

INDEX

The references are to pages. For *determinoid* and *determinant* the abbreviations *detoid* and *detant* are used.

- Abbreviated notations:
 for affects, 66;
 for detoids, 17–19, 347;
 for matrices, 1, 5–12, 347;
 augmented matrices, 8;
 a scalar matrix, 178;
 a unit matrix, 11, 177.
- Active and passive rows, defined:
 in a product of two matrices, 158;
 in any matrix product, 190.
 Corresponding rows, 160, 163, 191.
- Active rows, properties of:
 in a product of two matrices, 168;
 in any matrix product, 199, 222.
- Activities of a matrix product, 158, 191.
- Addition of detoids, 107.
- Addition and subtraction of matrices:
 similar matrices, 155;
 dissimilar matrices, 156;
 is commutative and associative, 155–6.
 Equivalence of conventionally equal matrices, 156.
- Affected elements of a matrix, vi.
- Affected derived products, 24.
 Sums of, 27, 134, 136.
- Affected minor detants:
 Sums of, 115, 134–7, 209, 221, 244.
 Sums of products of, 210, 217; 225, 228; 244.
 Complete matrices of, 252–3, 258.
- Affected simple minor detoids:
 Sums of, 123, 134–7.
- Affects, horizontal, vertical and total:
 vi, 24, 56, 86.
 Signs determined by, vi, 24.
 Relative affects, 24, 56.
- Affects of derived or minor matrices:
 defined by affects of sequences, 86;
 obtained by counting moves, 91;
 compared with affect of leading product, 89, 91;
 unaltered by extension or completion, 91;
 of complementary minors, 102.
 Theorems concerning, 91–104.
- Affects of derived products:
 defined by counting steps, 13, 24;
 relative affects, 24;
- Affects of derived products (*cont.*):
 obtained by counting moves, 50;
 unaltered by extension or completion, 29;
 reduced to affects of sequences, 83.
 Rules for determining, 25–30.
 Theorems concerning, 96–8, 100.
 Affects of complete derived products, 27.
 Signs determined by affects, 24;
 independent of order of factors, 47.
 Changes in, due to interchange of two parallel rows, 30, 38;
 to interchange of two suffixes, 36, 44;
 to reversals of rows, 50.
- Affects of derived or minor sequences:
 defined by counting steps, 56;
 relative affects, 56;
 obtained by counting inversions, 60;
 obtained by counting moves, 62;
 unaltered by extension or completion, 56;
 obtained by reducing the fundamental sequence to a sequence of natural numbers, 60.
 Expressions for, 57–61.
 Theorems concerning, 62–71.
 of complete derived sequences, 59;
 of coranged minor sequences, 58, 59, 61;
 of complementary minor sequences, 70, 71.
 Changes in, due to an interchange of two elements, 72, 76.
- Algebraical sum, *see* Sum.
- Algebraic equations, *see* Systems of linear algebraic equations.
- Associative law:
 for addition and subtraction of matrices, 155–6;
 for matrix products, 184, 186, 188.
- Augmented matrices:
 abbreviated notations for, 8;
 of matrix equations, 312, 327, 337;
 of systems of linear algebraic equations, 366, 374.
 Ranks of, 287.
 Generalisation of, 347.
- Augmented solutions of linear equations, 395, 399, 411.

- Backward steps, 4, 16, 55;
moves, 49.
- Bordered detants, 240;
detoids, 241.
- Cancellation of a matrix factor:
in a matrix equation, 357; 325, 336, 354;
in a matrix identity, 358;
of a scalar factor in a matrix equation, 157.
- Chain of matrix factors, 190–3.
- Changes in the affect and sign of a derived
product caused by an interchange of
two rows or two suffixes, 30, 36, 38, 44;
caused by inversions in the orders of ar-
rangement of the rows, 50.
- Changes in the affects of derived sequences
caused by an interchange of two ele-
ments or two suffixes, 72, 76.
- Classification of simple minors, 115.
- Co-factor of an element of a matrix, 106;
of a minor detant, 119;
of a minor detoid, 141;
of a simple minor detoid, 145;
Sum of products of an element and its
co-factor, 107, 140;
of a minor detant and its co-factor,
119, 138;
of a minor detoid and its co-factor, 143.
Generalisations, 108, 123.
- Commutative law:
for addition of matrices, 155–6;
not true for matrix products, 174, 208.
- Commutative products, 175, 181–3, 262–3:
Product of a square matrix and its conju-
gate reciprocal, 181, 263.
Product of two square matrices when the
product is a non-zero scalar matrix,
262–3, 266, 309.
Product of an undegenerate square matrix
and its inverse, 183.
- Complement of a simple minor detoid, 145.
- Complementary minor matrices or detoids:
defined, 102.
Affects of, 102, 104.
Sums of affected products of complementary
minor detoids, 140; 116, 138.
Complementary coranged minor detants of
a square matrix have like signs, 104.
- Complementary minor sequences:
defined, 56.
Affects of, 70, 71.
- Complete derived products:
defined, 3, 22.
Affects and signs of, 27.
Sum of, 12, 27.
Signs independent of order of factors, 13, 27, 47.
- Complete derived sequences:
defined, 55.
Affects of, 59, 62–3.
- Complete matrices of the minor detants of a
given fundamental matrix, 248:
of unaffected minor detants, 249;
of affected minor detants, 252.
Ranks of, 289.
Schemes of formation for, 249, 252.
Standard schemes, 251.
Common schemes, 250.
Standard complete matrix of minor detants,
251.
- Complete matrices of the minor detants of a
given matrix product, 253:
Corresponding schemes of formation for,
256.
Standard equations for, 258.
- Complete sets of unconnected solutions of a
system of homogeneous linear equa-
tions, 386:
Determination of all possible, 388.
Complete sets of mutually orthogonal solu-
tions, 406–7.
- Complete sets of unconnected solutions of any
system of linear equations, 395, 397:
Determination of all possible, 399.
Complete sets of mutually orthogonal solu-
tions, 411.
- Completed derived products, 22, 29:
Affects not altered by completion, 29.
- Completed minor matrices and detoids, 90, 91:
Affects not altered by completion, 91.
- Completed minor sequences, 56:
Affects not altered by completion, 56.
- Conditions:
that a matrix may have rank r , 288;
for solvability of a matrix equation, 301–2;
314, 327; 329, 337; 341, 357;
for non-zero solutions of a homogeneous
matrix equation, 325, 336, 354;
for solvability of a system of linear alge-
braic equations, 366, 374, 380–1.
