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Complexity Science

The Warwick Master's Course

Edited by
ROBIN BALL
University of Warwick

VASSILI KOLOKOLTSOV
University of Warwick

ROBERT S. MACKAY
University of Warwick

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Preface

Complexity Science is the study of systems with many interdependent components.

There is an urgent global need from industry, commerce, research institutions, academia, government and public services for a new generation trained to understand how complex systems behave, how to live with them, to control them and to design them well. We see this in public service management, transport, public opinion, epidemics, riots, terrorism, weather and climate. Relevant technological developments include distributed computing, data management, process control, personalised medicine, disease management, environmental sensor swarms, complex materials and nanobiotechnology.

Stimulated by problems from such a wide range of scientific disciplines, it presents great challenges and opportunities for Mathematics. Mathematics is essential for a deep understanding of complex systems and how to quantify their behaviour, for conclusions of genuine value to end-users, because of its powers for description, abstraction, deduction and prediction.

A range of Complexity Science concepts unify the field across disciplines: dynamics and diffusion, interacting agents and networks, coherent structures, emergence and self-organisation, upscaling and model reduction, quantification of complexity, scaling and extreme events, probabilistic modelling and statistical inference, feedback and control, diversity, optimisation and evolution.

This volume presents coherent introductions to the mathematical treatment of some areas of Complexity Science. It is based on some of the lecture modules of the Warwick EPSRC Doctoral Training Centre in Complexity Science.

Chapter 1 by Mario Nicodemi, Yu-Xi Chau, Christopher Oates, Anas

Rana and Leigh Robinson introduces the key themes of Self-Organisation and Emergence. It presents some of the basic examples and tools to illustrate and analyse these phenomena.

Chapter 2 by Yulia Timofeeva treats Complexity in Deterministic Dynamical Systems. Dynamical systems are represented by mathematical models describing phenomena whose instantaneous state changes over time. Examples are mechanics in physics, population dynamics in biology and chemical kinetics in chemistry. One basic goal of the mathematical theory of dynamical systems is to determine or characterise the long-term behaviour of the system using methods for analysing differential equations and iterated mappings. This chapter introduces some of the techniques used in the modern theory of dynamical systems and the concepts of chaos and strange attractors, and illustrates a range of applications to problems in the physical, biological and engineering sciences.

Chapter 3 by Stefan Grosskinsky treats Stochastic Dynamics of Interacting Particle Systems. These are lattice-based stochastic models of complex systems, describing the time evolution of a large number of interacting components or agents, which are simply called particles. The notes provide an introduction to their mathematical description using Markov semigroups and generators, and to basic probabilistic tools for their analysis. The techniques are used to understand collective phenomena and phase transitions as a result of local motion and interaction of the particles for several classes of models. This discussion is mainly example-based. It involves the role of symmetries and conservation laws and provides a connection to concepts from equilibrium statistical mechanics discussed in Chapter 4.

Chapter 4 by Ellák Somfai treats Statistical Mechanics of Complex Systems. This chapter starts by introducing equilibrium statistical mechanics via the maximum entropy principle. This is followed by a phenomenological description of phase transitions and various applications where dynamics plays a critical role, including interface growth and collective biological motion.

Chapter 5 by Colm Connaughton treats Numerical Simulation of Continuous Systems. This chapter provides a foundation in practical methods of obtaining numerical solutions of partial differential equations that arise in complexity science applications. The focus is on understanding the advantages and limitations of numerical methods generally and on selecting and validating an appropriate numerical algorithm when faced with a particular problem. It starts with a basic outline of timestepping methods for ordinary differential equations, then proceeds to cover finite difference methods for hyperbolic and parabolic equations, explicit versus implicit timestepping, issues related to stability, stiffness and singularities, fast Fourier transform and pseudo-spectral methods. It is example-based.

Chapter 6 by Vassili Kolokoltsov is on Stochastic methods in Economics and Finance. It presents theory for utility, risk, optimisation, portfolios, derivatives, fat tails, option pricing and credit risk.

Chapter 7 by Robert MacKay is on Space-Time Phases. The objective is to put the concept of "emergence" onto a firm foundation in the context of dynamics on large networks. The key notion is space-time phases: probability distributions for state as a function of space and time that can arise in systems that have been running for a long time. The chapter has two sections, the first treating the stochastic case of probabilistic cellular automata and the second the deterministic case of coupled map lattices.

Chapter 8, also by Robert MacKay is on Selfish Routing. The chapter is a summary of the very interesting theory of the gap between free market and centrally controlled solutions for many agent systems in an idealised case of traffic flow, following the excellent book by Roughgarden.

We are most grateful to Dayal Strub for typing up the notes of RSM, preparing the figures for RSM and VNK, putting all the files together into the required style and sorting out many issues with typesetting.

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R.C. Ball
V.N. Kolokoltsov
R.S. MacKay

Contributors

Mario Nicodemi*, Yu-Xi Chau†, Christopher Oates†, Anas Rana† and Leigh Robinson†
Istituto Nazionale Fisica Nucleare, Napoli, 80126, Italy
†*Centre for Complexity Science, University of Warwick, Coventry, CV4 7AL, UK*

Yulia Timofeeva *Department of Computer Science and Centre for Complexity Science, University of Warwick, Coventry, CV4 7AL, UK*

Stefan Grosskinsky *Mathematics Institute and Centre for Complexity Science, University of Warwick, Coventry, CV4 7AL, UK*

Ellák Somfai *Wigner RCP SZFI, H-1121 Budapest, Konkoly-Thege M. u. 29-33, Hungary*

Colm Connaughton *Mathematics Institute and Centre for Complexity Science, University of Warwick, Coventry, CV4 7AL, UK*

Vassili N. Kolokoltsov *Department of Statistics, University of Warwick, Coventry, CV4 7AL, UK*

Robert S. MacKay *Mathematics Institute and Centre for Complexity Science, University of Warwick, Coventry, CV4 7AL, UK*