

Cambridge University Press

978-1-107-65032-9 - The Collected Mathematical Papers of James Joseph Sylvester: Volume 1(1937–1853)

James Joseph Sylvester

Frontmatter

[More information](#)

---

# MATHEMATICAL PAPERS

Cambridge University Press

978-1-107-65032-9 - The Collected Mathematical Papers of James Joseph Sylvester: Volume 1(1937–1853)

James Joseph Sylvester

Frontmatter

[More information](#)

---

Cambridge University Press

978-1-107-65032-9 - The Collected Mathematical Papers of James Joseph Sylvester: Volume 1(1837–1853)

James Joseph Sylvester

Frontmatter

[More information](#)

THE COLLECTED  
MATHEMATICAL PAPERS

OF

JAMES JOSEPH SYLVESTER

F.R.S., D.C.L., LL.D., Sc.D.,

Honorary Fellow of St John's College, Cambridge;

Sometime Professor at University College, London; at the University of Virginia;  
at the Royal Military Academy, Woolwich; at the Johns Hopkins University, Baltimore  
and Savilian Professor in the University of Oxford

VOLUME I

(1837—1853)

Cambridge

At the University Press

1904

Cambridge University Press  
978-1-107-65032-9 - The Collected Mathematical Papers of James Joseph Sylvester: Volume 1(1937–1853)  
James Joseph Sylvester  
Frontmatter  
[More information](#)

---

CAMBRIDGE UNIVERSITY PRESS  
Cambridge, New York, Melbourne, Madrid, Cape Town,  
Singapore, São Paulo, Delhi, Tokyo, Mexico City

Cambridge University Press  
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

[www.cambridge.org](http://www.cambridge.org)  
Information on this title: [www.cambridge.org/9781107650329](http://www.cambridge.org/9781107650329)

© Cambridge University Press 1904

This publication is in copyright. Subject to statutory exception  
and to the provisions of relevant collective licensing agreements,  
no reproduction of any part may take place without the written  
permission of Cambridge University Press.

First published 1904  
First paperback edition 2011

*A catalogue record for this publication is available from the British Library*

ISBN 978-1-107-65032-9 Paperback

Cambridge University Press has no responsibility for the persistence or  
accuracy of URLs for external or third-party internet websites referred to in  
this publication, and does not guarantee that any content on such websites is,  
or will remain, accurate or appropriate.

## PREFATORY NOTE.

THE object aimed at in this volume has been to present a faithful record of the course of the author's thought, without such additions as recent developments of the subjects treated of might have afforded, and without any alterations other than that considerable number involved in the attempt to make the algebraical symbols read as the writer intended. While, for the reader's convenience, the author's references to his own papers have been accompanied by cross references to the pages of this volume, placed in square brackets.

By far the longest paper in the volume is No. 57, "On the Theory of the Syzygetic Relations of two Rational Integral Functions, comprising an application to the Theory of Sturm's Functions," and to this many of the shorter papers in the volume are contributory.

The volume contains also Sylvester's dialytic method of elimination (No. 9, etc.), his Essay on Canonical Forms (No. 34), and early investigations in the Theory of Invariants (Nos. 42, 43, etc.).

It contains also celebrated theorems as to Determinants (Nos. 37, 39, 48, etc.) and investigations as to the Transformation of Quadratic Forms (the *Law of Inertia*, No. 47, and the recognition of the Invariant factors of a matrix, Nos. 22, 24, 36).

A full table of contents is prefixed.

H. F. BAKER.

ST JOHN'S COLLEGE, CAMBRIDGE.

*April*, 1904.

