CHAPTER I

THE REDUCTION OF INSTRUMENTAL TONES TO A SINGLE SERIES OF PURE TONES

THE world of sound is bounded by the two extremes of pure tone and mere noise. The home of music lies in the lands around the ideal of tone. This ideal forms the first problem of musical science.

It is properly termed an ideal because pure tones rarely, if ever, occur under natural circumstances. That is evident from the familiar fact that, however perfectly a musical instrument may be played, its tones are easily distinguishable from those of other instruments, even though they may be of the same pitch. The same series of tones of exactly the same pitches, e.g. the diatonic scale on a c^1 of 264 vibrations per second, may be given by an indefinite number of instruments, and will be recognised as different on each, in spite of the sameness that is obviously common to all.

This peculiar complexity of tone has been explained by modern research in a way that at least in principle is complete and final. However pure and beautiful an instrumental tone may be, it can be analysed into audible parts by special means of two kinds. In the first the ear is provided with instruments which will increase the intensity of certain parts of the tone—if they are present in the tone to be analysed—beyond that of the other parts. When the resonator is placed against the ear, it seems to be full of the magnified sound, whose pitch may be surprisingly different from that of the tone it comes from. But its presence in the resonator is easily shown to depend upon the studied tone. Whenever the resonator is placed against the ear, it appears; and it only appears in the resonator when the tone being analysed is sounded, unless, of course, it is given from some other source at the same time. Any such experimental error can be easily avoided in most cases. After a little practice with the resonator, the partial tone will often be distinguishable in the whole tone even when the resonator has not been placed upon the ear. This is not the result of imagination or illusion. It only means that the ear has now been trained for this particular case to expect a certain partial tone and to direct observation specially upon it, so that it appears to be more or less abstracted from the whole.

A generalisation of this procedure gives the second method of analysis. The ear is first prepared for special observation (or abstraction) by listening for some time to the tone expected to occur. The tone to be analysed is then presented and if it contains the prepared tone as a partial, the latter will probably be heard sounding faintly through the whole. If the ear has been prepared for a tone whose pitch is not quite the same as that of the partial actually present in the whole, the listener will not hear the pitch he expects, but another that lies in pitch near the one expected. When the ear has thus been trained for many or for all the partials of a tone, it may be able to run through them all in sequence without any special preparation. And in the course of time it may learn to do this for any sound of a tonal nature. Even then, however, the tones of well played musical instruments do not cease to be the beautifully perfect unities they were before. They do not fall to pieces, as it were, permanently, but only when the attention is concentrated and moved from one of the partials to another.

The occurrence of these partials is due to the fact that most musical instruments when brought into a certain rate of vibration—n times per second,—fall at the same time into various rates of vibrations that may be any whole multiple of n : $2n$, $3n$, $4n$, $5n$, etc. The pitches corresponding to these ratios of vibration are : octave, octave and fifth, double octave, double octave and third, double octave and fifth ($6n$), a pitch slightly flatter than the $b^\flat$ above ($7n$), the triple octave ($8n$), then the d ($9n$) and the e ($10n$) above this c, and so on. The pitch of any partial may easily be reckoned out from a knowledge of the ratios for the chromatic scale and by approximations thereto. These ratios are :

c,	$d^\flat$,	d,	$e^\flat$,	e,	f,	$f\sharp$,	g,	$a^\flat$,	a,	$b^\flat$,	b,	c^1
24,		27,		30,	32,		36,		40,		45,	48
$\frac{1}{1}$	$\frac{16}{15}$	$\frac{9}{8}$	$\frac{6}{5}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{45}{32}$	$\frac{3}{2}$	$\frac{8}{5}$	$\frac{5}{3}$	$\frac{16}{9}$	$\frac{15}{8}$	$\frac{2}{1}$

All of these ratios can be derived from those of the octave (2 : 1), fifth (3 : 2), and major third (5 : 4) that have played so important a part in the history of musical theory.

