

CONTENTS.

INTRODUCTION.

| §§ | | PAGE |
|---------|---|------|
| 1. | General range of the subject | 1 |
| 2. | Early beginnings | 1 |
| 3. | Newton, John Bernoulli | 2 |
| 4. | Euler and Lagrange | 3 |
| 5. | Legendre | 4 |
| 6. | Jacobi | 5 |
| 7, 8. | General notion of variations | 6 |
| 9. | <i>Weak</i> variations : <i>special</i> variations : <i>strong</i> variations | 7 |
| 10, 11. | Weierstrass | 8 |
| 12. | Assumed limitations on functionality | 9 |

CHAPTER I.

INTEGRALS OF THE FIRST ORDER : MAXIMA AND MINIMA FOR SPECIAL WEAK VARIATIONS : EULER TEST, LEGENDRE TEST, JACOBI TEST.

| | | |
|---------|---|----|
| | Preliminary note | 11 |
| 13. | Types of problems arising from integrals with maxima or minima | 11 |
| 14. | Integral of the first order: its variation, with fixed limits | 12 |
| 15, 16. | Modification of the first variation | 13 |
| 17. | Characteristic equation, and curve; the Euler test | 15 |
| 18. | Quadratic terms in the variation: 'second' variation | 17 |
| 19–21. | Subsidiary characteristic equation; the Legendre test | 18 |
| 22–25. | Primitive of the subsidiary equation, with some properties | 22 |
| 26, 27. | The Jacobi test; conjugate points on a characteristic, with geometrical interpretation | 26 |
| 28. | Summary of the tests | 28 |
| 29. | Conjugate as actual limit of a range | 29 |
| 30. | Examples; the catenoid, and its properties connected with the tests | 30 |
| 31. | Mobile limits of an integral; terminal conditions | 36 |
| 32. | Complete variation of an integral with mobile limits | 36 |
| 33, 34. | Conditions for a maximum or minimum, consisting of the characteristic equation and terminal relations: examples, including brachistochrone, geodesic on paraboloid of revolution, and a failure | 38 |
| 35. | First integral of the characteristic equation: Hilbert's theorem | 48 |

CHAPTER II.

INTEGRALS OF THE FIRST ORDER: GENERAL WEAK VARIATIONS:
 • THE METHOD OF WEIERSTRASS.

| §§ | PAGE |
|--|------|
| Preliminary note | 52 |
| 36. Weak variations in general | 52 |
| 37. Modification of integral, in which both variables become dependent on a new variable | 53 |
| 38. Identities satisfied by new subject of integration F , the integral form being covariantive | 54 |
| 39. General variation of the integral | 55 |
| 40. The first variation | 55 |
| 41-43. Two equations in the range, and terminal relations, to be satisfied, in order that the first variation may vanish | 57 |
| 44, 45. The two critical equations are equivalent to a single characteristic equation, on account of identity satisfied by F | 59 |
| 46. Invariance of characteristic equation, and of one critical quantity | 61 |
| 47. Two quantities which remain continuous in passage through any free place of discontinuity in direction | 62 |
| 48. But the number of such free places in a finite range must be limited | 65 |
| 49. Primitive of characteristic equation involves two essential arbitrary constants | 65 |
| 50. The 'second' variation; the <i>deviation</i> due to a small variation | 67 |
| 51. Relations among the second derivatives of F | 68 |
| 52, 53. Normal form of the second variation | 69 |
| 54. Summary of assumptions made during construction of normal form | 72 |
| 55, 56. Discussion of this normal form: the Legendre test | 73 |
| 57-59. Subsidiary characteristic equation | 75 |
| 60, 61. Primitive of the subsidiary equation: can be derived from the primitive of the characteristic equation | 79 |
| 62. Properties of any two independent integrals of the subsidiary equation | 82 |
| 63, 64. An integral $Z(t, t')$ of the subsidiary equation, affecting the second variation: the Jacobi test | 82 |
| 65. Conjugate places in a range of integration; geometrical meaning of the Jacobi test | 84 |
| 66. The function $Z(t, t')$ vanishes at conjugates and changes sign in passing through any zero | 86 |
| 67. A range, bounded by two conjugates, does not contain any similarly bounded range | 87 |
| 68, 69. Consecutive characteristic curves: properties | 89 |
| 70-72. Characteristic curve can be drawn through two assigned points, as well as through one point with an assigned direction, if the second point is not within the region of conjugates of the first | 93 |
| 73. Composite small variations: their effect | 95 |
| 74. Summary of results established | 97 |
| 75. Examples by the Weierstrass method: catenoid: geodesics on a sphere, and on surfaces of revolution | 98 |
| 76-80. Weierstrass's proof that an integral range cannot extend beyond a conjugate | 110 |

CHAPTER III.

