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NUMERICAL METHODS
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BY

P. G. GUEST

University of Sydney
Australia

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PREFACE

The aim of this book is to provide an introduction to the methods of treating series of observations. The field covered embraces portions of both statistics and numerical analysis, and one might adopt the sub-title 'The Combination of Observations', in the sense used by Brunt many years ago, to describe the contents. The book is intended primarily for students and graduates in physics, and the types of observation discussed will be those most commonly met with in routine work in the physical sciences. It is hoped that the book will be useful as a reference work for statisticians and biologists, since much of the material presented here does not find a place in statistical textbooks but is only available in the original literature.

Part I (Chapters 1 to 4) deals with observations of a single variable ('curve' of zero degree). Much of this material will, of course, be familiar to statisticians, and Part I is certainly not intended as a substitute for a good text on statistics, but rather as a rapid summary of those portions of statistics used in the reduction of routine physical measurements. Subjects such as analysis of variance, factorial design, etc., are deliberately omitted. The fitting of straight lines is dealt with in Part II (Chapters 5 and 6). Some of the results derived in this part are special cases of general results for polynomials of arbitrary degree, but the fact that the majority of 'curves' fitted are straight lines warrants the treating of the linear case separately. Part III (Chapters 7 to 12) deals with the fitting of polynomial curves and of special types of curve. In the final chapter a number of typical examples are worked out in detail. These examples are intended to serve as a guide for those who want to fit a curve without going into the underlying theory. In a book of reasonable size it is not possible to treat all the topics relating to curve fitting, but it is hoped that the major topics have been covered and that other work can be located with the aid of the bibliography.

There has been a tendency in recent years for books on numerical analysis to omit numerical examples illustrating the applications of the methods. In the present work an attempt has been made to obtain a better balance between theory and practice. Each method is illustrated not only with an example but also with a full calculating scheme, so that the beginner can proceed along well-tried paths. However, it is also intended that the book

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should cover the theoretical aspects of curve fitting, and full derivations of all formulae are given. A knowledge of the calculus is presumed, and this background should enable most of the derivations to be followed. Some use is made of matrix notation in establishing a few of the more complicated results, but only the very simplest matrix operations are required.

The calculating schemes are designed primarily for a desk calculating machine, although most of the calculations in the first two parts can be done without the aid of a machine. In some cases a number of different schemes are given, for it is not wise to be dogmatic about which scheme is 'best'; the choice of the best scheme depends very much on the computing facilities and on the particular problem. In Section 12.1 there is a guide to assist in the choice of the calculating scheme most suited to any one of the commonly occurring types of problem. With high-speed automatic computers the routines for curve fitting will be very similar to those given for desk machines. Two routines for use with high-speed computers are discussed in Section 7.2. However, each computer has its own staff and manual, and the advice and instructions given by these should certainly be followed.

This book was written in England and Canada while the author was on sabbatical leave from the University of Sydney. The author wishes to express his thanks to the librarians of the various universities he visited for the facilities placed at his disposal. He also wishes to acknowledge his indebtedness to his wife Elizabeth for her patience and care in the typing of the manuscript.

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