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978-1-107-63638-5 - Moduli Spaces

Edited by Leticia Brambila-Paz, Oscar García-Prada, Peter Newstead and Richard P. Thomas

Table of Contents

[More information](#)

Contents

<i>Preface</i>	<i>page</i> ix
<i>List of contributors</i>	xi
1 Introduction to algebraic stacks	1
<i>K. Behrend</i>	
Introduction	3
1.1 Topological stacks: triangles	8
1.1.1 Families and their symmetry groupoids	8
1.1.2 Continuous families	13
1.1.3 Classification	16
1.1.4 Scalene triangles	23
1.1.5 Isosceles triangles	24
1.1.6 Equilateral triangles	26
1.1.7 Oriented triangles	26
1.1.8 Stacks	32
1.1.9 Versal families	34
1.1.10 Degenerate triangles	43
1.1.11 Change of versal family	58
1.1.12 Weierstrass compactification	66
1.2 Formalism	74
1.2.1 Objects in continuous families: categories fibered in groupoids	74
1.2.2 Families characterized locally: prestacks	79
1.2.3 Families which can be glued: stacks	82
1.2.4 Topological stacks	82
1.2.5 Deligne–Mumford topological stacks	89
1.2.6 Lattices up to homothety	96
1.2.7 Fundamental groups of topological stacks	99

1.3	Algebraic stacks	104
1.3.1	Groupoid fibrations	104
1.3.2	Prestacks	107
1.3.3	Algebraic stacks	113
1.3.4	The coarse moduli space	117
1.3.5	Bundles on stacks	120
1.3.6	Stacky curves: the Riemann–Roch theorem	124
2	BPS states and the $P = W$ conjecture	132
	<i>W.-Y. Chuang, D.-E. Diaconescu, and G. Pan</i>	
2.1	Introduction	132
2.2	Hausel–Rodriguez-Villegas formula and $P = W$	138
2.2.1	Hausel–Rodriguez-Villegas formula	139
2.2.2	Hitchin system and $P = W$	140
2.3	Refined stable pair invariants of local curves	141
2.3.1	TQFT formalism	142
2.3.2	Refined invariants from instanton sums	142
2.4	HRV formula as a refined GV expansion	145
3	Representations of surface groups and Higgs bundles	151
	<i>Peter B. Gothen</i>	
3.1	Introduction	151
3.2	Lecture 1: Character varieties for surface groups and harmonic maps	152
3.2.1	Surface group representations and character varieties	152
3.2.2	Review of connections and curvature in principal bundles	153
3.2.3	Surface group representations and flat bundles	155
3.2.4	Flat bundles and gauge equivalence	156
3.2.5	Harmonic metrics in flat bundles	157
3.2.6	The Corlette–Donaldson theorem	159
3.3	Lecture 2: G -Higgs bundles and the Hitchin–Kobayashi correspondence	160
3.3.1	Lie theoretic preliminaries	160
3.3.2	The Hitchin equations	161
3.3.3	G -Higgs bundles, stability and the Hitchin–Kobayashi correspondence	163
3.3.4	The Hitchin map	166
3.3.5	The moduli space of $SU(p, q)$ -Higgs bundles	166

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Table of Contents

[More information](#)

	<i>Contents</i>	vii
3.4	Lecture 3: Morse–Bott theory of the moduli space of G -Higgs bundles	168
3.4.1	Simple and infinitesimally simple G -Higgs bundles	168
3.4.2	Deformation theory of G -Higgs bundles	169
3.4.3	The \mathbb{C}^* -action and topology of moduli spaces	170
3.4.4	Calculation of Morse indices	172
3.4.5	The moduli space of $\mathrm{Sp}(2n, \mathbb{R})$ -Higgs bundles	174
4	Introduction to stability conditions	179
	<i>D. Huybrechts</i>	
4.1	Torsion theories and t-structures	181
4.1.1	μ -stability on curves (and surfaces): recollections	181
4.1.2	Torsion theories in abelian categories	186
4.1.3	t-structures on triangulated categories	187
4.1.4	Torsion theories versus t-structures	190
4.2	Stability conditions: definition and examples	192
4.2.1	Slicings	192
4.2.2	Stability conditions	194
4.2.3	$\mathrm{Aut}(\mathcal{D})$ -action and $\widetilde{\mathrm{GL}}^+(2, \mathbb{R})$ -action	198
4.2.4	Stability conditions on curves	200
4.3	Stability conditions on surfaces	203
4.3.1	Classification of hearts	204
4.3.2	Construction of hearts	208
4.4	The topological space of stability conditions	210
4.4.1	Topology of $\mathrm{Slice}(\mathcal{D})$	210
4.4.2	Topology of $\mathrm{Stab}(\mathcal{D})$	213
4.4.3	Main result	214
4.5	Stability conditions on K3 surfaces	216
4.5.1	Main theorem and conjecture	216
4.5.2	Autoequivalences	219
4.5.3	Building up $\mathrm{Stab}(X)^\circ$	220
4.5.4	Moduli space rephrasing	224
4.6	Further results	226
4.6.1	Non-compact cases	226
4.6.2	Compact cases	227
5	An introduction to d-manifolds and derived differential geometry	230
	<i>Dominic Joyce</i>	
5.1	Introduction	231

Cambridge University Press

978-1-107-63638-5 - Moduli Spaces

Edited by Leticia Brambila-Paz, Oscar García-Prada, Peter Newstead and Richard P. Thomas

Table of Contents

[More information](#)

viii

Contents

5.2	C^∞ -rings and C^∞ -schemes	234
5.2.1	C^∞ -rings	234
5.2.2	C^∞ -schemes	236
5.2.3	Modules over C^∞ -rings, and cotangent modules	239
5.2.4	Quasicoherent sheaves on C^∞ -schemes	240
5.3	The 2-category of d-spaces	243
5.3.1	The definition of d-spaces	243
5.3.2	Gluing d-spaces by equivalences	246
5.3.3	Fibre products in dSpa	249
5.4	The 2-category of d-manifolds	250
5.4.1	The definition of d-manifolds	250
5.4.2	‘Standard model’ d-manifolds, 1- and 2-morphisms	251
5.4.3	The 2-category of virtual vector bundles	255
5.4.4	Equivalences in dMan , and gluing by equivalences	257
5.4.5	Submersions, immersions and embeddings	259
5.4.6	D-transversality and fibre products	262
5.4.7	Embedding d-manifolds into manifolds	264
5.4.8	Orientations on d-manifolds	266
5.4.9	D-manifolds with boundary and corners, d-orbifolds	269
5.4.10	D-manifold bordism, and virtual cycles	272
5.4.11	Relation to other classes of spaces in mathematics	274
A	Basics of 2-categories	277
6	13/2 ways of counting curves	282
	<i>R. Pandharipande and R. P. Thomas</i>	
0	Introduction	283
$\frac{1}{2}$	Naive counting of curves	287
$1\frac{1}{2}$	Gromov–Witten theory	289
$2\frac{1}{2}$	Gopakumar–Vafa / BPS invariants	294
$3\frac{1}{2}$	Donaldson–Thomas theory	298
$4\frac{1}{2}$	Stable pairs	306
$5\frac{1}{2}$	Stable unramified maps	314
$6\frac{1}{2}$	Stable quotients	319
	Appendix: Virtual classes	324