- Conjugate matrix or detoid: defined, 5.
Self-conjugate or symmetrical, 5.
Rank of, 266.
Rank of a product of two mutually conju-
gate matrices, 288.
A detoid and its conjugate are equal, 16.
- Conjugate of a matrix product, 175, 205.
- Conjugate reciprocal matrix or detoid, 111, 180.
Product of a matrix and its conjugate
reciprocal, 180.
(See Reciprocal matrix.)
- Conjugate reciprocal of a square matrix, 181,
267, 309:
Rank of, 289.
Product of a square matrix and its conju-
gate reciprocal, 181, 263.
(See Reciprocal of a square matrix.)
- Conjugate reciprocal of a standard product of
square matrices, 261.
- Connections between the rows of a matrix, 269:
of a functional matrix, 294.
Unconnected connections, 326, 337.
Theorems concerning, 271–8, 287.
- Connections between linear equations, 364,
412, 415;
between linear functions, 364.
- Conventional equality of matrices, 154.
- Conventionally equal matrices, 154–7, 161,
173, 204:
are equivalent, 156, 173, 204.
- Coranged and deranged minors, 2, 55.
- Coranged minors of a matrix, 2:
Affects of complementary, 102, 104.
Complementary coranged minor detants of
a square matrix have like signs, 104.
- Coranged minors of a sequence, 55:
Affects of complementary, 70, 71.
Affects of coranged minors of a sequence
of natural numbers, 61.

- Corresponding rows in a matrix product, 160, 163, 191.
- Corresponding schemes of formation of matrices of minor detants, 256.
- Criteria for the rank of a matrix, 279, 280, 286, 288.
- Degenerate matrix: defined, 265; has no inverse, 267.
- Dependences between functions, 415.
- Derived determinants:
 Theorems concerning affects of, 96–8, 100–1. (See Minor determinants, Derived determinoids.)
- Derived matrices and determinoids:
 defined, 2, 16;
 coranged and deranged, 2;
 extended and completed, 90, 91;
 brought to leading position by moves, 91–3;
 complementary, 102, 104.
 Affects of, 86.
 Theorems concerning affects, 91–104.
 Affects unaltered by extension or completion, 91.
 (See Minor matrices, Affects of derived matrices.)
- Derived products:
 defined, 3, 22;
 complete, 12, 13;
 complete and incomplete, 3, 4, 22, 27, 47;
 extended and completed, 22, 29;
 affected and unaffected, 24;
 reduced to leading product by moves, 48.
 Affects of, relative affects, 24.
 Theorems concerning affects, 96–8, 100.
 Affects not altered by extensions, 29.
 Signs determined by affects, 24;
 independent of order of factors, 47.
 Sum of affected complete products, 12, 27.
 Sum of affected products of order k , 48, 134, 136–7.
- Derived sequences:
 defined, 55;
 coranged and deranged, 56;
 coranged, 58–9, 61, 65, 70–1;
 complete and incomplete, 55;
 complete, 59;
 complementary, 56, 70, 71;
 extended and completed, 56;
 brought to leading position by moves, 62–4.
 Affects of, 56;
 not altered by extensions, 56.
 Theorems concerning, 62–71.
 Inversions in, 60.
 Signs determined by affects of all distinct coranged minors of given order, 61.
 (See Minor sequences, Affects of derived sequences.)
- Determinant, defined to be the detoid of a square matrix, 12, 15.
 Agreement with usual definition, 50.
 Leading diagonal, order, as in square matrix, 1.
 Conjugate detant, self-conjugate or symmetrical detant, 5.
 Reciprocal detant, 110, 267.
 Conjugate reciprocal detant, 111, 180.
 Bordered detant, expansion of, 240.
- Determinant (*cont.*):
 Detant of any matrix product whose two activities are equal, 220, 232, 236.
 Detant of a product of three matrices whose two activities are equal, 236.
 (See Determinoid.)
- Determinoid of a rectangular matrix:
 defined as sum of affected complete derived products of the matrix, 24.
 Rule of signs for, 12.
 Abbreviated notations for, 17–19, 347.
- Determinoid, definitions of terms used in connection with:
 Affects, coranged and deranged minors, derangements, derived detoids, derived matrices, derived products, derived element, leading line, leading position, leading rows, long and short rows, minor detoids, minor matrices, moves, orders, rank, steps have the same signification for the determinoid of a matrix as for the matrix itself.
 (See Rectangular matrix, Affects.)
- Determinoid, use of terms conjugate and reciprocal:
 The conjugate detoid of a matrix is the detoid of the conjugate matrix, 5.
 A detoid and its conjugate have equal values, 17.
 The reciprocal detoid of a matrix is the detoid of the reciprocal matrix, 110.
 The conjugate reciprocal detoid of a matrix is the detoid of the conjugate reciprocal matrix, 111.
 The terms ‘conjugate of,’ ‘reciprocal of,’ ‘determinoid of’ are commutative with one another.
- Determinoid, properties of, 19–21, 36, 43, 48, 52–4, 91–104, 105–52.
 Properties of:
 long rows, 19–21, 36, 43–4, 105–9;
 short rows, 111–3, 140;
 long-cut minors, 116–23, 136–8;
 short-cut minors, 113–5, 123–7, 134–6, 138–40;
 superior minors, 113–5, 123–7;
 inferior minors, 116–23, 134–40.
- Determinoid of a product of two matrices, 209–20:
 when both matrices are square, 210;
 when passivity < efficiency, 209, 212;
 when passivity = efficiency, 213;
 when passivity < efficiency, 215–7.
 General formula for, 210.
 Rules for evaluation of, 217.
 Special cases:
 one matrix a unit matrix, 219;
 one matrix a scalar matrix, 219;
 two inversely similar matrices, 220.
- Determinoid of any matrix product, 221–32:
 when one passivity < efficiency, 221;
 when no passivity < efficiency, 225.
 Theorems concerning, 221.
 General formulae for, 227, 228.
 Progressive development of, 229.
 Expressions for,
 when the activities are equal, 232, 236;
 when they are unequal, 243.

- Determinoid of any matrix product (*cont.*):
 Special cases:
 one factor a unit matrix, 231;
 one factor a scalar matrix, 231;
 one factor numerical, 231.
 Determinoid, generalisation of, 48, 134, 136–7, 244.
 (*See* Sum of affected derived products of order k .)
 Determinoids: Addition of, 107.
 Multiplication by a scalar number, 19, 108.
 Interchange of two long rows, 20, 36, 43.
