# LOGIC

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### DEMONSTRATIVE INFERENCE: DEDUCTIVE AND INDUCTIVE

BY

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# INTRODUCTION TO PART II

§ 1. BEFORE introducing the topics to be examined in Part II, I propose to recapitulate the substance of Part I, and in so doing to bring into connection with one another certain problems which were there treated in different chapters. I hope thus to lay different emphasis upon some of the theories that have been maintained, and to remove any possible misunderstandings where the treatment was unavoidably condensed.

In my analysis of the proposition I have distinguished the natures of substantive and adjective in a form intended to accord in essentials with the doctrine of the large majority of logicians, and as far as my terminology is new its novelty consists in giving wider scope to each of these two fundamental terms. *Prima facie* it might be supposed that the connection of substantive with adjective in the construction of a proposition is tantamount to the metaphysical notions of substance and inherence. But my notion of substantive is intended to include, besides the metaphysical notion of substance —so far as this can be philosophically justified—the notion of occurrences or events to which some philosophers of the present day wish to restrict the realm of reality. Thus by a substantive *proper* I mean an existent; and the category of the existent is divided into the two subcategories: what continues to exist, or the continuant; and what ceases to exist, or the occurrent, every occurrent being referrible to a continuant. To exist is to be

in temporal or spatio-temporal relations to other existents; and these relations between existents are the fundamentally external relations. A substantive proper cannot characterise, but is necessarily characterised; on the other hand, entities belonging to any category whatever (substantive proper, adjective, proposition, etc.) may be characterised by adjectives or relations belonging to a special adjectival sub-category corresponding, in each case, to the category of the object which it characterises. Entities, other than substantives proper, of which appropriate adjectives can be predicated, function as quasi-substantives.

§ 2. The term adjective, in my application, covers a wider range than usual, for it is essential to my system that it should include relations. There are two distinct points of view from which the treatment of a relation as of the same logical nature as an adjective may be defended. In the first place the complete predicate in a relational proposition is, in my view, relatively to the subject of such proposition, equivalent to an adjective in the ordinary sense. For example, in the proposition, 'He is afraid of ghosts,' the relational component is expressed by the phrase 'afraid of'; but the complete predicate 'afraid of ghosts' (which includes this relation) has all the logical properties of an ordinary adjective, so that for logical purposes there is no fundamental distinction between such a relational predicate and an irrational predicate. In the second place, if the relational component in such a proposition is separated, I hold that it can be treated as an adjective predicated of the substantive-couple 'he' and 'ghosts'. In other words, a relation cannot be identified with a *class* of couples, i.e. be

conceived extensionally; but must be understood to *characterise* couples, i.e. be conceived intensionally. It seems to me to raise no controvertible problem thus to include relations under the wide genus adjectives. It is compatible, for example, with almost the whole of Mr Russell's treatment of the proposition in his *Principles of Mathematics*; and, without necessarily entering into the controvertible issues that emerge in such philosophical discussions, I hold that some preliminary account of relations is required even in elementary logic.

§ 3. My distinction between substantive and adjective is roughly equivalent to the more popular philosophical antithesis between particular and universal; the notions, however, do not exactly coincide. Thus I understand the philosophical term particular not to apply to quasi-substantives, but to be restricted to substantives proper, i.e. existents, or even more narrowly to occurrents. On the other hand, I find a fairly unanimous opinion in favour of calling an adjective predicated of a particular subject, a particular—the name universal being confined to the abstract conception of the adjective. Thus red or redness, abstracted from any specific judgment, is held to be universal; but the redness, manifested in a particular object of perception, to be itself particular. Furthermore, *qua* particular, the adjective is said to be an existent, apparently in the same sense as the object presented to perception is an existent. To me it is difficult to argue this matter because, while acknowledging that an adjective may be called a universal, I regard it not as a mere abstraction, but as a factor in the real; and hence, in holding that the objectively real is properly construed into an adjective

characterising a substantive, the antithesis between the particular and the universal (i.e. in my terminology between the substantive and the adjective) does not involve separation within the real, but solely a separation *for thought*, in the sense that the conception of the substantive apart from the adjective, as well as the conception of the adjective apart from the substantive, equally entail abstraction.

