
Contents

	<i>Preface</i>	page ix
	<i>Acknowledgements</i>	xiv
	PART I FUNDAMENTALS AND BASIC APPLICATIONS	1
1	Introduction	3
	1.1 Solitons: Historical remarks	11
	Exercises	14
2	Linear and nonlinear wave equations	17
	2.1 Fourier transform method	17
	2.2 Terminology: Dispersive and non-dispersive equations	19
	2.3 Parseval's theorem	22
	2.4 Conservation laws	22
	2.5 Multidimensional dispersive equations	23
	2.6 Characteristics for first-order equations	24
	2.7 Shock waves and the Rankine–Hugoniot conditions	27
	2.8 Second-order equations: Vibrating string equation	33
	2.9 Linear wave equation	35
	2.10 Characteristics of second-order equations	37
	2.11 Classification and well-posedness of PDEs	38
	Exercises	42
3	Asymptotic analysis of wave equations: Properties and analysis of Fourier-type integrals	45
	3.1 Method of stationary phase	46
	3.2 Linear free Schrödinger equation	48
	3.3 Group velocity	51

3.4	Linear KdV equation	54
3.5	Discrete equations	61
3.6	Burgers' equation and its solution: Cole–Hopf transformation	68
3.7	Burgers' equation on the semi-infinite interval	71
	Exercises	73
4	Perturbation analysis	75
4.1	Failure of regular perturbation analysis	76
4.2	Stokes–Poincaré frequency-shift method	78
4.3	Method of multiple scales: Linear example	81
4.4	Method of multiple scales: Nonlinear example	84
4.5	Method of multiple scales: Linear and nonlinear pendulum	86
	Exercises	96
5	Water waves and KdV-type equations	98
5.1	Euler and water wave equations	99
5.2	Linear waves	103
5.3	Non-dimensionalization	105
5.4	Shallow-water theory	106
5.5	Solitary wave solutions	118
	Exercises	128
6	Nonlinear Schrödinger models and water waves	130
6.1	NLS from Klein–Gordon	130
6.2	NLS from KdV	133
6.3	Simplified model for the linear problem and “universality”	138
6.4	NLS from deep-water waves	141
6.5	Deep-water theory: NLS equation	148
6.6	Some properties of the NLS equation	152
6.7	Higher-order corrections to the NLS equation	156
6.8	Multidimensional water waves	158
	Exercises	167
7	Nonlinear Schrödinger models in nonlinear optics	169
7.1	Maxwell equations	169
7.2	Polarization	171
7.3	Derivation of the NLS equation	174
7.4	Magnetic spin waves	182
	Exercises	185

<i>Contents</i>		vii
	PART II INTEGRABILITY AND SOLITONS	187
8	Solitons and integrable equations	189
	8.1 Traveling wave solutions of the KdV equation	189
	8.2 Solitons and the KdV equation	192
	8.3 The Miura transformation and conservation laws for the KdV equation	193
	8.4 Time-independent Schrödinger equation and a compatible linear system	197
	8.5 Lax pairs	198
	8.6 Linear scattering problems and associated nonlinear evolution equations	199
	8.7 More general classes of nonlinear evolution equations	205
	Exercises	210
9	The inverse scattering transform for the Korteweg–de Vries (KdV) equation	214
	9.1 Direct scattering problem for the time-independent Schrödinger equation	215
	9.2 Scattering data	219
	9.3 The inverse problem	222
	9.4 The time dependence of the scattering data	224
	9.5 The Gel'fand–Levitan–Marchenko integral equation	225
	9.6 Outline of the inverse scattering transform for the KdV equation	227
	9.7 Soliton solutions of the KdV equation	228
	9.8 Special initial potentials	232
	9.9 Conserved quantities and conservation laws	235
	9.10 Outline of the IST for a general evolution system – including the nonlinear Schrödinger equation with vanishing boundary conditions	239
	Exercises	256
	PART III APPLICATIONS OF NONLINEAR WAVES IN OPTICS	259
10	Communications	261
	10.1 Communications	261
	10.2 Multiple-scale analysis of the NLS equation	269
	10.3 Dispersion-management	274
	10.4 Multiple-scale analysis of DM	277

10.5	Quasilinear transmission	298
10.6	WDM and soliton collisions	303
10.7	Classical soliton frequency and timing shifts	306
10.8	Characteristics of DM soliton collisions	309
10.9	DM soliton frequency and timing shifts	310
11	Mode-locked lasers	313
11.1	Mode-locked lasers	314
11.2	Power-energy-saturation equation	317
	<i>References</i>	334
	<i>Index</i>	345