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David A. Brannan, Matthew F. Esplen and Jeremy J. Gray
Frontmatter
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Geometry

SECOND EDITION

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In memory of Wilson Stothers

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Preface

Geometry! For over two thousand years it was one of the criteria for recognition as an educated person to be acquainted with the subject of geometry. Euclidean geometry, of course.

In the golden era of Greek civilization around 400 BC, geometry was studied rigorously and put on a firm theoretical basis – for intellectual satisfaction, the intrinsic beauty of many geometrical results, and the utility of the subject. For example, it was written above the door of Plato's Academy 'Let no-one ignorant of Geometry enter here!' Indeed, Archimedes is said to have used the reflection properties of a parabola to focus sunlight on the sails of the Roman fleet besieging Syracuse and set them on flame.

Plato (c. 427–347 BC) was an Athenian philosopher who established a school of theoretical research (with a mathematical bias), legislation and government.

Archimedes (c. 287–212 BC) was a Greek geometer and physicist who used many of the basic limiting ideas of differential and integral calculus.

For two millennia the children of those families sufficiently well-off to be educated were compelled to have their minds trained in the noble art of rigorous mathematical thinking by the careful study of translations of the work of Euclid. This involved grasping the notions of axioms and postulates, the drawing of suitable construction lines, and the careful deduction of the necessary results from the given facts and the Euclidean axioms – generally in two-dimensional or three-dimensional Euclidean space (which we shall denote by $\mathbb{R}^2$ and $\mathbb{R}^3$, respectively). Indeed, in the 1700s and 1800s popular publications such as *The Lady's and Gentleman's Diary* published geometric problems for the consideration of gentlefolk at their leisure. And as late as the 1950s translations of Euclid's *Elements* were being used as standard school geometry textbooks in many countries.

Euclid (c. 325–265 BC) was a mathematician in Hellenistic Alexandria during the reign of Ptolemy I (323–283 BC), famous for his book *The Elements*.

We give a careful algebraic definition of $\mathbb{R}^2$ and $\mathbb{R}^3$ in Appendix 2.

Just as nowadays, not everyone enjoyed Mathematics! For instance, the German poet and philosopher Goethe wrote that 'Mathematicians are like Frenchmen: whatever you say to them, they translate into their own language, and forthwith it is something entirely different!'

Johann Wolfgang von Goethe (1749–1832) is said to have studied all areas of science of his day except mathematics – for which he had no aptitude.

The Golden Era of geometry came to an end rather abruptly. When the USSR launched the Sputnik satellite in 1957, the Western World suddenly decided for political and military reasons to give increased priority to its research and educational efforts in science and mathematics, and redeveloped the curricula in these subjects. In order to make space for subjects newly developed or perceived as more 'relevant in the modern age', the amount of geometry taught in schools and universities plummeted. Interest in geometry languished: it was thought 'old-fashioned' by the fashionable majority.

Nowadays it is being realized that geometry is still a subject of abiding beauty that provides tremendous intellectual satisfaction in return for effort put into its study, and plays a key underlying role in the understanding, development and applications of many other branches of mathematics. More and more universities are reintroducing courses in geometry, to give students a 'feel' for the reasons for studying various areas of mathematics (such as Topology), to service the needs of Computer Graphics courses, and so on. Geometry is having a revival!

Topics in computer graphics such as 'hidden' surfaces and the shading of curved surfaces involve much mathematics.

Since 1971, the Open University in the United Kingdom has taught mathematics to students via specially written correspondence texts, and has traditionally given geometry a central position in its courses. This book arises from those correspondence texts.

We adopt the Klein approach to geometry. That is, we regard the various geometries as each consisting of an underlying set together with a group of transformations acting on that set. Those properties of the set that are not altered by any of the transformations are called *the properties of that geometry.*

Following a historical review of the development of the various geometries, we look at conics (and at the related quadric surfaces) in Euclidean geometry. Then we address a whole series of different geometries in turn. First, affine geometry (that provides simple proofs of some results in Euclidean geometry). Then projective geometry, which can be regarded as the most basic of all geometries; we divide this material into a chapter on projective lines and a chapter on projective conics. We then return to study inversive geometry, which provides beautiful proofs of many results involving lines and circles in Euclidean geometry. This leads naturally to the study of hyperbolic geometry in the unit disc, in which there are two lines through any given point that are parallel to a given line. Via the link of stereographic projection, this leads on to spherical geometry: a natural enough concept for a human race that lives on the surface of a sphere! Finally we tie things together, explaining how the various geometries are inter-related.

