made easily except in a few of the final results where the indirect action is insensible or is separated from the direct action. For the large inequality due to Venus he obtains a coefficient of $14''\cdot83$ while mine is $14''\cdot55$; a portion of the difference is probably due to certain terms of the second order relative to the ratios of the masses of Venus and of the earth to the sun which Professor Newcomb has included. On the other hand, he states that the possible errors arising in his method may be of the order of this difference, while such errors are excluded from my result. His results and mine for the annual mean motions of the perigee and node agree within $0''\cdot01$, which is the limit of accuracy to which I have obtained these quantities.

* "Investigation of Inequalities in the Motion of the Moon produced by the Action of the Planets." Carnegie Institution, *Publication* 72, Washington, D.C., June, 1907.

E. W. B.

NEW HAVEN, CONN., U.S.A.
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