

Introduction to the Theory of Thermal Neutron Scattering

Since the advent of the nuclear reactor, thermal neutron scattering has proved a valuable tool for studying many properties of solids and liquids, and research workers are active in the field at reactor centres and universities throughout the world.

This classic text provides the basic quantum theory of thermal neutron scattering and applies the concepts to scattering by crystals, liquids and magnetic systems. Other topics discussed are the relation of the scattering to correlation functions in the scattering system, the dynamical theory of scattering and polarisation analysis. No previous knowledge of the theory of thermal neutron scattering is assumed, but basic knowledge of quantum mechanics and solid-state physics is required.

The book is intended for experimenters rather than theoreticians, and the discussion is kept as informal as possible. A number of examples, with worked solutions, are included as an aid to the understanding of the text.

G. L. SQUIRES (1924–2010) was a Lecturer in Physics at the University of Cambridge and a Fellow of Trinity College Cambridge from 1956. He published two other books with Cambridge University Press: *Practical Physics* and *Problems in Quantum Mechanics with Solutions*, wrote an article on quantum mechanics for the *Encyclopaedia Britannica* and contributed extensively to leading scientific journals. From his retirement in 1991 until his death in 2010, Dr Squires was the curator of the Cavendish Laboratory Museum and wrote a number of historical articles on scientists and scientific discoveries in Cambridge.





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G. L. SQUIRES (1924–2010)

Lecturer in Physics at the University of Cambridge and Fellow of Trinity College Cambridge





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PREFACE

This book arose from some lectures given in Cambridge in 1973 at a Summer School organised by the Neutron Scattering Group of the Institute of Physics and the Faraday Society. It is intended for experimenters in the field of thermal neutron scattering who wish to see the theoretical ideas developed in a not too formal manner. But I hope it may be of interest to students and research workers in related fields.

I assume no previous knowledge of the theory of thermal neutron scattering, but a familiarity with the basic concepts of quantum mechanics and solid state physics is necessary for a proper understanding of the text. The required results in these subjects are summarised in appendices. The latter also contain proofs of some of the mathematical results.

Some problem examples have been given at the ends of chapters. They are intended to illustrate the text, and the reader is advised to glance at them even if he is not inclined to try to solve them. Their purpose is partly, as with some of the appendices, to remove some mathematical material from the main body of the text, and partly to stimulate the reader to a more active understanding of the subject.

Thermal neutron scattering is being applied in more and more areas of science. But this book does not attempt to cover the theory of all the applications. Instead I have confined myself to the basic ideas of the theory. My aim is to bring the reader to the point where he can tackle the theoretical expositions of the specialised branches given in more advanced textbooks and in theoretical papers.

It is a pleasure to thank Mr Cole, Dr Dore, Dr Howie, Professor Joannopoulos, Dr de Vallera, and Dr Zeilinger for reading parts of the manuscript and making useful comments.

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G. L. SQUIRES