Chapter 0
Chapter 1
Chapter 2
Chapters 3 and 4
Chapter 5
Chapter 6
Chapter 7
Chapter 8

Study Guide

The book assumes a basic knowledge of Group Theory and of Linear Algebra, as these are used throughout. However, for completeness and students' convenience we give a very rapid review of both topics in the appendices.

Appendices 1 and 2.

The book follows many of the standard teaching styles of The Open University. Thus, most chapters are divided into five sections (each often further divided into subsections); sections are numbered using two digits (such as 'Section 3.2') and subsections using three digits (such as 'Subsection 3.2.4'). Generally a section is considered to be about one evening's hard work for an average student.

We number in order the theorems, examples, problems and equations within each section.

We use wide pages with margins in which we place various historical notes, cross-references, teaching comments and diagrams; the cross-references need not be consulted by students unless they wish to remind themselves of some point on that topic, but the other margin notes should be read carefully. We use boxes in the main text to highlight definitions, strategies, and the statements of theorems and other key results. The end of the proof of a theorem is indicated by a solid symbol '■', and the end of the solution of a worked example by a hollow symbol '□'. Occasionally the text includes a set of 'Remarks'; these are comments of the type that an instructor would give orally to a class, to clarify a definition, result, or whatever, and should be read carefully. There are many worked examples within the text to explain the concepts being taught, and it is important that students read these carefully as they contain many key teaching points; in addition, there is a good stock of in-text problems to reinforce the teaching, and solutions to these are given in Appendix 3. At the end of each chapter there are exercises covering the material of that chapter, some of which are fairly straight-forward and some are more challenging; solutions are not given to the exercises.

Our philosophy is to provide clear and complete explanations of all geometric facts, and to teach these in such a way that students can understand them without much external help. As a result, students should be able to learn (and, we hope, to enjoy) the key concepts of the subject in an uncluttered way.

Most students will have met many parts of Chapter 1 already, and so can proceed fairly quickly through it. Thereafter it is possible to tackle Chapters 2 to 4 or Chapters 5 and 6, in either order. It is possible to omit Chapters 7 or 8, if the time in a course runs short.

Notation for Functions as Mappings

Suppose that a function f maps some set A into some set B, and that it maps a typical point x of A onto some *image point* y of B. Then we say that A is the *domain* (or *domain of definition)* of f, B the *codomain* of f, and denote the function f as a *mapping* (or *map)* as follows:

Note that we use two different arrows here, to distinguish between the mapping of a set and the mapping of an element.

$$\begin{aligned} f : A &\to B \\ x &\mapsto y \end{aligned}$$

We often denote y by the expression $f(x)$ to indicate its dependence on f and x.

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Without the assistance and the forbearance of our families, the writing of the original OU course and its later rewriting in this form would have been impossible. It was Michael Brannan's idea to produce it as a book.

Changes in the Second Edition

In addition to correcting typos and errors, the authors have changed the term 'gradient' to 'slope', and avoided the use of 'reversed square brackets' — so that, for instance, the interval $\{x : 0 < x \leq 1\}$ is now written as (0,1] rather than]0,1]. Also, they have clarified the difference between a geometry and models of that geometry; in particular, the term 'non-Euclidean' geometry has now been largely replaced by 'hyperbolic' geometry, and the term 'elliptic' geometry has been introduced where appropriate. The problems and exercises have been revised somewhat, and more exercises included. Each chapter now includes a summary of the material in that chapter, and before the appendices there are now lists of symbols and suggestions for further reading.

Solutions to the exercises appear in an Instructors' Manual available from the publisher.

The authors have taken the opportunity to add some new material to enrich the reader's diet: a treatment of conics as envelopes of tangent families, barycentric coordinates, Poncelet's Porism and Ptolemy's Theorem, and planar maps. Also, the treatment of a number of existing topics has been significantly changed: the geometric interpretation of projective transformations, the analysis of the formula for hyperbolic distance, and the treatment of asymptotic d-triangles.

The authors appreciate the warm reception of the first edition, and have tried to take on board as many as possible of the helpful comments received. Special thanks are due to John Snygg and Jonathan I. Hall for invaluable comments and advice.

Instructors' Manual

Complete solutions to all of the end-of-chapter exercises are available in an Instructors' Manual, which can be downloaded from www.cambridge.org/9781107647831.