INTEGRALS INVOLVING DERIVATIVES OF THE SECOND ORDER: SPECIAL
 WEAK VARIATIONS, BY THE METHOD OF JACOBI; GENERAL WEAK
 VARIATIONS, BY THE METHOD OF WEIERSTRASS.

| §§ | | PAGE |
|-----------|---|------|
| | Preliminary note | 116 |
| 81. | Integrals involving second derivatives of one dependent variable | 116 |
| 82. | Full variation of the integral with mobile limits | 117 |
| 83, 84. | Vanishing of the first variation: characteristic equation | 118 |
| 85. | Subsidiary characteristic equation, and its primitive | 120 |
| 86. | Terminal conditions: four cases | 121 |
| 87. | Results for integrals of order n | 124 |
| 88–90. | Second variation for special variations: its normal form | 126 |
| 91, 92. | Properties of integrals of subsidiary characteristic equation | 129 |
| 93, 94. | Determination of two quantities, required for the normal form | 130 |
| 95. | Normal form of second variation: the Legendre test | 132 |
| 96. | The Jacobi test; with summary of tests | 134 |
| 97. | General weak variations: the Weierstrass method: transformation of integral: two fundamental identities satisfied by integrand | 135 |
| 98. | First variation of the integral | 136 |
| 99. | The two characteristic equations | 138 |
| 100, 101. | The conditions at the limits: four cases: comparison with § 86 | 138 |
| 102. | Continuity of four magnitudes through a free discontinuity of direction and curvature on characteristic curve | 142 |
| 103. | The two characteristic equations in § 99 are equivalent to one only, owing to the fundamental identities of § 97 | 145 |
| 104. | Certain invariant forms | 146 |
| 105. | Relations among the second derivatives of the integrand, including the single characteristic equation of § 103 | 148 |
| 106, 107. | Preliminary normal form of the second variation | 153 |
| 108. | The subsidiary characteristic equations | 157 |
| 109. | Equivalent single subsidiary equation, with the 'deviation' as variable | 158 |
| 110. | Final normal form of the second variation | 160 |
| 111, 112. | Primitive of the characteristic equations, with one essential property | 161 |
| 113. | Primitive of the subsidiary equation | 163 |
| 114, 115. | Properties of integrals of the subsidiary equation | 165 |
| 116. | Consecutive characteristic curve | 167 |
| 117–119. | Conjugate points; a range, limited by conjugates, cannot include within itself a range similarly limited | 169 |
| 120. | Adjacent characteristic curves: sub-consecutive curves | 175 |
| 121, 122. | Equations of sub-consecutive characteristic | 176 |
| 123. | Consecutive characteristic can be drawn through assigned contiguous point | 178 |
| 124, 125. | Discussion of normal form of second variation | 179 |
| 126. | The Jacobi test | 181 |
| 127. | The Legendre test | 182 |
| 128. | Summary of conditions for weak variations | 183 |
| 129. | Example, in which tests are satisfied: yet there is not a real minimum | 184 |

CHAPTER IV.

 INTEGRALS INVOLVING TWO DEPENDENT VARIABLES AND THEIR
 FIRST DERIVATIVES: SPECIAL WEAK VARIATIONS.

| §§ | | PAGE |
|-----------|--|------|
| 130. | Integrals of the first order in two dependent variables | 189 |
| 131. | Special variations imposed on the integral | 190 |
| 132. | Mobile limits | 191 |
| 133, 134. | The first variation : characteristic equations, and terminal conditions | 192 |
| 135. | Hamilton form of the characteristic equations | 193 |
| 136, 137. | Nature of the primitive characteristic equations : determination of arbitrary constants : various cases | 194 |
| 138. | Special forms when a first integral can be obtained : examples | 197 |
| 139. | Subsidiary characteristic equations, and their primitive | 204 |
| 140. | Lemma auxiliary to the establishment of this primitive | 206 |
| 141–143. | Relation between two integral-sets of the subsidiary equation | 209 |
| 144. | The second variation, under special variations | 212 |
| 145, 146. | Equations for its transformation, and their resolution in terms of two integral-sets of the subsidiary equation | 213 |
| 147. | Normal form of the second variation | 216 |
| 148. | The Legendre test | 217 |
| 149. | Conjugate points : the Jacobi test | 218 |
| 150, 151. | Invariantive property of fundamental group of integral-sets of the sub- sidiary equation | 220 |
| 152, 153. | Critical function for the Jacobi test | 222 |
| 154. | Summary of results for special weak variations | 224 |

CHAPTER V.