 Two long rows identical, 20, 44.
 Vanishing of a long row, 107.
 Addition to one long row of scalar multiples of other long rows, 107–9.
 Reversals of rows, 53.
 Affected minor detant invariant for all derangements of its rows, 101.
 Affected minor detoid invariant for all derangements of its long rows, 101.
 Reciprocal of a detoid whose orders differ by 1, 268.
 Bordered detoid, 241.
 Determinoids, expansions of:
 in terms of the elements of a long row, 105;
 generalisation, 110;
 in terms of simple minor detants, 113;
 generalisation, 123;
 in terms of the simple minor detants of a long-cut minor matrix, 116;
 generalisations, 121–3; 143, Case I.
 (*See* Sum.)
 Dissimilar matrices:
 Equality of, 154.
 Addition and subtraction of, 156.
 Distinct minors of a sequence, 232, 248;
 standard arrangement of, 251;
 minor detants, 115, 249;
 horizontal and vertical minor matrices, 249.
 Distributive law:
 for multiplication by a scalar number, 157;
 for matrix products, 173, 184, 188–9.
 Double-suffix notation:
 for matrices, 5–9;
 for detoids, 17, 18.
 Efficiency of a matrix (or detoid), 1;
 of a matrix product, 191.
 Elements of a matrix (or detoid), 1.
 Affects of the elements, affected and unaffected elements, vi;
 relative affects, 24.
 Expressions for the elements of a product matrix, 159, 162–3, 194.
 Leading element, 1.
 Elements of a sequence; affects of the elements, relative affects, leading element, 55–6.
 Equality of matrices: 153–5;
 identical (similar matrices), 154;
 conventional (dissimilar matrices), 154.
 Conventionally equal matrices, 154–6, 161, 173, 204;
 are equivalent, 156, 173, 204.
 Equations: Linear algebraic equations, 364–415.
 Connections between, 364, 412, 415.
 Extravagant equations, 388–92.
 Equations with only extravagant solutions, 391, 409–10.
 (*See* Systems of linear algebraic equations.)
 Equations: Matrix equations, 156, 176, 299–358.
 Cancellation of factors in, 357, 325, 336, 354.
 Manipulation of, 156–7, 176–7, 357.
 Matrix equations of the first degree:
 $X = A$, 300;
 $X + A = B$, 302;
 $AX = C$, 302–5, 312–27;
 $XB = C$, 306–9, 327–37;
 $AXB = C$, 309–11, 337–57;
 $AX = 0$, 324–6;
 $XB = 0$, 335–7;
 $AXB = 0$, 354–6.
 (*See* Matrix equations of the first degree.)
 Equipotent matrices, 285, 297;
 have equal ranks, 283, 286, 297.
 Equivalence of conventionally equal matrices, 156, 173, 204.
 Expansion of bordered detant, 240;
 of bordered detoid, 241.
 Expansions of a detoid:
 in terms of elements of a long row, 105;
 generalisation, 110;
 in terms of simple minor detants, 113;
 generalisation, 123;
 in terms of simple minor detants of a long-cut minor matrix, 116;
 generalisations, 121–3; 143, Case I.
 (*See* Determinoid, Sum.)
 Expressions for:
 affects of derived sequences, 57;
 detoid of a product of two matrices, 210, 215–6, 220;
 detoid of any matrix product, 228, 233, 243;
 elements of a product matrix, 159, 162–3, 194.
 Extended derived matrix, 90–1;
 derived product, 22, 29;
 derived sequence, 56.
 Affects unaltered by extension, 91, 29, 56.
 Extravagant equations, 388–92.
 Extravagant matrix, defined, 288.
 Rank of product of two mutually conjugate extravagant matrices, 289.
 Extravagant solutions of linear equations, 404;
 are self-orthogonal, 405.
 Examples of, 390–1.
 Condition that all solutions shall be extravagant, 409;
 Systems having only extravagant solutions, 391, 409, 410.
 Factor matrices, defined, 158, 190.
 Factors, cancellation of:
 in a matrix equation, 357; 325, 336, 354;
 in a matrix identity, 358.
 Factors of a derived product, 3;
 relative affects of, 24.
 Sign of product independent of the order of its factors, 13, 47.

- Factors of special form in a matrix product:
 zero matrix, 207;
 unit matrix, 177, 207, 219;
 scalar matrix, 178, 207, 219;
 quasi-scalar matrix, 179;
 scalar number, 157, 206;
 square matrix whose order is equal to the efficiency, 224.
- Formation, *see* Schemes of formation.
- Forward moves: defined, 49, 57;
 applied, 49, 50; 62; 91.
- Forward steps: defined, 4, 55;
 applied, 12, 24; 55, 56.
- Functional dependences, 415.
- Functional matrices:
 Rank and connections of, 295.
 Equipotent, 297.
- Fundamental matrix or detoid, 86, 248;
 sequence, 55.
- General solution of a matrix equation, formulae for:
 $AX=C$, 317, 319; 303. $AX=0$, 324.
 $XB=C$, 330, 331; 306. $XB=0$, 336.
 $AXB=C$, 345, 349; 310. $AXB=0$, 354.
- General solution of a system of linear algebraic equations, formulae for:
 any system, 368–9, 375–6; 400–3;
 homogeneous system, 381–2; 392.
- Generalisation of a determinoid, 48, 134, 136–7, 244.
 (*See* Sum of affected derived products of order k .)
- Generalisation of Laplace's development of a determinant, 119, 123; 143, Case I.
 (*See* Expansions of a detoid, Sum.)
- Homogeneous linear equations, *see* System.
- Horizontal suffixes, 12;
 affects, vi, 24;
 moves, 48;
 steps, 4;
 minors, 233–4, 249.
- Identical equality of matrices, 154.
- Identical long rows in a detoid, 20, 44.
- Identity: Cancellation of matrix factors in a matrix identity, 358.
- Incomplete derived sequence, 55;
 extended or completed, 56;
 affect not altered, 56.
- Incomplete derived product, 4, 22;
 extended or completed, 22, 23;
 affect not altered, 29.
- Independence of functions, condition for, 416.
- Inferior simple minors, 115, 134, 136–7, 143.
 Sum of similar affected inferior simple minor detoids:
 short-cut, 134; long-cut, 136.
 (*See* Simple minor, Sum.)
- Inner elements of a sequence, 76;
 rows and elements of a matrix, 39, 44.