## TABLE OF CONTENTS

|  | PAGES |
|--|-------|
| 1. <i>Analytical development of Fresnel's optical theory of crystals</i> . . . . .<br>(Philosophical Magazine 1837, 1838)  | 1—27  |
| 2. <i>On the motion and rest of fluids</i> . . . . .<br>(Philosophical Magazine 1838)  | 28—32 |
| 3. <i>On the motion and rest of rigid bodies</i> . . . . .<br>(Philosophical Magazine 1839)  | 33—35 |
| 4. <i>On definite double integration, supplementary to a former paper on the motion and rest of fluids</i> . . . . .<br>(Philosophical Magazine 1839)              | 36—38 |
| 5. <i>On an extension of Sir John Wilson's theorem to all numbers whatever</i> . . . . .<br>(Philosophical Magazine 1838)  | 39    |
| 6. <i>Note to the foregoing</i> . . . . .<br>(Philosophical Magazine 1839)   | 39    |
| 7. <i>On rational derivation from equations of coexistence, that is to say, a new and extended theory of elimination, Part I.</i><br>(Philosophical Magazine 1839) | 40—46 |
| 8. <i>On derivation of coexistence, Part II., being the theory of simultaneous simple homogeneous equations</i> . . . . .<br>(Philosophical Magazine 1840)         | 47—53 |
| 9. <i>A method of determining by mere inspection the derivatives from two equations of any degree</i> . . . . .<br>(Philosophical Magazine 1840)                   | 54—57 |
| 10. <i>Note on elimination</i> . . . . .<br>(Philosophical Magazine 1840)  | 58    |

## CONTENTS.

vii

|   | PAGES   |
|---|---------|
| 11. <i>On the relation of Sturm's auxiliary functions to the roots of an algebraic equation</i> . . . . .<br>(Plymouth British Association Report 1841)                 | 59, 60  |
| 12. <i>Examples of the dialytic method of elimination as applied to ternary systems of equations</i> . . . . .<br>(Cambridge Mathematical Journal 1841)                 | 61—65   |
| 13. <i>Introduction to an essay on the amount and distribution of the multiplicity of the roots of an algebraic equation</i> . . . . .<br>(Philosophical Magazine 1841) | 66—68   |
| 14. <i>A new and more general theory of multiple roots</i> . . . . .<br>(Philosophical Magazine 1841)   | 69—74   |
| 15. <i>On a linear method of eliminating between double, treble, and other systems of algebraic equations.</i> . . . . .<br>(Philosophical Magazine 1841)               | 75—85   |
| 16. <i>Memoir on the dialytic method of elimination, Part I.</i> . . . . .<br>(Philosophical Magazine 1842)   | 86—90   |
| 17. <i>Elementary researches in the analysis of combinatorial aggregation</i> . . . . .<br>(Philosophical Magazine 1844)  | 91—102  |
| 18. <i>On the existence of absolute criteria for determining the roots of numerical equations</i><br>(Philosophical Magazine 1844)                                      | 103—106 |
| 19. <i>An account of a discovery in the theory of numbers relative to the equation</i><br>$Ax^3 + By^3 + Cz^3 = Dxyz$ . . . . .<br>(Philosophical Magazine 1847)        | 107—109 |
| 20. <i>On the equation in numbers</i><br>$Ax^3 + By^3 + Cz^3 = Dxyz,$<br><i>and its associate system of equations</i> . . . . .<br>(Philosophical Magazine 1847)        | 110—113 |
| 21. <i>On the general solution, in certain cases, of the equation</i> $x^3 + y^3 + z^3 = Mxyz$ . . . . .<br>(Philosophical Magazine 1847)                               | 114—118 |

|   | PAGES    |
|---|----------|
| 22. <i>On the intersections, contacts, and other correlations of two conics expressed by indeterminate coordinates</i> . . . . .  | 119—137  |
| (Cambridge and Dublin Mathematical Journal 1850)  |          |
| 23. <i>An instantaneous demonstration of Pascal's theorem by the method of indeterminate coordinates</i> . . . . .  | 138      |
| (Philosophical Magazine 1850)   |          |
| 24. <i>On a new class of theorems in elimination between quadratic functions</i> . . . . .  | 139—144  |
| (Philosophical Magazine 1850)   |          |
| 25. <i>Additions to the articles 'On a new class of theorems,' and 'On Pascal's theorem,'</i> . . . . .   | 145—151  |
| (Philosophical Magazine 1850)   |          |
| 26. <i>On the solution of a system of equations in which three homogeneous quadratic functions of three unknown quantities are respectively equated to numerical multiples of a fourth non-homogeneous function of the same</i> . . . . . | 152—154  |
| (Philosophical Magazine 1850)   |          |
| 27. <i>On a porismatic property of two conics having with one another a contact of the third order</i> . . . . .  | 155, 156 |
| (Philosophical Magazine 1850)   |          |
| 28. <i>On the rotation of a rigid body about a fixed point</i> . . . . .  | 157—161  |
| (Philosophical Magazine 1850)   |          |
| 29. <i>On the intersections of two conics</i> . . . . .   | 162—164  |
| (Cambridge and Dublin Mathematical Journal 1851)  |          |
| 30. <i>On certain general properties of homogeneous functions</i> . . . . .   | 165—180  |
| (Cambridge and Dublin Mathematical Journal 1851)  |          |
| 31. <i>Reply to Professor Boole's observations on a theorem contained in last November number of this Journal.</i> . . . . .  | 181—183  |
| (Cambridge and Dublin Mathematical Journal 1851)  |          |



