MATRICES

AND

DETERMINOIDS

University of Calcutta
Readership Lectures

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AND

DETERMINOIDS

BY

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PREFACE

THE present work is an amplification of a course of lectures given for the University of Calcutta in the winter of 1909–10. Its chief feature is that it deals with *rectangular matrices* and *determinoids* as distinguished from *square matrices* and *determinants*, the determinoid of a rectangular matrix being related to it in the same way as a determinant is related to a square matrix. An attempt is made to set forth a complete and consistent theory or calculus of rectangular matrices and determinoids.

The first volume contains the most fundamental portions of the theory, and concludes with the solution of any system of linear algebraic equations, which is treated as a special case of the solution of a matrix equation of the first degree.

A second volume, which is nearly ready, will contain further developments of the general theory, including a discussion of matrix equations of the second degree. It will also contain a large number of applications to Algebra and to the Analytical Geometry of space of two, three and n dimensions.

A third volume, if opportunity should occur for its completion, would deal chiefly with applications to Vector Analysis and the Theory of Invariants. The complete exposition was in fact undertaken with a view to these last-mentioned applications.

A first attempt of this kind must necessarily contain many imperfections. In particular it may be pointed out that in the present volume no complete account has been given of the properties of the reciprocal of a rectangular matrix, and that the discussion of extravagant solutions of a system of linear algebraic equations is incomplete.

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Owing to the omission of an article at the beginning of Chapter II, the *affects* of the elements of a matrix have not been formally defined. This omission is briefly supplied here:

§ 5*a*. Affects of the elements of a matrix.

When we pass from the leading element a_{11} of the matrix

$$[a]_m^n = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1y} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2y} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{x1} & a_{x2} & \dots & a_{xy} & \dots & a_{xn} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{my} & \dots & a_{mn} \end{bmatrix}$$

to any other element a_{xy} by vertical and horizontal forward steps, the number of vertical steps must always be $x-1$, and the number of horizontal steps must always be $y-1$.

The *affect of the element* a_{xy} of the matrix $[a]_m^n$ is defined to be the number ω given by the equation

$$\omega = (x-1) + (y-1).$$

The numbers $x-1$ and $y-1$ are called respectively the *vertical affect* and the *horizontal affect* of a_{xy}, these being respectively the number of vertical steps and the number of horizontal steps which are taken. Their sum, the number ω, is called the *total affect* or simply the *affect* of the element a_{xy}.

The *sign determined by the affect* ω is + or − according as $(-1)^\omega = +1$ or -1, i.e. according as ω is even or odd.

Further $(-1)^\omega a_{xy}$ will be called the *affected element* corresponding to the *unaffected element* a_{xy}.

If in passing from a_{11} to a_{xy} both forward and backward steps are permissible, then the number of vertical steps taken differs from $x-1$ by an even number, and the number of horizontal steps taken differs from $y-1$ by an even number; also the total number of steps taken differs from ω by an even number.

My thanks are due to Sir Asutosh Mukhopadhyay, Vice-Chancellor of the University of Calcutta, for encouragement in the prosecution of this work; to the Syndicate of the University of Calcutta for their liberality in defraying the cost of publication; and to the officials of the Cambridge University Press for the great care which has been exercised in the printing.

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