- Interchange of
 two parallel rows in a matrix, 30, 38;
 inner and outer elements and rows, 39;
 two suffixes in a matrix, 36, 44;
 inner and outer elements and rows, 44;
- Interchange of (*cont.*)
 two elements in a sequence, 72, 76;
 inner and outer elements, 76;
 two elements in a derived product, 13, 27, 47;
 two long rows in a detoid, 20, 36, 43;
 two short rows in a detoid, 44.
- Interchange of two long rows changes the sign of:
 the fundamental detoid, 20, 36, 43;
 every complete derived product, 44;
 every simple minor detant, 44;
 every superior short-cut minor detoid, 44.
- Invariance of the sign of a derived product, 47;
 of a complete derived product, 13.
- Inverse matrices: defined, 183;
 principal inverse matrix, 182;
 inverse post-factors, 183, 322;
 inverse pre-factors, 183, 335;
 unique inverse of undegenerate square matrix, 183, 267, 309;
 inverse of non-singular matrix, 267.
 Degenerate matrix has no inverse, 322, 335.
 Product of undegenerate square matrix and its inverse is commutative, 183.
- Inversions in a derived sequence, 60.
 Affect determined by counting them, 60.
- Irreducible matrix equations:
 defined, 312, 328, 338.
 Solutions when finitely solvable, 323, 333, 352.
- Laplace's development of a determinant, generalisations of, 119, 123; 143, Case I.
- Law of equality for matrices, 153.
 Commutative law for addition of matrices, 155–6;
 not true for multiplication, 174, 208.
- Associative law for addition of matrices, 155–6;
 for matrix products, 184, 186, 188.
- Distributive law for multiplication by a scalar number, 157;
 for matrix products, 173, 184, 188–9.
- Leading element, line, row, diagonal, position, 1, 2, 16, 55.
- Moves which bring to leading position:
 a derived product, 49;
 a derived sequence, 62–4;
 a derived matrix, 91–4.
- Linear algebraic equations: 364–415.
 Connections between, 364, 412, 415.
 (*See* Systems of linear algebraic equations.)
- Linear functions, connections between, 364.
- Long rows: defined, 1, 16.
 Properties of, 19–21, 36, 43–4, 105–9.
 Addition of detoids, 107.
 Interchange of two long rows, 20, 36, 43.
 Two long rows identical, 20, 44.
 Vanishing of a long row, 107.
 Multiplication of a detoid by a scalar number, 19, 108.
 Expansion of a detoid in terms of elements of a long row, 105.
 Property of any two long rows, 108, 110.
 Addition to one long row of scalar multiples of other long rows, 109.

- Long-cut minors: 115–23, 136–8.
 Expansion of a detoid in terms of the simple minor detants of a long-cut minor, 119.
 Property of any two similar long-cut minors, 122, 123.
 Sum of long-cut simple minor detoids of given reduced order, 136.
- Manipulation of matrix equations, 156–7, 176–7, 357.
- Matrices: augmented and unaugmented, 8, 287, 312, 327, 337, 366, 374;
 degenerate and undegenerate, 265;
 equipotent, 238, 297;
 inverse, 182–3;
 similar, 153–5;
 singular and non-singular, 266.
 Addition and subtraction of, 155–6.
 Equality of, 153–5.
 Multiplication of, 158–64, 184–90.
 (*See also* sub-headings.)
- Matrices of minor detants, 248, 253;
 standard, 251.
 (*See* Complete matrices of minor detants.)
- Matrix: defined, 1, 153–64, 184–90;
 cannot be equated to a non-zero scalar number, 179;
 conjugate, 5;
 conjugate reciprocal, 111, 180;
 functional, 295;
 inverse, principal inverse, 182–3;
 quasi-scalar, 179;
 reciprocal, 110–11;
 rectangular, 1;
 scalar, 178;
 self-conjugate or symmetrical, 5;
 square, 1;
 undegenerate square, 183, 267, 289, 309;
 unit, 11, 177;
 zero, 265.
 (*See* Rectangular matrix, Square matrix and other sub-headings.)
- Matrix equations: 156, 176, 299–358.
 Cancellation of factors in, 357.
 Manipulation of, 156–7, 176–7, 357.
- Matrix equations of the first degree: 299–358;
 defined, 299, 300.
 The equation $X=A$, 300.
 The equation $X+A=B$, 302.
 The equations $AX=C$, $XB=C$, $AXB=C$:
 Solutions when A and B are undegenerate square matrices, 302, 306, 309.
 Augmented and unaugmented matrices, 312, 327, 337.
 Reduction to irreducible equations, 312, 328, 338.
 Conditions for finite solvability, 314, 329, 341.
 Solutions when irreducible and finitely solvable, 323, 333, 352.
 Formulae for general solutions, 317, 319; 330, 331; 345, 349.
 Simplest particular solutions, 317, 319; 330, 332, 345, 349.
 Infinite solutions, 326, 337, 356.
 The equations $AX=0$, $XB=0$, $AXB=0$:
 Conditions for non-zero solutions, 325, 336, 354.
- Matrix equations of the first degree (*cont.*):
 The equations $AX=0$, $XB=0$, $AXB=0$:
 General solutions, 304, 324; 307, 335; 310, 354.
 Ranks of solutions, 325, 336, 355–6.
- Matrix factors: defined, 158, 190.
 Cancellation of, 357–63.
 Special factors:
 quasi-scalar matrix, 179;
 scalar matrix, 178, 207, 219;
 unit matrix, 177, 207, 219;
 zero matrix, 207;
 square matrix whose order is equal to the efficiency of the product, 224.
- Matrix identities, cancellation of factors in, 358–63.
- Matrix products: defined, 158–64, 184–90;
 are associative, 186, 188;
 are distributive, 173, 188–9;
 are in general not commutative, 174, 208.
 Standard forms of, 158, 184, 192.
 Conjugates of, 175, 205.
 Reciprocals of standard products of square matrices, 261.
 Properties of passive rows, 164–8, 195–9.
 Properties of active rows, 168–72, 199–204.
 Distribution over partial products, 164, 167, 195, 199.
 (*See* Product.)
- Minor determinants:
 Affects of, theorems concerning, 96–8, 100–1.
 Affected minor detant is invariant for all derangements of its rows, 101.
 Co-factors of, 119.
 Complete matrices of, 248, 253.
 distinct, 115, 249;
 of a long-cut minor matrix, 122–3;
 of a short-cut minor matrix, 138.
 Sum of affected simple minor detants, 115, 209;
 affected minor detants of order k , 134, 136–7, 244.
 (*See* Minor matrices, Sum.)