## CONTENTS.

ix

|  | PAGES               |
|--|---------------------|
| 32. <i>Sketch of a memoir on elimination, transformation and canonical forms</i> . . . . .   | 184—197             |
| (Cambridge and Dublin Mathematical Journal 1851)   |                     |
| 33. <i>On the general theory of associated algebraical forms</i> . . . . .   | 198—202             |
| (Cambridge and Dublin Mathematical Journal 1851)   |                     |
| 34. <i>An essay on canonical forms, supplement to a sketch of a memoir on elimination, transformation and canonical forms</i> . . . . .  | 203—216             |
| (George Bell, Fleet Street, 1851)  |                     |
| 35. <i>Explanation of the coincidence of a theorem given by Mr Sylvester in the December number of this Journal with one stated by Professor Donkin in the June number of the same</i> . . . . . | 217, 218            |
| (Philosophical Magazine 1851)  |                     |
| 36. <i>An enumeration of the contacts of lines and surfaces of the second order</i> . . . . .  | 219—240             |
| (Philosophical Magazine 1851)  |                     |
| 37. <i>On the relation between the minor determinants of linearly equivalent quadratic functions</i> . . . . .   | 241—250             |
| (Philosophical Magazine 1851)  | [See p. 647 below.] |
| 38. <i>Note on quadratic functions and hyperdeterminants</i> . . . . .   | 251                 |
| (Philosophical Magazine 1851)  |                     |
| 39. <i>On a certain fundamental theorem of determinants</i> . . . . .  | 252—255             |
| (Philosophical Magazine 1851)  |                     |
| 40. <i>Extensions of the dialytic method of elimination</i> . . . . .  | 256—264             |
| (Philosophical Magazine 1851)  |                     |
| 41. <i>On a remarkable discovery in the theory of canonical forms and of hyperdeterminants</i>   | 265—283             |
| (Philosophical Magazine 1851)  |                     |
| 42. <i>On the principles of the calculus of forms</i> . . . . .  | 284—327             |
| (Cambridge and Dublin Mathematical Journal 1852)   |                     |
| 43. <i>On the principles of the calculus of forms</i> . . . . .  | 328—363             |
| (Cambridge and Dublin Mathematical Journal 1852)   |                     |

|   | PAGES    |
|---|----------|
| 44. <i>Sur une propriété nouvelle de l'équation qui sert à déterminer les inégalités séculaires des planètes</i> . . . . .  | 364—366  |
| (Nouvelles Annales de Mathématiques 1852)   |          |
| 45. <i>On a remarkable theorem in the theory of equal roots and multiple points</i> . . . .   | 367—369  |
| (Philosophical Magazine 1852)   |          |
| 46. <i>Observations on a new theory of multiplicity</i> . . . . .   | 370—377  |
| (Philosophical Magazine 1852)   |          |
| 47. <i>A demonstration of the theorem that every homogeneous quadratic polynomial is reducible by real orthogonal substitutions to the form of a sum of positive and negative squares</i> . . . . . | 378—381  |
| (Philosophical Magazine 1852)   |          |
| 48. <i>On Staudt's theorems concerning the contents of polygons and polyhedrons, with a note on a new and resembling class of theorems</i> . . . . .  | 382—391  |
| (Philosophical Magazine 1852)   |          |
| 49. <i>On a simple geometrical problem, illustrating a conjectured principle in the theory of geometrical method</i> . . . . .  | 392—395  |
| (Philosophical Magazine 1852)   |          |
| 50. <i>On the expression of the quotients which appear in the application of Sturm's method to the discovery of the real roots of an equation</i> . . . . .   | 396—398  |
| (Hull British Association Report 1853)  |          |
| 51. <i>On a theorem concerning the combination of determinants</i> . . . . .  | 399—401  |
| (Cambridge and Dublin Mathematical Journal 1853)  |          |
| 52. <i>Note on the calculus of forms</i> . . . . .  | 402, 403 |
| (Cambridge and Dublin Mathematical Journal 1853)  |          |
| 53. <i>On the relation between the volume of a tetrahedron and the product of the sixteen algebraical values of its superficies</i> . . . . .   | 404—410  |
| (Cambridge and Dublin Mathematical Journal 1853)  |          |