§ 4. Again, taking the whole proposition constituted by the connecting of substantive with adjective, I have maintained that in a virtually similar sense the proposition is to be conceived as abstract. But, whereas the characterising tie may be called constitutive in its function of connecting substantive with adjective to construct the proposition, I have spoken of the assertive tie as epistemic, in the sense that it connects the thinker with the proposition in constituting the unity which may be called an act of judgment or of assertion. When, however, this act of assertion becomes in its turn an object of thought, it is conceived under the category of the existent; for such an act has temporal relations to other existents, and is necessarily referrible to a thinker conceived as a continuant. Though, relatively to the primary proposition, the assertive tie must be conceived as epistemic; yet, relatively to the secondary proposition which predicates of the primary that it has been asserted by A, the assertive tie functions constitutively.

§ 5. In view of a certain logical condition presupposed throughout this Part of my work, I wish to remind the reader of that aspect of my analysis of the proposition, according to which I regard the subject as that which is *given* to be determinately characterised

by thought. Now I hold that for a subject to be characterised by some adjectival determinate, it must first have been presented as characterised by the corresponding adjectival determinable. The fact that what is given is characterised by an adjectival determinable is constitutive; but the fact that it is *presented* as thus characterised is epistemic. Thus, for a surface to be characterised as red or as square, it must first have been constructed in thought as being the kind of thing that has colour or shape; for an experience to be characterised as pleasant or unpleasant, it must first have been constructed in thought as the kind of thing that has hedonic tone. Actually what is given, is to be determined with respect to a conjunction of several specific aspects or determinables; and these determine the category to which 'the given' belongs. For example, on the dualistic view of reality, the physical has to be determined under spatio-temporal determinables, and the psychical under the determinable consciousness or experience. If the *same* being can be characterised as two-legged and as rational, he must be put into the category of the physico-psychical.

§ 6. The passage from topics treated in Part I to those in Part II, is equivalent to the step from implication to inference. The term inference, as introduced in Part I, did not require technical definition or analysis, as it was sufficiently well understood without explanation. It was, however, necessary in Chapter III to indicate in outline one technical difficulty connected with the paradox of implication; and there I first hinted, what will be comprehensively discussed in the first chapter of this Part, that implication is best conceived

as potential inference. While for elementary purposes implication and inference may be regarded as practically equivalent, it was pointed out in Chapter III that there is nevertheless one type of limiting condition upon which depends the possibility of using the relation of implication for the purposes of inference. Thus reference to the specific problem of the paradox of implication was unavoidable in Part I, inasmuch as a comprehensive account of symbolic and mechanical processes necessarily included reference to all possible limiting cases; but, apart from such a purely abstract treatment, no special logical importance was attached to the paradox. The limiting case referred to was that of the permissible employment of the compound proposition 'If $p$ then $q$,' in the unusual circumstance where knowledge of the truth or the falsity of $p$ or of $q$ was already present when the compound proposition was asserted. This limiting case will not recur in the more important developments of inference that will be treated in the present part of my logic.

It might have conduced to greater clearness if, in Chapters III and IV, I had distinguished—when using the phrase *implicative proposition*—between the primary and secondary interpretations of this form of proposition. Thus, when the compound proposition 'If $p$ then $q$' is rendered, as Mr Russell proposes, in the form 'Either not-$p$ or $q$,' the compound is being treated as a *primary* proposition of the same type as its components $p$ and $q$. When on the other hand we substitute for 'If $p$ then $q$' the phrase '$p$ implies $q$,' or preferably '$p$ would imply $q$,' the proposition is no longer primary, inasmuch as it predicates about the proposition $q$ the adjective 'implied by $p$' which renders the compound a secondary

proposition, in the sense explained in Chapter IV[1]. Now whichever of these two interpretations is adopted, the inference which is legitimate under certain limiting conditions is the same. Thus given the compound 'Either not-$p$ or $q$' conjoined with the assertion of '$p$,' we could infer '$q$'; just as given '$p$ implies $q$' conjoined with the assertion of '$p$,' we infer '$q$.' It is for this reason that the two interpretations have become merged into one in the ordinary symbolic treatment of compound propositions; and in normal cases no distinction is made in regard to the possibility of using the primary or secondary interpretation for purposes of inference. The normal case, however, presupposes that $p$ and $q$ are entertained hypothetically; when this does not obtain, the danger of petitio principii enters. The problem in Part I was only a very special and technical case in which this fallacy has to be guarded against; in Part II, it will be dealt with in its more concrete and philosophically important applications.