- Minor matrices and determinoids:
 defined, 2, 16.
 Affects of, defined, 86–90;
 Theorems concerning, 90–104;
 Affected minor detoid invariant for derangements of its long rows, 101.
 brought to leading position by moves, 91–4;
 coranged and deranged, 2, 16;
 complementary, 102–4;
 extended and completed, 90, 91;
 affects unaltered by extension, 91.
 Co-factors of minor detoids, 141; 106, 119;
 of simple minor detoids, 145.
 Simple minors: 3, 115, 116;
 long-cut, 116, 123, 136;
 short-cut, 116, 123, 134, 138;
 superior, 116, 123;
 inferior, 116, 134, 136;
 horizontal and vertical, 233, 234, 249;
 reduced and unreduced orders of, 116.
 Sum of affected:
 simple minor detants, 115, 209;
 minor detants of order k , 134, 136–7; 244;
 superior simple minor detoids, 123;
 inferior simple minor detoids, 134, 136.

- Minor matrices and determinoids (*cont.*):
 Sum of products of a minor detoid and its co-factor belonging to two fixed complementary simple minors, 140–3; 107, 116, 138.
 (See also Affects of derived matrices, Sum.)
- Minor sequences: defined, 55.
 Affects of, defined, 56;
 Theorems concerning, 62–71.
 of complementary minors, 70.
 brought to leading position by moves, 62–4;
 coranged and deranged, 55;
 complete and incomplete, 55;
 complete, 59;
 complementary, 56, 70, 71;
 distinct, 61;
 standard arrangement of, 251;
 extended and completed, 56;
 affects unaltered by extension, 56.
 Expressions for affects, 57–62;
 of coranged minors, 59, 61;
 of coranged minors of a sequence of natural numbers, 61.
 Inversions in, 60.
 (See Affects of derived sequences.)
- Moves in a matrix: defined, 48;
 horizontal and vertical, 48;
 forward and backward, 49;
 moves bringing to leading position:
 a derived matrix, 91–3;
 affect thereby determined, 91;
 a derived product, 49;
 affect thereby determined, 49.
- Moves in a sequence: defined, 57;
 forward and backward, 57;
 minor sequence brought to leading position by moves, 49;
 its affect thereby determined, 49.
- Multiplication of a matrix by a scalar number, 157;
 is commutative and distributive, 157;
 of a matrix product by a scalar number, 206;
 of a determinoid by a scalar number, 19, 108.
- Multiplication of matrices, 158–64, 184–90;
 is associative and distributive but in general not commutative, 173–4, 184–90, 208.
 (See Product.)
- Mutually orthogonal solutions of
 any system of linear equations, 411;
 a system of homogeneous equations, 404.
 Systems all of whose solutions are mutually orthogonal, 409.
- Natural numbers, sequences of, 60;
 affect of any minor, 60;
 affect of a coranged minor, 61.
- Non-commutative property of matrix products, 174, 208.
- Non-extravagant equations, 388–92, 399.
- Non-extravagant mutually orthogonal solutions of a system of homogeneous linear equations, 405–7.
- Non-singular matrix: defined, 266.
 Reciprocal, conjugate reciprocal and inverse of, 267.
- Non-zero solutions, conditions for, 325, 336, 354, 381.
- Notation: for matrices, 1, 5, 347;
 for augmented matrices, 8;
 for detoids, 15, 17, 347;
 for sequence, 55, 57;
 for a scalar matrix, 178;
 for a unit matrix, 11, 177;
 $Q_m^n = \Sigma (-1)^\omega$, 127.
- Order of a square matrix and detant, 1, 16.
- Orders of a matrix, horizontal and vertical, 1;
 of a detoid, 1, 16.
 Reduced and unreduced orders of a simple minor, 116.
- Orthogonal solutions of a system of linear algebraic equations: defined, 405, 412;
 complete sets of, 406, 407, 411.
 Condition that all solutions shall be mutually orthogonal, 409.
- Outer elements of a sequence, 76;
 elements and rows of a matrix, 39.
- Parallel rows: Interchanges of, 30–44, *see* Interchange.
- Partial products: Matrix product expressed as a sum of, 164, 167; 195, 199.
- Passive rows:
 in a product of two matrices, 158;
 in any matrix product, 190.
 Properties of, 164–8, 195–9, 222.
- Passivities of a matrix product, and of matrix factors, 158, 191.
 Conditions for non-zero solutions, 325, 336, 354, 381.
 Conditions for cancellation, 357.
- Post-factors:
 inverse, of a matrix, 322;
 applied to solve the irreducible matrix equations $XB = C$, $AXB = C$, 333, 352;
 applied to solve a system of unconnected linear equations, 371–2.
 Cancellation of:
 in matrix equations, 357–8, 336, 354;
 in matrix identities, 358–63.
- Pre-factors:
 inverse of a matrix, 335;
 applied to solve the irreducible matrix equations $AX = C$, $AXB = C$, 323, 352;
 applied to solve a system of unconnected linear equations, 371.
 Cancellation of:
 in matrix equations, 357–8, 325, 354;
 in matrix identities, 358–63.
- Principal inverse of a matrix, 182.
 (See Inverse.)
- Product matrix: defined, 158, 161, 187, 189.
 Form of, 159, 163, 190.
 Expressions for elements of, 159, 162–3, 194–5.
 Conjugate of, 175, 205.
 Reciprocal and conjugate reciprocal of, 261.
- Product of a matrix and a scalar number, 157;
 is commutative and distributive, 157.
- Product of two matrices: 158–84, 209–20.
 Active rows, 158; 168–72, 217.
 Properties of, 168–72, 217.

- Product of two matrices (*cont.*):
 Activities, defined, 158.
 Conjugate of, 175.
 Conjugate reciprocal, 261.
 Corresponding rows, 160, 163–4.
 Detoid of, 209–20.
 is in general not commutative, 174.
 Commutative products, 175, 181–3, 262–3.
 is always distributive, 173.
 Efficiency, defined, 191.
 Elements of product matrix, expressions for, 159, 162–3.
 Equivalence of conventionally equal matrices, 173.
 Partial products, product expressed as sum of, 164, 167.
 Passive rows, 158, 191; 164–8, 217.
 Properties of, 164–8, 217.
 Passivities, defined, 158.
 Product matrix, form of, 159, 163.
 Rank of product matrix, 281–4, 288–9, 293–4.
 Reciprocal and conjugate reciprocal of a standard product of two square matrices, 261.