The existence of partial tones has been confirmed objectively in a number of ways. In various cases the presence of vibrations corresponding to the pitch of the partials heard can be demonstrated to vision. A long stretched string may be seen to vibrate, not only in its whole length, but also in parts of such length as would give the various partials as independent tones, if these parts of the string were made to vibrate separately from the rest of the string (cf. 35, 91 f.). In other cases the motion of a minute mirror standing in connexion with a vibrating membrane may be photographed with so little error that the result may be taken as representing the motions of the air that excite the vibrating membrane (40, 78 ff.). (Phonography is dependent upon such a vibrating membrane, and everyone is aware how good a reproduction of sound may be obtained thereby.) When the photographs so obtained are subjected to mechanical 'harmonic analysis,' the partial tones that result correspond very closely with those that can be heard by the most careful analysis with resonators or with prepared attention.

Such studies as these show that tones of the same nominal pitch from different instruments differ only in respect of the group of partials from the full series (of possible multiples of *n*) that they contain and in the relative strength of these. Some tones like those of tuning-forks and of the flute contain very few partials, perhaps only the first. Others, such as pianoforte tones, are rich in the lower partials. Others again, like those of the trumpets, contain a host of high partials in great strength, which give them their peculiar brightness and brilliancy. And so on. The results of this line of study will be found extensively in special treatises and in text-books of physics (40, 175 ff.; 20, 118 f., etc.).

Partials may be eliminated from the tones of any instrument by the method of physical interference. That consists in principle of the conduction of a sound containing at least one partial other than the fundamental component of the tone, along a tube which for a certain length is double and then unites again to enter the ear. The one doubled part is made longer than the other by half a wave-length of the partial to be eliminated. When the parts unite, each will bring this partial in exactly opposite phase to the other. If the one is at the phase of maximal condensation of the air, the other will be at that of maximal rarefaction; and the result will be the elimination of that component of the aerial disturbance. If there are many partials in the tone, there will have to be a special device of this kind for the elimination of each, so that the apparatus for the production of a pure tone from an instrumental one is apt to be somewhat complicated, unless special care is taken to begin with a tone containing very few partials, such as that of a tuning-fork.

In spite of the difficulties that thus face any complete generalisation, no one doubts for a moment that the series of perfectly pure tones, each consisting of a fundamental with no upper partials at all, thus isolated,

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is one and the same, no matter from what instrument it may have been derived. This is in perfect accordance with the results of the attentional analysis of instrumental tones. For the partials separated by the attention do not seem to differ from the (pure) tones of identical pitch, otherwise produced, in any such way as would make us believe that the series of pure tones is not the same whatever its source may be.

Thus we obtain a simpler starting-point for our study of tone—the series of pure tones, each one of which corresponds to a certain fixed rate of aerial vibrations, unmixed with any other rate. Now this series is perfectly continuous. If we start from any ordinary pitch, e.g. middle *c* of 264 vibrations per second, we can raise or lower the pitch of tone gradually, producing differences as minute as the mechanical means at our disposal will allow. There is no reason in the nature of tone why we should select any one pitch or rate of vibration for our *c* or *a* rather than any other. And even when a standard pitch has been adopted for practical purposes, minor variations due to change of temperature, mis-tuning, etc., are inevitable. It has been claimed that the vibratory rate of 256 should be taken as the standard of 'philosophic pitch,' because $256 = 2^8$; i.e. if we imagine a tone of one vibration per second, the (fictional) tone of two vibrations would be the octave of it, four vibrations would give the double octave, and so on, so that the eighth octave would give us 256 vibrations (50, 33 ff.). It is certainly very useful to have a commonly accepted standard for convenience of reference. Then we know what rate of vibration is implied by any nominal pitch, e.g. A_1, without having to give it separately. But the standard now perhaps most commonly in use is a c^1 of 264 vibrations per second. One advantage of this basis (although a slight one), is that it is a multiple of 24, and so can be readily used in connexion with the diatonic series of ratios stated above. I shall use this standard throughout the following pages unless some other standard is specially indicated. The usually current nomenclature of octaves may be looked upon as starting from 'middle *c*,' the *c* common to the baritone and contralto voice, which is called c^1. Above that the octaves are c^2, c^3, c^4, c^5 (the highest note on the large concert grand piano), c^6, etc.; below we have c^0, C, C_1, C_2 (A_2 is the lowest note on the same instrument). Plain letters will thus indicate absolute pitch, italicised letters relative pitch.