## CONTENTS.

xi

|   | PAGES   |
|---|---------|
| 54. <i>On the calculus of forms, otherwise the theory of invariants</i> . . . . .   | 411—422 |
| (Cambridge and Dublin Mathematical Journal 1853)  |         |
| 55. <i>Théorème sur les limites des racines réelles des équations algébriques</i> . . . . .   | 423     |
| (Nouvelles Annales de Mathématiques 1853)   |         |
| 56. <i>Nouvelle méthode pour trouver une limite supérieure et une limite inférieure des racines réelles d'une équation algébrique quelconque</i> . . . . .  | 424—428 |
| (Nouvelles Annales de Mathématiques 1853)   |         |
| 57. <i>On a theory of the syzygetic relations of two rational integral functions, comprising an application to the theory of Sturm's functions, and that of the greatest algebraical common measure</i> . . . . . | 429—586 |
| (Philosophical Transactions of the Royal Society of London 1853)  |         |
| 58. <i>On the conditions necessary and sufficient to be satisfied in order that a function of any number of variables may be linearly equivalent to a function of any less number of variables</i> . . . . .      | 587—594 |
| (Philosophical Magazine 1853)   |         |
| 59. <i>On Mr Cayley's impromptu demonstration of the rule for determining at sight the degree of any symmetrical function of the roots of an equation expressed in terms of the coefficients</i> . . . . .        | 595—598 |
| (Philosophical Magazine 1853)   |         |
| 60. <i>A proof that all the invariants to a cubic ternary form are rational functions of Aronhold's invariants and of a cognate theorem for biquadratic binary forms</i> . . . . .                                | 599—608 |
| (Philosophical Magazine 1853)   |         |
| 61. <i>On a remarkable modification of Sturm's theorem</i> . . . . .  | 609—619 |
| (Philosophical Magazine 1853)   |         |

|   | PAGES    |
|---|----------|
| 62. <i>Note on a remarkable modification of Sturm's theorem, and on a new rule for finding superior and inferior limits to the roots of an equation</i> . . . . . | 620—626  |
| (Philosophical Magazine 1853)   |          |
| 63. <i>On the new rule for finding superior and inferior limits to the real roots of any algebraical equation</i> . . . . .                                       | 627—629  |
| (Philosophical Magazine 1853)   |          |
| 64. <i>Note on the new rule of limits</i> . . . . .   | 630—633  |
| (Philosophical Magazine 1853)   |          |
| 65. <i>The algebraical theory of the secular inequality determinative equation generalised</i> . . . . .  | 634—636  |
| (Philosophical Magazine 1853)   |          |
| 66. <i>On the explicit values of Sturm's quotients</i> . . . . .  | 637—640  |
| (Philosophical Magazine 1853)   |          |
| 67. <i>On a fundamental rule in the algorithm of continued fractions</i> . . . . .  | 641—644  |
| (Philosophical Magazine 1853)   |          |
| 68. <i>On a generalisation of the Lagrangian theorem of interpolation</i> . . . . .   | 645, 646 |
| (Philosophical Magazine 1853)   |          |
| NOTE ON SYLVESTER'S THEOREMS ON DETERMINANTS<br>IN THIS VOLUME. . . . .   | 647—650  |