 Redundant final passive rows, 161, 165.
 Standard form, 158.
 Reduction to, 162.
 (*See also* Determinoid.)
 Product of two matrices.—Special cases:
 One factor a unit matrix, 177;
 a scalar matrix, 178;
 a quasi-scalar matrix, 179;
 an undegenerate square matrix, 283.
 A matrix and its conjugate reciprocal, 180.
 An undegenerate matrix and its principal inverse, 181.
 Two mutually inverse matrices, 182–3, 267.
 Two inversely similar matrices, 183.
 Two mutually conjugate matrices, 288–9, 389.
 Commutative products:
 A square matrix and its conjugate reciprocal, 181, 263, 309.
 An undegenerate square matrix and its inverse, 183, 309.
 Two square matrices whose product is a non-zero scalar matrix, 266, 309.
 Product of three matrices: 184–8, 236–43;
 is associative, 186;
 is distributive, 188;
 is in general not commutative, 174, 208.
 Rank of, in special cases, 285.
 Ranks of two equipotent matrices, 286.
 Reduction to standard form, 184–6; 192.
 Product formed by any chain of matrix factors: 188–208, 221–36, 243–4, 244–7, 253–60.
 Active rows, 191, 199–204, 222–3.
 Properties of, 199–204, 222–3.
 Activities, defined, 191.
 Conjugate of, 205.
 Conjugate reciprocal, 261.
 Corresponding rows, 191–2.
 Detoid of, 221, 236.
 Generalisation of detoid, 253–60.
 is associative, 188–9;
 is distributive, 189–90;
 Product formed by any chain of matrix factors (*cont.*):
 is in general not commutative, 189; 174, 208.
 Efficiency, defined, 191.
 Elements of product matrix, expressions for, 194–5.
 Equivalence of conventionally equal matrices, 204–5.
 Multiplication by a scalar number, 206.
 Partial products, product expressed as a sum of, 195–9, 222.
 Passive rows, 191, 195–9, 222.
 Properties of, 195–9, 222.
 Passivities, defined, 191.
 Product matrix, form of, 190.
 Rank of product, superior limit to, 283.
 Reciprocal and conjugate reciprocal of a standard product of square matrices, 261.
 Standard form of product, 192;
 Reduction to, 192.
 (*See also* Determinoid.)
 Product formed by a chain of matrix factors.—Special cases:
 One factor a zero matrix, 207;
 a unit matrix, 207, 231;
 a scalar matrix, 207, 231.
 Standard product of square matrices, reciprocal and conjugate reciprocal of, 261.
 Products, derived, 22–52.
 (*See* Derived products.)
 Properties of
 active rows, 168–72, 199–204, 217, 222–3;
 passive rows, 164–8, 195–9, 217, 222;
 long rows, 19–21, 36, 43–4, 105–9;
 short rows, 111–3, 140;
 long-cut minors, 116–23, 136–8;
 short-cut minors, 113–5, 123–7, 134–6, 138–40;
 superior minors, 113–5, 123–7;
 inferior minors, 116–23, 134–40;
 the number Q_m^n , 127–34.
 Quasi-scalar matrix, 179.
 Q_m^n , defined, 127.
 Properties of, 127–34.
 Table of values, 131.
 Rank of a matrix: defined, 265.
 Conditions that rank shall be r , 279, 288.
 Criteria for rank, 279, 280, 286, 288.
 Theorems concerning, 278–89.
 Theorems concerning connections and, 271–8.
 unaltered by derangements, 266;
 unaltered when the matrix is multiplied by or into an undegenerate square matrix, 283.
 Zero rank, 265.
 Rank of augmented matrix, 287;
 conjugate matrix, 266;
 functional matrix, 294;
 reciprocal of a square matrix, 267, 289;
 complete matrix of minor detants, 289;
 solutions of the equations $AX=0$, $XB=0$, $AXB=0$, 325, 336, 355.

- Rank of a product of two matrices:
 cannot exceed rank of either factor, 281.
 Special cases:
 one factor is an undegenerate square matrix, 283;
 both factors are undegenerate and one has rank equal to its passivity, 282, 293;
 two mutually conjugate real matrices, 288;
 two mutually conjugate undegenerate matrices, 288–9.
- Rank of a product of three matrices:
 Three undegenerate matrices with a common passivity, 285.
 Equipotent matrices have equal ranks, 285.
- Rank of any matrix product:
 cannot exceed rank of any factor, 283.
- Real linear equations, systems of, 387, 389, 399, 406, 411.
 Complete sets of mutually orthogonal real unit solutions, 406, 411.
- Reciprocal matrix and detoid: defined, 110;
 conjugate reciprocal defined, 111.
 Fundamental property of the reciprocal or conjugate reciprocal matrix, 110–1, 180–1.
 Reciprocal detant of a square matrix, 267.
 Reciprocal detoid of a matrix whose orders differ by 1, 268.
 Reciprocal of a non-singular matrix is undegenerate, 267.
 Reciprocal of an undegenerate square matrix is undegenerate, 267.
 Rank of the reciprocal of a square matrix, 289.
 Reciprocal and conjugate reciprocal of a standard product of square matrices, 261.
- Rectangular matrix: defined, 1.
 Standard notations for, 1, 5, 10.
 Abbreviated notations, 1, 5–12, 347.
- Rectangular matrix, definitions of terms used in connection with:
 Elements, horizontal and vertical orders, efficiency, long and short rows, 1.
 Derived products: complete and incomplete, 3, 4.
 Order of a derived product, 4.
 Steps: forward, backward, horizontal and vertical, 4.
 Moves: forward, backward, horizontal and vertical, 48–9.
 Inner and outer rows and elements, 39, 44.
 Derangements of the matrix, 2.
 Minor matrices: corranged and deranged, 2.
 Simple minor matrices:
 long-cut and short-cut, 116;
 horizontal and vertical, 233–4, 249;
 superior and inferior, 116;
 reduced and unreduced orders of, 116.
 Derived matrices, 2.
 Leading element, leading line, leading row, leading position, 1, 2.
 Affects of elements and derived products, vi, 24.
 Affects of derived or minor matrices, 86–8.
 Determinoid or determinant of the matrix, 12, 24.
- Rectangular matrix, definitions of terms used in connection with (*cont.*):
 Derived detoids and detants, minor detoids and detants, 16.
 Affects of derived or minor detoids and detants, 86–8.
 Complementary minor matrices or minor detoids, 102.
 Co-factor of:
 an element, 106;
 a minor detant, 119;
 a minor detoid, 141.