 INTEGRALS INVOLVING TWO DEPENDENT VARIABLES AND THEIR
 FIRST DERIVATIVES: GENERAL WEAK VARIATIONS.

| | | |
|-----------|--|-----|
| 155, 156. | The method of Weierstrass : a fundamental identity | 226 |
| 157. | Relations among the second derivatives of the subject of integration | 228 |
| 158. | First variation of the integral : three characteristic equations : character- istic curve : terminal conditions | 229 |
| 159. | Continuity of certain derivatives at free discontinuities on the curve | 231 |
| 160. | The three characteristic equations equivalent to two | 232 |
| 161. | Primitive of the characteristic equations : subsidiary equations | 234 |
| 162, 163. | The second variation : preliminary modified form | 236 |
| 164. | Remark on change of method from the method of Chapter III | 238 |
| 165, 166. | The three subsidiary equations reducible to two equations, with modified variables | 239 |
| 167. | One selected integral of the two subsidiary equations | 242 |
| 168. | Final normal form of the second variation | 244 |
| 169, 170. | Primitive of the subsidiary equations, with note as to the dependent variables | 246 |
| 171. | Property of fundamental sets of integrals of subsidiary equation | 250 |
| 172. | Discussion of the second variation : the Legendre test | 251 |

CONTENTS

xvii

| §§ | | PAGE |
|--|---|------|
| 173. | Conjugates on a characteristic curve: the Jacobi test | 253 |
| 174. | Critical equation for the determination of a conjugate | 253 |
| 175. | Example: minimum reduced path of a ray in geometrical optics | 256 |
| 176. | A conjugate-bounded range contains no similar range | 262 |
| 177. | Contiguous characteristic curves | 265 |
| 178. | Integrals involving n dependent variables, with first derivatives | 267 |
| CHAPTER VI. | | |
| INTEGRALS WITH TWO DEPENDENT VARIABLES AND DERIVATIVES OF THE SECOND ORDER: MAINLY SPECIAL WEAK VARIATIONS. | | |
| 179. | First variation of an integral for special variations | 271 |
| 180, 181. | Characteristic equations: terminal conditions | 272 |
| 182. | Form of the primitive of the characteristic equations | 274 |
| 183. | Properties of the primitive | 276 |
| 184, 185. | Subsidiary characteristic equations: their primitive | 278 |
| 186. | Invariant property of a fundamental group of integral-sets of the subsidiary equations | 281 |
| 187. | Particular properties of combinations of integral-sets of these equations | 282 |
| 188. | Six fundamental relations among four selected integral-sets | 284 |
| 189. | First variation by the method of Weierstrass: general weak variations | 286 |
| 190, 191. | Three characteristic equations: terminal conditions | 289 |
| 192. | The three equations are equivalent to two, in virtue of two identities | 291 |
| 193. | Terminal conditions agree with the conditions in § 181 | 292 |
| 194. | Continuity of certain derivatives through any free isolated discontinuity on the characteristic curve | 293 |
| 195. | Second variation, for special weak variations | 294 |
| 196. | Relations for reduction to normal form | 295 |
| 197. | Preliminary normal form of second variation | 297 |
| 198. | Use of four integral-sets of the subsidiary equations, for the determination of the coefficients in the first normal form | 298 |
| 199. | Resolution of equations of reduction in § 196 by means of the six relations in § 188 | 301 |
| 200. | Final normal form of second variation: the Legendre test | 304 |
| 201. | Limitation of range: the Jacobi test | 306 |
| 202, 203. | Critical equation in connection with the Jacobi test | 307 |
| 204. | A range, bounded by two conjugates, cannot enclose a similar range | 310 |
| 205. | Summary of tests for special variations | 311 |
| 206. | General weak variation: statement (without proof) of ten results concerning the second variation | 311 |

CHAPTER VII.