§ 7. The mention of this fallacy immediately suggests Mill's treatment of the functions and value of the syllogism; but, before discussing his views, I propose to consider what his main purpose was in tackling the charge of petitio principii that had been brought against the whole of formal argument, including in particular the syllogism. In the first section of his chapter, Mill refers to two opposed classes of philosophers—the one of whom regarded syllogism as the universal type of all logical reasoning, the other of whom regarded syllogism

[1] The interpretation of the implicative form '$p$ implies $q$' as secondary is developed in Chapter III, § 9, where the modal adjectives *necessary*, *possible*, *impossible*, are introduced.

as useless on the ground that all such forms of inference involve *petitio principii*. He then proceeds: 'I believe both these opinions to be fundamentally erroneous,' and this would seem to imply that he proposed to relieve the syllogism from the charge. I believe, however, that all logicians who have referred to Mill's theory—a group which includes almost everyone who has written on the subject since his time—have assumed that the purport of the chapter was to *maintain* the charge of petitio principii, an interpretation which his opening reference to previous logicians would certainly not seem to bear. His subsequent discussion of the subject is, verbally at least, undoubtedly confusing, if not self-contradictory; but my personal attitude is that, whatever may have been Mill's general purpose, it is from his own exposition that I, in common with almost all his contemporaries, have been led to discover the principle according to which the syllogism can be relieved from the incubus to which it had been subject since the time of Aristotle. In my view, therefore, Mill's account of the philosophical character of the syllogism is incontrovertible; I would only ask readers to disregard from the outset any passage in his chapter in which he appears to be contending for the annihilation of the syllogism as expressive of any actual mode of inference.

Briefly his position may be thus epitomised. Taking a typical syllogism with the familiar major 'All men are mortal,' he substituted for 'Socrates' or 'Plato' the minor term 'the Duke of Wellington' who was then living. He then maintained that, going behind the syllogism, certain instantial evidence is required for establishing the major; and furthermore that the validity

of the conclusion that the Duke of Wellington would die depends ultimately on this instantial evidence. The interpolation of the universal major 'All men will die' has undoubted value, to which Mill on the whole did justice; but he pointed out that the formulation of this universal adds nothing to the positive or factual data upon which the conclusion depends. It follows from his exposition that a syllogism whose major is admittedly established by induction from instances can be relieved from the reproach of begging the question or circularity if, and only if, the minor term is not included in the ultimate evidential data. The Duke of Wellington being still living could not have formed part of the evidence upon which the universal major depended. It was therefore part of Mill's logical standpoint to maintain that there were principles of induction by which, from a limited number of instances, a universal going beyond these could be logically justified. This contention may be said to confer constitutive validity upon the inductive process. It is directly associated with the further consideration that an instance, not previously examined, may be adduced to serve as minor premiss for a syllogism, and that such an instance will always preclude circularity in the formal process. Now the charge of circularity or petitio principii is epistemic; and the whole of Mill's argument may therefore be summed up in the statement that the epistemic validity of syllogism and the constitutive validity of induction, both of which had been disputed by earlier logicians, stand or fall together.

In order to prevent misapprehension in regard to Mill's view of the syllogism, it must be pointed out that he virtually limited the topic of his chapter to cases in

which the major premiss would be admitted by all logicians to have been established by means of induction in the ordinary sense, i.e. by the simple enumeration of instances; although many of them would have contended that such instantial evidence was not by itself sufficient. Thus all those cases in which the major was otherwise established, such as those based on authority, intuition or demonstration, do not fall within the scope of Mill's solution. Unfortunately all the commentators of Mill have confused his view that universals cannot be intuitively but only empirically established, with his specific contention in Chapter IV. I admit that he himself is largely responsible for this confusion, and therefore, while supporting his view on the functions of the syllogism, I must deliberately express my opposition to his doctrine that universals can only ultimately be established empirically, and limit my defence to his analysis of those syllogisms in which it is acknowledged that the major is thus established. Even here his doctrine that all inference is from particulars to particulars is open to fundamental criticism; and, in my treatment of the principles of inductive inference which will be developed in Part III, I shall substitute an analysis which will take account of such objections as have been rightly urged against Mill's exposition.

[NOTE. There are two cases in which the technical terminology employed in Part II differs from that in Part I. (1) The phrase *primitive proposition*, in Part I, is to be understood psychologically; in Part II, logically as equivalent to *axiom*. (2) *Counter-implicative*, in Part I, applies to the form of a compound proposition; in Part II, to a principle of inference.]