

## Index

- Abel integral equation, 72
- adiabatic invariant, 87, 89
- admissibility conditions, 29
- Airy function, 55, 59, 60
- Airy's equation, 59
- AKNS method, 201
- anomalous dispersion, 314, 319
- associated flux, 194
- asymptotic expansion, 49
- asymptotic sequence, 49
- averaged equation, 324
- Benney–Luke equation, 127
  - KP equation, 128
    - with surface tension, 127
- Benney–Roskes equations, *see* BR equations
- Bernoulli equation, 101
- bi-soliton, 330
- blow-up, 163, 317
- bound state, 245
- Boussinesq equation, 5, 15, 109, 167, 210
  - linearized, 109
- Boussinesq model
  - coupled, 110
- BR equations, 158, 165, 182
- breaking time, 26
- Burgers' equation, 30, 199
  - generalized, 32
  - inviscid, 6, 21, 26, 32, 71
  - linearized, 73
  - semi-infinite interval, 71
  - viscous, 27, 31, 68
- Camassa–Holm equation, 212
- centro-symmetric material, 171
- CFL condition, 68
- channel, wavelength, 303
- characteristics, 36, 37
- characteristics, method of, 25, 26
- chirp, 289, 314, 330
- cnoidal function, 13, 120
- cnoidal wave, 8
- Cole–Hopf transformation, 68, 197
- collison-induced time shift, 262
- conservation law, 23, 194
- conservation of wave equation, 52
- conserved density, 194
- convective derivative, 100
- convolution theorem, 42
- CW, 266
- CW wave, 179
- D'Alembert's solution to wave equation, 36, 43
- Davey–Stewartson (DS) equation, 159
  - generalized, (GDS), 159
- deep-water dispersion relationship, 104
- deep-water limit, 148
- density of a conserved quantity, 23
- direct scattering problem, 198, 214
- Dirichlet–Neumann map methods, 124
- Dirichlet–Neumann relationship, 71
- discrete eigenvalue, *see* eigenvalue, proper
- discrete wave equation, 4
- dispersion
  - anomalous, 178
  - normal, 178
- dispersion following the loss profile, 276
- dispersion parameter, 106
- dispersion relation, 18
  - discrete, 62
- dispersive equation, 20

- linear
  - multidimensional, 24
  - nonlinear wave, 24
- DM, 275
- DM map strength, 279
- DM system, 262, 313
- DM-PES equation, 324
- DN map, *see* Dirichlet–Neumann relationship
- dromion, 160, 161
- DS equation, *see* Davey–Stewartson equation
- DSI system, 159
- DSII system, 161
- EDFA, 261, 266
- eigenvalue
  - proper, 245
- energy integral, 91
- entropy-satisfying conditions, *see*
  - admissibility conditions
- Euler equations, 99
- fan, 31
- fast variable, 81
- ferromagnetic, 171
- flux of a conserved quantity, 23
- four-wave mixing, 273
- four-wave mixing instability, 262
- Fourier transform, 18, 63
  - discrete, 64
  - inverse discrete, 64
- FPU (Fermi–Pasta–Ulam) model, 3
- FPU equation, 4
- FPU model, 123, 192
- FPU problem, 14, 107, 124
- Fredholm alternative, 92
- frequency, 19
- frequency-shift method, 78
  - failure of, 81
- FWHM, 262, 263
- FWM, 271
- Gel'fand–Levitan–Marchenko (GLM)
  - equation, 258
- Gel'fand–Levitan–Marchenko (GLM) integral
  - equations, 224, 225, 250
- Gel'fand–Levitan–Marchenko normalization
  - constants, 230
- Ginzburg–Landau equation, 318
- group velocity, 20, 48, 51, 53
  - bounded, 65
- group-velocity frame, 132
- guiding center, 270
- half-derivative, 71
- hard spring, 86
- Harry–Dym equation, 212
- Hilbert transform, 157
- hodograph transformation, 213
- integrable equation, 123
- inverse Z transform, 63
- inverse scattering problem, 198, 214
- inverse scattering transform (IST), 10, 194, 200
- irrotational flow, 100
- isospectral equation, 212
- isospectral flow, 201, 225
- isotropic material, 172
- Jost solutions, 241
- Kadomtsev–Petviashvili equation, *see* KP equation
- KdV equation, 6, 13, 15, 56, 112, 189, 193, 203, 214
  - 1 + 1 dimension, 99
  - concentric, 211
  - generalized, 14, 15
    - blow-up, 167
  - integrable, 123
  - linear, 54, 73, 139
  - linearized, 20, 58
    - conservation law, 23
  - modified, 14, 15, 167, 195, 200, 203, 253
  - NLS equation, 133, 137
    - solved by IST, 227
- Kerr effect, 315
- Kerr material, 172
- kinematic condition, 102
- kink solutions, 15
- Klein–Gordon equation, 16, 40, 42, 66, 137
  - nonlinear, 130
- KP equation, 99, 116, 118, 137, 167, 211
  - Benney–Luke equation, 128
  - linear, 117
  - linearized, 122
  - unidirectional, 128
- KPI equation, 118, 162
- KPII equation, 118, 162
- $L_2$ -norm, 22
- Laplace transform, 71
- Lax pair, 199, 239
- Lax's equation, 199
- leading-order equation, 76