CHAPTER II

ANALYTIC DESCRIPTION AND THEORY OF THE SERIES OF PURE TONES

HAVING thus reduced the usual tonal material of music to its simplest components, we have now to describe this continuous series. The terms of our description, to be scientifically useful and explanatory, must be such as will bring tones into systematic connexion with as many other similar objects as possible.

Being dependent upon the working of a sense organ—the cochlea of the ear,—tones are classified in psychology as sensations. We naturally expect them to show great similarity to the sensations we get from our other sense-organs, such as the eye (vision), the tongue (taste), the skin (touch, temperature, pain), and various others, such as hunger and thirst. The similarity of all these to one another is certainly not at first striking. And it has usually been thought that the differences are far more numerous and important than any resemblances there may happen to be. Many men, indeed, judging by the perennial failure of the attempt to bring our different senses into systematic connexion, have adopted a standpoint of extreme scepticism towards any such claim or expectation. But since psychology became, some decades ago, an experimental science, the study of the sensations has been pursued most carefully and exhaustively, and the real relations of resemblance and of structure between our various sensations have gradually grown clearer. Beyond this nothing is required but a frank and determined rejection of the old prejudice and a whole hearted effort to work out the inner similarity of sound and of the few other senses we have. The problem then is to describe the tonal series so as to show the inner connexion not only between all the parts of the series, but between tones and the sensory objects of the other senses.

The first and most obvious feature of sounds is that which distinguishes them from the sensations of other senses. No kind of sound or group of sounds is ever confused with a sight or with a touch. Psychologists call this a difference of quality. The word quality is often used by musicians to designate that difference of tones of the same pitch which is due to the peculiar blend of partials they contain. It is better, however, to call this the (pitch) blend of tones. For practical purposes that word is the best which most readily suggests the thing named, or its cause, or the like. The word blend is in common use as a name for similar differences in objects that appeal to other senses, especially to taste and smell, of which the latter is the more important. The blend is here due to the mixture of the components. Similarly the blend of a tone will be the difference due to the admixture of partials, which a trained ear can learn to pick out and name, as a practised palate will detect the components of a tea or the varying flavours of a wine. This word 'blend' seems better than the French word 'timbre,' which does not fit into our language either in its native pronunciation or in ours.

The second attribute that is found in all tones and in the sensations of other senses is intensity. Both scientific and popular usage agree as to the meaning of this term. The word loudness is not so useful for classification, because it is inapplicable to the other senses and so does not serve to indicate any variant common to them all. One word of frequent occurrence must be carefully avoided in this connexion, namely, volume. We think of loudness as great volume when many instruments sound together as in an orchestra and so make a very intense sound or a mass of very many sounds. Having thus associated many sounds with much sound, we often use the word where there is obviously only one sound present, as when we speak of the great volume of a singer's voice, especially of a contralto's or a bass's. Here a touch of the scientific usage begins to appear. But that highly justifiable usage does not tolerate any confusion of volume with loudness.

Volume is properly used to distinguish that difference between tones of different pitch that makes the low tone great, massive, all-pervasive, and the high tone small, thin, and light. The other words we use to designate differences of pitch have the same sort of association. Sharp and flat are closely akin to thin and broad, or small and large. The Latin and French words gravis, grave, acutus, aigu, bear the same implications. In the eighth Problem on Music Aristotle asked: "Why does the low tone dominate the higher? Is it because the low is the greater? For it is like the obtuse angle, while the other resembles the acute angle." And the twelfth Problem answers: "Is it because the low tone is great and therefore more powerful and because the small is included in the great?" (65, 17, 19; cf. 16, 13, 19).

Although the words of these sentences are very suggestive to a theorist of the present day, it is doubtful whether Aristotle meant really to ascribe differences of size to the tones as mere sounds or sensations. His mind was very much impressed by the discovery that the low string gives out not only its own tone but the higher octave, so that, as we should say now also, the low tone contains 'the higher one'—its own first higher partial, the octave of itself. But Aristoxenus refers to "the blunder of Lasus and some of the school of Epigonus, who attribute breadth to tones" (1, 167). And many modern writers have inclined more or less tentatively towards this idea as an explicit description of tones as such. We must now certainly take the idea with complete seriousness and think of tones as of different size or mass or bulk, just as a visual sensation can be of different size in respect of its mass or area, or as pain and hunger can be large and massive, or as pain and touch can be small as sand or needles. There is every reason to believe that this difference of volume or extent is dependent upon the number of elementary sense-organs of hearing that are in action at the same time. But that is a question for physiology.