ORDINARY INTEGRALS UNDER STRONG VARIATIONS, AND THE WEIERSTRASS
TEST: SOLID OF LEAST RESISTANCE: ACTION.

| | | |
|-----------|---|-----|
| 207. | General notion of a small variation | 320 |
| 208. | Strong variations: fundamental constituent type | 321 |
| 209, 210. | Cumulative condition, arising from condition for the type | 322 |

| xviii | CONTENTS | PAGE |
|-----------|--|------|
| §§ | | |
| 211–213. | Strong variation of ordinary integral with first derivatives | 324 |
| 214. | Weierstrass E -function and Weierstrass test | 328 |
| 215. | Examples of the E -function as a test, with extension of the fundamental constituent type of variation; geodesics in general: example due to Weierstrass | 329 |
| 216. | Expression for E -function with a squared factor | 335 |
| 217. | Modified form of the E -test: it is distinct from the Legendre test. | 337 |
| 218. | When the subject of integration is rational, neither a maximum nor a minimum can exist | 338 |
| 219. | Solid of Least Resistance: Newton's problem: alternative laws | 340 |
| 220. | Extended type of strong variations | 347 |
| 221. | Modified E -function for extended type of strong variation | 350 |
| 222. | Strong variations when second derivatives occur | 353 |
| 223. | Form of E -function for these variations | 355 |
| 224. | Strong variations of integrals involving two dependent variables and their first derivatives | 356 |
| 225. | The E -function for skew curves | 357 |
| 226, 227. | Alternative form of the E -function of § 225: it is distinct from the Legendre test | 359 |
| 228. | Dogma of Least Action; expression for Action | 363 |
| 229. | Tests of Action, under special weak variations | 364 |
| 230. | Action is a minimum under special weak variations: example, from projectile | 368 |
| 231, 232. | Action is a minimum under general weak variations | 371 |
| 233. | Strong variations imposed on Action: expression for variation of Action under strong variations | 376 |
| 234. | Action is not a true minimum: the Weierstrass test not being satisfied | 381 |
| 235. | Representation (including the time) for configurations of moving systems; example of Action of a projectile, with strong variations | 382 |
| 236. | General review of all the tests | 384 |

CHAPTER VIII.

RELATIVE MAXIMA AND MINIMA OF SINGLE INTEGRALS: ISOPERIMETRICAL PROBLEMS.

| | | |
|-----------|---|-----|
| 237. | Types of problems in relative (or limited) maxima and minima | 387 |
| 238. | Isoperimetrical problems: simplest type, involving derivative of the first order, with a coexistent integral remaining constant | 388 |
| 239. | Selection of general weak variations leaving constant integral unchanged | 391 |
| 240, 241. | Vanishing of 'first' variation of variable integral | 393 |
| 242. | Characteristic equation: terminal conditions | 394 |
| 243. | Likewise for integrals, subjected only to special weak variations | 395 |
| 244. | Continuity of certain functions through free discontinuities on the characteristic curve, these being finite in number | 396 |
| 245. | Comparison with, and divergence from, results in §§ 37–49 | 397 |
| 246. | 'Second' variation: normal form | 399 |
| 247–249. | Subsidiary equation, and its primitive | 401 |
| 250. | Discussion of the second variation: the Legendre test | 407 |

CONTENTS

xiX

| §§ | | PAGE |
|-----------|--|------|
| 251, 252. | The Jacobi test: its analytical expression | 407 |
| 253. | A conjugate-bounded range contains no similar range | 409 |
| 254. | Consecutive characteristics | 410 |
| 255. | Examples: maximum area with assigned length of contour: solid of maximum attraction: solid of maximum volume with assigned superficies | 412 |
| 256. | Strong variations for problems in relative maxima and minima, with derivatives of first order: examples | 419 |
| 257. | Isoperimetrical problems, with derivatives of order higher than the first | 425 |
| 258, 259. | Characteristic equations: terminal conditions | 426 |
| 260. | Second variation | 428 |
| 261. | Isoperimetrical problems, with several dependent variables; example | 428 |
| 262, 263. | Relative maxima and minima, limited by equations of non-integral type; three types of frequent occurrence | 433 |
| 264. | Method of proceeding | 436 |
| 265. | Simplest type of problem, with limiting equation not of the integral type | 436 |
| 266. | Expression of the first variation of an integral, by variations limited through an imposed coexistent equation | 437 |
| 267. | Characteristic equation: terminal conditions | 439 |
| 268. | Characteristic equations generally reducible in number by one unit | 440 |
| 269. | Form of results for special weak variations; examples relating to geodesics | 441 |
| 270. | Statement of results when second derivatives occur | 447 |
| 271. | Reduction in the number of characteristic equations | 449 |
| 272. | Form of results for special weak variations: shortest curve of constant circular curvature: brachistochrone in resisting medium | 451 |
| 273. | Postulation of more than one non-integral coexistent equation | 455 |