- leading-order solution, 76
- Legendre equation, associated, 257
- Lenard hierarchy, 208
- linear index of refraction, 176
- linear susceptibility, 172
- long-wave expansion, 110
- lump soliton, 121
- lump solution, 161
- lumped model, 266
- magnetization, 170
- Marchenko equation, 257
- master equation, 317
  - normalized, 317
- material derivative, 100
- maximal balance, 107
- Maxwell's equations, 169
- method of multiple scales
  - linear case, 81
  - nonlinear case, 84
- method of stationary phase, 46, 55
- method of steepest descent, 56
- Miura transformation, 195
- mode-locking, 315
- Navier–Stokes equations
  - incompressible, 99
- NLS equaiton
  - deep-water waves, 152
- NLS equation, 130, 132, 136, 138, 141, 156, 169, 174, 177, 200, 203, 206, 212, 240, 253, 261, 269
  - damped, 168
  - deep water, 148
  - defocusing, 132, 136, 178
  - dispersion-managed, 278, 285, 316
    - convolution form, 279
    - focusing, 132, 151, 178
  - Galilean invariance, 153
  - lossless, 270
  - normalized, 263
  - PES equation, relation to, 314
    - vs PES equation, 321
  - NLS equation with mean (NLMS), 182
  - NLS systems, 127
  - nonlinear optics, 169, 261
  - nonlinear Schrödinger equation, *see* NLS equation
  - non-local equation, 124
  - norming constant, 246
  - numerical ill-posedness, 68
- $O(\epsilon)$  correction, 77
- optics variant, 140
- Painlevé equation
  - second, 15
  - third, 16
- Parseval's theorem, 22
- pendulum
  - linear, 87
  - nonlinear, 90
- period, 19
- perturbing contribution, 317
- PES
  - dispersion-managed, 324
- PES equation, 314, 317
  - constant dispersion, 319
  - vs NLS equation, 321
- phase, 19
- phase contours, 24
- phase speed
  - multidimensional, 24
- phase velocity, 20
- polarization, 170
- power-energy-saturation equation, *see* PES equation
- propagation constant, 320
- pulse
  - quasilinear, 298
- quasilinear mode, 299
- quasilinear pulse, 298
- quasimonochromatic assumption, 137
- Rankine–Hugoniot relations, 29
- rarefaction wave, 31
- reduced map strength, 297
- reflection coefficients, 217, 245
- reflectionless potential, 228
- regular perturbation analysis, 76
- residual timing shift, 306
- resonant term, *see* secular term
- retarded frame, 133
- Riemann–Hilbert boundary value problem, 217
- Riemann–Hilbert problem, 248
- satury power terms, 313
- scattering data, 198, 214
- scattering problem, 240
- Schrödinger equation, 48
  - differential–difference, 73
  - doubly-discrete, 66

- linear, 48, 50, 51, 53
- linear, semi-discrete, 61
- semi-discrete, 65
- time-independent, 197, 215
- Schrödinger scattering problem, 207
- second harmonic resonance, 181
- secular term, 78, 85
- self-phase-modulation, 180
- self-similar, 16, 49
- shallow-water dispersion relationship, 104
- shallow-water wave, 107
- shock conditions, *see* Rankine–Hugoniot equations
- shocks, 28
- sideband frequency, 139
- sideband wavenumber, 139
- similarity solution, 58
  - slowly varying, 51
- simple harmonic oscillator, 74
- sine-Gordon (SG) equation, 14, 16, 167, 200, 204, 254
- singular perturbation, 78
- sinh–Gordon equation, 204, 254
- slow variable, 81
- slowly varying envelope, 131
- soft spring, 86
- solitary, 6
- solitary wave, 14
  - elastic collision, 9
  - train of, 7
- soliton, 10, 11, 14, 119, 193
  - bi-, 330
  - black, 153
  - bright, 132, 152, 178, 246
  - dark, 132, 153, 178, 246
  - dispersion-managed, 285
  - gray, 153
  - return-to-zero, 305
- soliton collision, 307
  - DM, 309
- soliton instability, 156
- soliton shape, 320
- soliton state, 320
- soliton string, 321
- solvability condition, 92, 93
- spectral renormalization, 286, *see* SPRZ method
  - SPM, 271
  - SPRZ method, 314
- stationary point, 46
- steepest descent contour, 56
- Stokes' water wave
- instability, 156
- Stokes–Poincaré frequency-shift method, 148
- Stokes–Poincaré method, 78
- surface tension, 116
- susceptibility
  - linear, 172
  - third-order, 172
- third-order susceptibility, 172
- time equation, 240
- Toda lattice, 124
- torque equation, 183
- Townes modes, 164
- transmission coefficients, 217
- u*-4 model, 130
- universality, 141
- vibrating string, 33
- virial equation, 168
- virial theorem, 164
- vorticity, 100
- vorticity equation, 100
- water wave
  - shallow, 107
- water wave equations, 102
  - non-dimensional, 106
- water waves
  - deep-water limit, 149
  - multidimensional, 158
- water waves variant, 140
- wave equation
  - conservation of, 52
- wavelength, 19
  - multidimensional, 24
- wavelength channel, 303
- wavenumber, 19
  - multidimensional, 24
- WDM, 270
- WDM system, 261
- weak transverse variation, 117
- well-posedness, 39, 40
- WKB approximation, 52
- WKB method, 89
- Wronskian, 216
- XPM, 271
- Z transform, 63
- Z transform, inverse, 63
- Zakharov variables, 124