The only other property of pure tones is what we commonly call their pitch. By pitch tones fall into a definite order or series. This is not naturally a discrete series: like the ordinal numbers, of which each one is an individual separated from the next by a unit of space, into which other numbers of a fractional nature may be fitted; or like the pitches of the diatonic scale. It is a continuous series: we can pass from any one point of it to any other by gradations that are not distinguishable from those that lie next to them on either side, but that are distinguishable from those that lie a certain distance away on either side. And the series is ordinal, not because it can be considered *conceptually* as a continuous series of positions, but because it appears so to us phenomenally, as is often said, or merely as sensation. The series of colours of the spectrum, merely as colours (i.e. apart from their position in the dispersed spectrum, and from the wave-length they depend upon) can be treated in an ordinal way, although that series is, as sensation, really a series of qualities. But the pitch series is, as sensation, itself really ordinal[1]. It presents itself to us as ordinal and

[1] Aristoxenus wrote: "Tension is the continuous transition of the voice from a lower position [τόπου] to a higher," etc. (1, 102. 172; 14, 83f.). The Greek term is highly suggestive. But it does not seem certain that he meant by it more than cessation of change of the voice, a permanence of one kind of activity. It is, of course, significant that in this case we naturally incline towards the term 'place' or to the idea of the voice's 'moving.' We do not incline to say that the weather 'moves,' or the colours of leaves 'move' in the autumn, when we mean only that they change. No doubt Aristoxenus used the terms 'place' and 'motion' at the suggestion of the ordinal and motional aspects of tone. Nevertheless his concepts of position and motion of the voice probably did not include more than what he might have attributed to a thing that only changes, i.e. progress of change and arrest of change Cf. 80, 221f.

calls for ordinal names, whether we know anything about the wave-lengths that cause it or not.

The attributes of tones thus far enumerated are: quality, intensity, volume and pitch. The relations between these four are an important problem. It has been suggested that intensity is a sort of density of sensation, as it were. Just as a gas may fill a certain volume and yet be very thinly scattered throughout it, so it is thought a sensation may be of one and the same quality, and volume, and pitch, and yet be more or less dense—or intense. Suggestive arguments in favour of this view have been advanced, but they do not yet seem sufficient for their purpose.

The relation between pitch and volume is much clearer. When tones are compared with noises, a marked difference is apparent. Tones, as everyone feels and knows, are smooth and regular, noises are rough and irregular. Tones may also be said to be balanced and symmetrical, while noises are chaotic and disorderly. These descriptions obviously refer to the volume of tones, not to their pitches. A pitch has only a definite position or place; it is not smooth or balanced. But pitch gives tone a position as a whole; it is by means of pitch that tones are brought into a definite and accurate series, and their volumes along with them. The question then arises: what position has pitch itself in the tone's volume?

This is not an absurd question, but a very natural one. For if tones have an aspect of volume and can be arranged in a very definite and single series by means of pitch—a property that is distinguishable from volume; and if pitch is not only thus really ordinal, but is also felt as ordinal or appears to us so as a property of sensation; it is perfectly natural to suppose that what is thus ordinal is a part of the tone's volume and to ask in consequence—which part of the volume constitutes the pitch?

Of course, one's habits of thought may oppose this line of inquiry. One of the greatest obstacles to the advance of knowledge is the opposition our minds offer by the mere force of unfamiliarity to the application of old and simple notions to common objects to which they have not hitherto been applied. The mind seems to refuse to establish the desired connexion. All sorts of excuses and objections are offered to the new invitation. "Metaphors are so misleading." But it is not a case of metaphors now. Pitch is no mere analogy; the ordinal status and arrangement of tones is one of the bed-rock facts of music. And 'volume' is no more a mere simile than is interval or concord or discord. It is as much there as any fact could possibly be. Psychologists admit it more and more frequently, and it is only a matter of time till everyone who considers the subject will agree with them. Nor is it 'mystical' to suggest that pitch has a position in volume. A line of well-founded and logical thought is only mystical to those who do not take the trouble to follow it carefully. A mystic is one who claims to have special insight or experience which he has discovered by accident or providential good-will and which he is powerless to reveal to others either because it defies all description or because, not knowing how he himself attained it, he is unable to lead thither all who would share it with him. But there is nothing mystical about pitch or tonal volume; nor are the ordinary logical processes of inference held to be the special privilege of a few minds.