 Rank of the matrix, 265, 295.
 Connections between the rows, 269, 295.
- Rectangular matrix.—Allied matrices defined:
 Conjugate matrix, 5.
 Reciprocal matrix, 110.
 Conjugate reciprocal matrix, 111.
 Inverse, principal inverse, 182–4.
 Complete matrices of minor detants of given order, 248.
 Standard complete matrix of minor detants, 251.
- Rectangular matrix.—Special forms:
 Square matrix, 1, 15;
 order of, 1;
 leading diagonal, 1;
 detant of, 15.
 Self-conjugate or symmetrical matrix, 5.
 Scalar matrix, 178.
 Quasi-scalar matrix, 179.
 Unit matrix, 11–12, 177.
 Zero matrix, 155.
- Reduced order of a simple minor, 116.
- Reduction of:
 affects of derived products to affects of sequences, 83;
 any fundamental sequence to a sequence of natural numbers, 60;
 a matrix equation to an irreducible matrix equation, 312, 328, 338;
 a matrix product to standard form, 162, 184, 192.
- Redundant final passive rows in a matrix product, 161, 163, 191.
- Relative affects:
 of elements of a derived sequence, 56;
 of factors of a derived product, 24.
- Reversals of rows in a detoid, 53.
- Rows:
 Active and passive, 158, 190.
 Properties of active, 168, 199, 222.
 Properties of passive, 164, 195, 222.
 Connected and unconnected, 269, 294.
 Theorems concerning connections, 271, 287.
 Corresponding, in a matrix product, 160, 163, 191.
 Interchange of two parallel, 30, 38;
 two long rows, 20, 36, 43;
 two short rows, 44.
 Long and short, 1, 16.
 Properties of long, 19–21, 36, 43–4, 105–9.
 Properties of short, 111–3, 140.
 Redundant final passive, 161, 163, 191.
- Rows of 0's, or zero rows:
 final, in any matrix, 154;
 active, in a matrix product, 200–1;

- Rows of 0's, or zero rows (*cont.*):
 passive, in a matrix product, 196–8;
 long, in a detoid, 107;
 short, in a detoid, 112.
 Rule of Signs for a detoid, 12.
 Rules for determining:
 the affect of a derived product, 25–30;
 the affect of a derived sequence, 57–62;
 changes in affect and sign of a derived product due to an interchange of rows or suffixes, 30, 36, 38, 44;
 change in affect of a derived sequence due to an interchange of elements, 72, 74, 77, 81.
 Rules for evaluating the detoid:
 of a product of two matrices, 217;
 of any matrix product, 222–5.
 Scalar matrix: defined, notation for, 178;
 occurring as factor:
 in a product of two matrices, 178, 219;
 in any matrix product, 207, 231.
 Scalar number:
 Multiplication of a matrix by, 157.
 Multiplication of a detoid by, 19, 108.
 occurring as a factor in any matrix product, 206, 231;
 cannot be equated to any non-zero matrix, 179.
 Schemes of formation for complete matrices of the minor detants of order k of a given fundamental matrix, 249–53.
 Standard scheme, 251.
 Common schemes, 250.
 Corresponding schemes for the formation of a complete matrix of the minor detants of a matrix product, 256.
 Self-conjugate or symmetrical matrix, self-conjugate determinant, 5.
 Short rows: defined, 1, 16.
 Properties of, 111–3, 140.
 of 0's, 112.
 Sum of products obtained by multiplying each element of a short row by its co-factor, 140.
 Short-cut minors: defined, 116.
 Properties of, 113–5, 123–7, 134–6, 138–40.
 Sums of similar affected short-cut minor detoids, 113, 123, 134, 137.
 Sums of products obtained by multiplying each simple minor detant of a given short-cut minor matrix by its co-factor, 138.
 Sign determined by affect, vi, 24.
 Change in sign of a derived product caused by an interchange of two rows or suffixes, 30, 36, 38, 44.
 Sign of derived product independent of order of arrangement of factors, 47; 13, 27.
 Interchange of two long rows changes the sign of a detoid, 20, 36, 43.
 Signs of all coranged minor sequences of given order, 61.
 Rule of Signs for a detoid, 12;
 self-consistency of, 13.
 Similar matrices: defined, 153.
 Addition of, 155.
 Inversely similar matrices, 153;
 product of, 179.
 Simple minor matrices and detoids:
 defined, classification, 3, 115.
 Long-cut, 116, 123, 136.
 Short-cut, 116, 123, 134, 138.
 Superior, 116, 123.
 Inferior, 116, 134, 136.
 Horizontal and vertical, 233, 234, 249.
 Reduced and unreduced orders of, 116.
 Sums of similar affected simple minor detoids:
 superior detoids, 123;
 inferior detoids, 134, 136;
 simple minor detants, 115, 209.
 (See Sum.)
 Single-suffix notations:
 for matrices, 10;
 for sequences and one-rowed matrices, 11, 55, 58;
 for detoids, 18, 19.
 Singular and non-singular matrices:
 defined, 266.
 Reciprocal, conjugate reciprocal and inverse of a non-singular matrix, 267.
 Solutions of equations.
 (See Matrix equations, Systems of linear algebraic equations.)
 Solvability of $AX=C$, $XB=C$, $AXB=C$:
 Conditions for finite solution, 314, 329, 341.
 Conditions for infinite solutions, 327, 337, 357.
 Solvability of $AX=0$, $XB=0$, $AXB=0$:
 Conditions for non-zero solutions, 325, 336, 354.
 Solvability of a system of linear algebraic equations, 366, 374.
 Condition for non-zero solutions of a homogeneous system, 381.
 Square matrices, products of:
 Commutative products:
 A square matrix and its conjugate reciprocal, 181, 263, 309.
 An undegenerate square matrix and its inverse, 183, 309.
 Two square matrices whose product is a non-zero scalar matrix, 262, 266, 309.
 Reciprocal and conjugate reciprocal of any standard product of square matrices, 261.
 Square matrix: Order, leading diagonal, 1.
 Determinant of reciprocal, 267.
 Rank of reciprocal, 289.
 Special forms:
 Unit matrix, 11, 177, 207, 231.
 Scalar matrix, 178, 207, 231.
 Quasi-scalar matrix, 179.
 Self-conjugate matrix, 5.
 Undegenerate square matrix:
 has a unique inverse, 183, 309;
 reciprocal is undegenerate, 267, 289;
 multiplication by or into, does not alter rank, 283.
 (See also Rectangular matrix.)