CHAPTER IX.

DOUBLE INTEGRALS WITH DERIVATIVES OF THE FIRST ORDER :
 WEAK VARIATIONS : MINIMAL SURFACES.

| | | |
|-----------|--|-----|
| | Preliminary note, mainly on minimal surfaces | 457 |
| 274. | Double integrals of the simplest type: special weak variations | 458 |
| 275. | Lemma in double integration: convention as to order of integration | 459 |
| 276. | Modification of expression for first variation, with fixed limits | 461 |
| 277. | Characteristic equation: characteristic surface | 462 |
| 278. | First variation, with mobile limits | 463 |
| 279–281. | Characteristic equation: boundary conditions | 466 |
| 282. | Cauchy's primitive of the characteristic equation under assigned data, with geometrical interpretation | 469 |
| 283. | Second variation for special variations: normal form | 471 |
| 284, 285. | Subsidiary characteristic equation, and its primitive | 473 |
| 286–288. | Discussion of normal form of second variation: the Legendre test; and the Jacobi test, in descriptive form | 475 |
| 289. | Differential equation of minimal surface, under special variations: the range on a catenoid | 479 |
| 290. | General weak variations: transformation of integral for completely independent variables | 481 |

| xx | CONTENTS | PAGE |
|-----------|--|------|
| §§ | | |
| 291. | Properties of the transformed integrand | 483 |
| 292. | Relations between second derivatives of the integrand | 486 |
| 293. | First variation of the modified integral under general weak variations | 487 |
| 294, 295. | Vanishing of the first variation: characteristic equations: boundary condition | 489 |
| 296. | Agreement of boundary conditions with result in § 280 | 491 |
| 297. | Continuity of certain derivatives through an isolated free discontinuity | 492 |
| 298. | The three characteristic equations equivalent to one | 495 |
| 299, 300. | The single characteristic the same as the equation of § 278 | 497 |
| 301. | Minimal surfaces: primitive of characteristic equations obtained | 498 |
| 302. | Boundary condition for minimal surfaces: general form | 501 |
| 303. | The equations of the characteristic provide an actual minimum for weak variations, subject to conjugates | 502 |
| 304. | Schwarz's theorem on the determination of minimal surfaces: examples | 503 |
| 305. | The second variation, under general weak variations | 507 |
| 306, 307. | Subsidiary characteristic equations | 508 |
| 308, 309. | Modification in the form of the second variation | 510 |
| 310. | Relations for the transformation | 513 |
| 311. | Use of subsidiary equations in the transformation | 514 |
| 312. | Remark on the process adopted, similar to § 164 | 516 |
| 313. | Discussion of the normal form: the Legendre test | 517 |
| 314. | The Jacobi test; special application to catenoid | 518 |
| 315. | General expression for small variation to a consecutive minimal surface | 521 |
| 316. | Equations of the consecutive characteristic: general application to catenoid, and to Enneper's minimal surface | 523 |

CHAPTER X.

STRONG VARIATIONS AND THE WEIERSTRASS TEST, FOR DOUBLE INTEGRALS INVOLVING FIRST DERIVATIVES: ISOPERIMETRICAL PROBLEMS.

| | | |
|-----------|---|-----|
| 317. | Strong variations of double integrals: constituent element for surface variations | 528 |
| 318. | The characteristic component of a strong variation | 529 |
| 319. | The component from the arbitrary surface | 530 |
| 320. | Total strong variation: the E_{Σ} -function, and the Weierstrass test | 532 |
| 321. | Alternative forms of the E_{Σ} -function | 534 |
| 322. | Types of strong variation that can be constructed from the elementary type | 537 |
| 323. | The four tests for a double integral | 538 |
| 324. | Examples under the Weierstrass test, including minimal surfaces | 539 |
| 325. | The integral cannot have a maximum or minimum, when the integrand is rational in the derivatives; example, Dirichlet's Principle, in two dimensions | 542 |
| 326. | Isoperimetical problems | 545 |
| 327. | First variation of variable integral and constant integral | 546 |
| 328, 329. | Variations permitted by the constant integral: characteristic equations: conditions at mobile boundary | 547 |
| 330. | Summary of requirements, from first variation | 550 |

