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978-1-107-66143-1 - The Collected Mathematical Papers of James Joseph Sylvester: Volume III (1870–1883)

James Joseph Sylvester

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# MATHEMATICAL PAPERS

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THE COLLECTED  
MATHEMATICAL PAPERS

OF

JAMES JOSEPH SYLVESTER

F.R.S., D.C.L., LL.D., Sc.D.,

Honorary Fellow of St John's College, Cambridge ;

Sometime Professor at University College, London ; at the University of Virginia ;  
at the Royal Military Academy, Woolwich ; at the Johns Hopkins University, Baltimore  
and Savilian Professor in the University of Oxford

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## PREFATORY NOTE.

THE present volume deals very largely with the Author's enumerative method of obtaining the complete system of concomitants of a system of quantics, with the help of generating functions; the brief but very luminous papers here reprinted, at the end of the volume, from the *Johns Hopkins University Circulars* shew the Author preparing his memoir on the Constructive Theory of Partitions, which begins the next, and last, volume of his *Mathematical Works*. The previous volume included the period of the Author's activity at the Military Academy, Woolwich; this volume nearly covers the time of that surely most interesting experiment in educational method when, at Baltimore, unhindered by traditional routine, and encouraged to give full rein to his invention, he was able, nay obliged, as he tells us (p. 76), to yield to the inquisitive student who would have the New Algebra, that or nothing; with results that are imperishable. The matter is seen so well from the Author's point of view in his Commemoration day Address at Johns Hopkins University (1877), that, after some hesitation, a reprint of this is included in the present volume (No. 10). The Remarks on Research, in *Nature*, vol. xvi. (1877), are from this Address. The present volume also includes the Author's investigations on Chemistry and Algebra (No. 24), the paper on Certain Ternary Cubic-Form Equations (No. 39), and the paper on Subinvariants and Perpetuants (No. 67). In connection with the enumerative methods in this volume the reader's attention may be directed to a paper, by F. Franklin, "On the Calculation of the Generating Functions and Tables of Groundforms for Binary Quantics," in the *American Journal of Mathematics*, III. (1880), pp. 128—153, to which, as to one or two other memoirs referring to matters dealt with in the text, I have ventured to add a reference at the appropriate place.

H. F. BAKER.

ST JOHN'S COLLEGE, CAMBRIDGE.

24 November 1909.

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