 Standard arrangement of distinct minor sequences of given order, 251.

- Standard complete matrix of minor detants, 251, 258.
 Standard equation for, 258.
 Standard schemes of formation, 251, 258.
- Standard notations for matrix and detoid:
 single-suffix, 10; 18; 11, 55, 58;
 double-suffix, 1, 5; 17.
- Standard product, standard form of matrix product: 158, 184, 192.
 Reduction to, 162, 184, 192.
 Expressions for elements of, 195.
- Steps in a matrix: forward, backward, horizontal and vertical, 4.
 Affects determined by counting, vi, 24; 12, 13.
- Steps in a sequence: forward and backward, 55.
 Affects determined by counting, 56.
- Subtraction of matrices, *see* Addition.
- Suffixes: vertical and horizontal, 6, 12.
 Double-suffix notations, 5–9, 17, 18.
 Single-suffix notations, 10, 18, 19; 11, 55, 58.
- Sum of affected:
 minor detants of order k , 134, 136, 137; 244;
 derived products of order k , 134, 136, 137; 244;
 simple minor detants, 115; 209;
 complete derived products, 12, 27.
- Sum of affected:
 superior short-cut simple minor detoids of reduced order k , 123;
 inferior short-cut simple minor detoids of reduced order k , 134, 137;
 (inferior) long-cut simple minor detoids of reduced order k , 136, 137.
- Sum of products obtained from a matrix or detoid by multiplying:
 a minor detoid Δ of given orders by its co-factor Δ' , Δ and Δ' belonging to two fixed complementary simple minor matrices, 140;
 a simple minor detant of a given long-cut minor matrix by its co-factor, 116;
 a simple minor detant of a given short-cut minor matrix by its co-factor, 138;
 an element of a given long row by its co-factor, 107;
 an element of a given short row by its co-factor, 140.
- Sum of products obtained from a matrix or detoid by multiplying:
 a simple minor detant of a given long-cut minor matrix by the co-factor of the corresponding simple minor detant of a similar long-cut minor matrix, 122, 123;
 an element of a given long row by the co-factor of the corresponding element of another long row, 107, 108, 110.
- Superior simple minors, 116, 119, 123, 140–3.
 Sum of similar affected, 119, 123.
 (*See* Sum.)
- Symmetrical matrix, *see* Self-conjugate.
- Symmetrical forms of the general solution of any system of linear algebraic equations, 399–404;
 of a homogeneous system, 392–4.
- System of homogeneous linear algebraic equations, 380, 386, 392, 404.
 Definitions:
 Matrix of the system, 380;
 orthogonal solutions, 405;
 extravagant and non-extravagant solutions, unit solutions, zero solutions, 404.
 Condition for non-zero solutions, 381.
 General solution:
 Ordinary forms, 381–2.
 Symmetrical form, 392–3.
 Extravagant solutions:
 are self-orthogonal, 405.
 Examples of, 390–1.
 Condition that all solutions shall be extravagant, 409.
 Mutually orthogonal solutions:
 Mutually orthogonal non-extravagant solutions are unconnected, 406.
 Complete sets of mutually orthogonal real solutions of a real system, 406.
 Condition that all solutions shall be mutually orthogonal, 409.
 Solutions connected or unconnected with the active rows of the matrix of the system, 389–91, 399.
 Unconnected solutions: 386.
 Determination of all possible complete sets of, 388.
- System of unconnected equations, 383.
- System of $n-1$ unconnected equations in n variables, 383–4.
- Systems of real equations, 387, 389, 406.
- Systems in which all solutions are extravagant or all solutions are mutually orthogonal, 391, 392, 409–10.
- Systems of linear algebraic equations in general, 365, 373, 395, 399–411.
 Definitions:
 Augmented and unaugmented matrices, 366, 374;
 augmented and unaugmented solutions, 395;
 finite solutions, 365;
 infinite solutions, 403;
 orthogonal solutions, 412.
 Conditions for solvability, for finite solutions, for infinite solutions, 366, 374.
 General solution:
 Ordinary forms, 368–9, 375–6.
 Symmetrical forms, 400–3.
 Infinite solutions, 403–4.
 Mutually orthogonal solutions, 411–12.
 Complete sets of mutually orthogonal real solutions of a real system, 411.
 Unconnected solutions, 395.
 Determination of all possible complete sets of, 388, 397, 399.
 System of unconnected equations, 371.
 System of n unconnected equations in n variables, 376–7.
- Total affects, vi, 24.
 (*See* Affects.)
- Unaffected elements, vi.
 derived products, 24.
 detants, 249, 252–60.

- Unaugmented matrices of the matrix equations $AX=C$, $XB=C$, $AXB=C$, 312, 327, 337;
 of systems of linear algebraic equations, 366, 374.
- Unaugmented solutions of systems of linear algebraic equations, 395, 397, 402, 411.
- Unconnected connections between the rows of a matrix, 326, 337.
- Unconnected linear equations, 365, 371, 376, 383, 415;
 linear functions, 365.
 rows of a matrix, 269.
 rows of a functional matrix, 295.
 (*See Connections.*)
- Unconnected solutions of systems of linear equations, 386, 395.
 Determination of complete sets of, 388, 397, 399.
- Undegenerate matrix: defined, 265.
 Complete matrices of its minor detants are undegenerate, 289.
 If the product of two matrices is a non-zero scalar matrix (or an undegenerate square matrix), the factor matrices are undegenerate, 267.
 The inverse and reciprocal of a non-singular matrix are undegenerate, 267.
- Special products of undegenerate matrices, 180–4, 262, 266–7, 282, 285, 288, 293, 309.
- Undegenerate square matrix:
 Reciprocal and conjugate reciprocal are undegenerate, 267, 289.
 has a unique undegenerate inverse, 183, 267, 309.
 Multiplication by or into it does not alter rank, 283.
 Special products of two undegenerate square matrices, 181, 183, 262, 266, 309.
- Unit matrix: defined, notation for, 11, 177;
 occurring as factor in a matrix product, 177, 207; 219, 231.
- Unreduced order of a simple minor, 116.
- Vertical affects, vi, 24, 87;
 minors, 233, 249;
 moves, 48;
 steps, 4;
 suffixes, 6, 12.
- Zero matrix, 155, 207.
- Zero rank, 265.
- Zeros, rows of:
 final, in any matrix, 154;
 active, in a matrix product, 200–1;
 passive, in a matrix product, 196–8;
 long, in a detoid, 107;
 short, in a detoid, 112.