CONTENTS

xxi

| §§ | PAGE |
|---|------|
| 331. Continuity of certain derivatives through isolated free discontinuities | 551 |
| 332. Examples : surface enclosing maximum volume within given area | 552 |
| 333. Second variation (weak) for isoperimetrical problem | 561 |
| 334. Strong variations: the E_2 -function : application to example in § 332 | 562 |

CHAPTER XI.

DOUBLE INTEGRALS, WITH DERIVATIVES OF THE SECOND ORDER :
 WEAK VARIATIONS.

| | |
|--|-----|
| 335. Special weak variations : 'first' variation | 567 |
| 336. The special variation and a mobile boundary | 569 |
| 337, 338. Characteristic equation : boundary condition | 570 |
| 339. Second variation | 572 |
| 340. Equations of relation, to modify the expression of the second variation | 573 |
| 341. Subsidiary characteristic equation | 574 |
| 342-346. Resolution of the equations of § 340 by means of integrals of the subsidiary equation | 575 |
| 347. Special relation between two integrals of the subsidiary equation | 582 |
| 348. Normal form of second variation : the Legendre test | 584 |
| 349. Characteristic variations | 585 |
| 350. Consecutive characteristic surface | 587 |
| 351. Conjugate of initial curve on a characteristic surface: the Jacobi test | 589 |
| 352. General weak variations : transformation of the integral | 590 |
| 353. Identities satisfied by the modified integrand | 591 |
| 354. 'First' variation of the integral : three characteristic equations | 594 |
| 355. General boundary condition | 596 |
| 356. The three characteristic equations in § 354 are equivalent to one equation | 597 |
| 357. Identification with equation in § 337 | 598 |

CHAPTER XII.

TRIPLE INTEGRALS WITH FIRST DERIVATIVES.

| | |
|--|-----|
| 358. Lemma in triple integration | 601 |
| 359. First variation of triple integral, under special variation | 603 |
| 360. Characteristic equation : boundary condition | 606 |
| 361. Primitive of the characteristic equation : Cauchy's existence-theorem | 608 |
| 362. Special forms of boundary condition | 610 |
| 363. Second variation | 611 |
| 364. Subsidiary characteristic equation | 613 |
| 365. Normal form: the Legendre test | 614 |
| 366. The Jacobi test: Dirichlet's Principle, expressed as an integral possessing a minimum under weak variations | 615 |
| 367. General weak variations | 618 |
| 368. Identities satisfied by transformed integrand in the triple integral | 619 |
| 369. General weak variation of the integral: the four characteristic equations: boundary surface condition | 621 |
| 370, 371. The four characteristic equations are equivalent to the single equation in § 360 | 622 |

| xxii | CONTENTS | PAGE |
|-----------|---|------|
| §§ | | |
| 372. | Identification of the boundary conditions in §§ 360 and 369 | 624 |
| 373. | Summary of conditions from first variation, under weak variations | 627 |
| 374. | The Legendre test stated, for general weak variations | 627 |
| 375. | Remark on the Jacobi test | 628 |
| 376. | Strong variations for triple integrals; geometrical representation of a volume in quadruple space by ordinary three-dimensional space | 629 |
| 377, 378. | Representation of volumes: (i) characteristic, (ii) rudimentary strong variation | 632 |
| 379–381. | Component of the variation of the integral from the characteristic constituent of the strong variation | 634 |
| 382. | Component of the variation of the integral, from the non-characteristic constituent of the strong variation | 638 |
| 383. | Total strong variation of the integral: the Weierstrass function | 639 |
| 384. | The Weierstrass test for triple integrals | 640 |
| 385. | Examples of the Weierstrass test: not satisfied by the Dirichlet integral of § 366, so that Dirichlet's Principle does not express a true minimum: the four-dimensional minimal volume, which satisfies all the tests | 641 |
| 386. | Conclusion | 645 |
| | INDEX | 647 |