We may therefore consider our question clear and reasonable. And the most likely answer follows naturally from the apparent balance and symmetry of tones as compared with noises. We may assume that pitch holds a central position in volume. And, as pitch is ordinal, while volume suggests a volume of parts or particles, we may go on to assume that pitch is constituted by a specially prominent or noticeable part of the volume of sound that makes up a tone. We certainly do not hear tone as a group of distinguishable particles like a handful of sand. We hear it as a continuously smooth closed volume. But nevertheless a part of this whole volume might well be more noticeable than the rest, just as a part of a variably 'toned' visual surface may be most deeply coloured—red, for example,—although we could not pick out and isolate any part of it that would be all, and nothing less or more than all, the reddest part. Yet no one doubts that a visual surface consists of a mass of minimal particles of colour surface, grouped into a continuous whole. These particles are presumably the minimal areas of colours given by single recipient visual organs—the cones (and rods) of the retina. There are also in the ear elementary receptors of sound; and these presumably afford us the minimal particles of sound that make up tone.

Now all (pure) tones are the same in symmetry and balance and smoothness. So we may consider this central position and predominance of pitch to be characteristic of pure tone as against all grades of noise, which are relatively rough and unbalanced, vague or indefinite in pitch or marked by many prominent points of pitch. Tones differ from one another in size of volume and in the ordinal position of their pitches relatively to one another. The pitch of a higher tone lies a little to one side of the pitch of a tone just lower in pitch; and the pitches of all tones together form a single linear series, having the tone of greatest volume at one end and the tone of least volume at the other. If we were to project the volumes and the pitches of all the tones of the series against one another in our thought, we should obtain a scheme of the following kind:

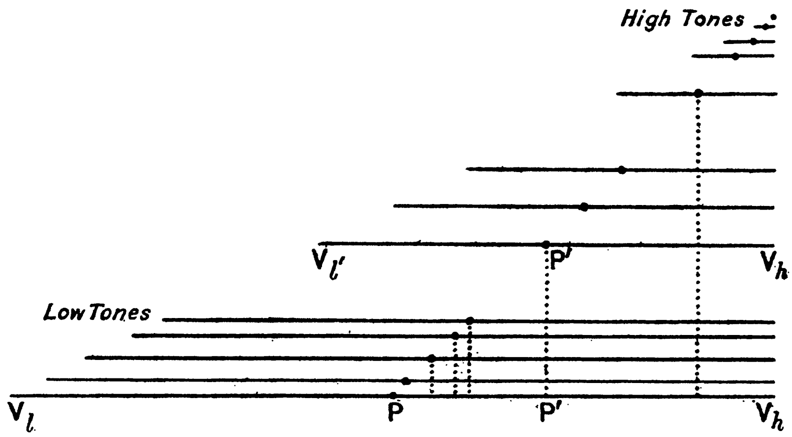

Fig. 1

If we placed all the pitch-points on a perpendicular line above one another, we should indeed represent the decrease of volume (as we go up) properly by the decrease in the breadth of the line used, while the symmetry and balance of tone would be indicated by the central position of the *P* point in the volume line (*Vl* = 'lower' end of volume, *Vh* = 'higher' end of volume). But we should not have given any representation of the fact that the pitch of a tone higher than another lies on one side of the pitch of the latter in an ordinal series. This series is quite properly indicated in our figure.

We have as yet no proof for the assumption that the *Vh* ends of all the volumes should lie perpendicularly above one another, or—whether in mere projection on the base line of the figure (which may be supposed to represent the greatest possible volume or the lowest possible tone), or in reality—should be the same point. It is conceivable that the pyramid of tones should be acute or obtuse angled rather than right-angled. But these alternatives are far from likely for various reasons of which the most important will be set forth immediately.