1 WORKING WITH NUMBERS

WHAT YOU NEED TO KNOW

- The definition of the different number sets:
  - $\mathbb{N}$ is the set of natural numbers $\{0, 1, 2, 3, \ldots\}$.
  - $\mathbb{Z}$ is the set of integers $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$; $\mathbb{Z}^+$ is the set of positive integers; $\mathbb{Z}^-$ is the set of negative integers.
  - $\mathbb{Q}$ is the set of all rational numbers. A rational number can be written as a fraction using two integers, where the denominator must not be zero. Examples include $-\frac{1}{2}, 0.54, 17, 2\frac{3}{4}$.
  - $\mathbb{R}$ is the set of all real numbers. These include the natural numbers, integers and rational numbers, as well as irrational numbers. An irrational number cannot be written as a fraction; examples include π, $\sqrt{3}$, $\sin 60°$.
- Numbers can be written to a fixed number of decimal places or significant figures:
  - To round to one decimal place, look at the digit in the second decimal place; to round to two decimal places, look at the digit in the third decimal place. If that digit is less than 5, round down; if the digit is 5 or more, round up.
  - To round to three significant figures, look at the digit to the right of the third significant figure. If that digit is less than 5, round down; if the digit is 5 or more, round up.
- Very small and very large numbers can be expressed in standard form (or scientific notation) $a \times 10^k$ where $1 \leq a < 10$ and k is an integer.
- To find the percentage error, use the formula $\varepsilon = \left|\frac{v_A - v_E}{v_E}\right| \times 100\%$, where v_E is the exact value and v_A is the approximate value of v.
- Converting to a larger unit means fewer of them, so divide. Converting to a smaller unit means more of them, so multiply.
- To change one currency to another, multiply by the appropriate exchange rate. When commission is charged, first work out the amount of commission paid. The exchange rate is then applied to the 'original amount – commission paid'.

EXAM TIPS AND COMMON ERRORS

- The square root of any number that is not a perfect square (e.g. $\sqrt{4}$) or the ratio of two perfect squares (e.g. $\sqrt{\frac{16}{25}}$) will be irrational. Many trigonometric ratios (e.g. $\sin 45°$) are irrational.
- Zeros at the beginning of a decimal (e.g. **0.000** 301) and at the end of a large number (e.g. 134 **000**) do not count as significant figures. The first significant figure of a number is the first non-zero digit in the number, counting from the left.

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1.1 DIFFERENT TYPES OF NUMBERS

WORKED EXAMPLE 1.1

Write down a number that is:

(a) a real number but not rational
(b) a rational number and in $\mathbb{N}$
(c) a rational number but not in $\mathbb{N}$.

(a) π

π is an irrational number as it cannot be written as a fraction.

(b) 3

All integers are rational numbers, e.g. $3 = \frac{3}{1}$ is a fraction with denominator 1.

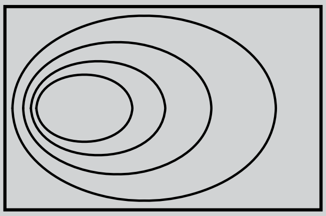
(c) $\frac{1}{2}$

Practice questions 1.1

1. Mark each cell to indicate which number set(s) the number belongs to. The first row has been completed for you.

	$\mathbb{N}$	$\mathbb{Z}$	$\mathbb{Q}$	$\mathbb{Z}$
–3	✗	✓	✓	✓
0.76				
cos 120°				
$\sqrt{5}$				
1.23×10^8				

2. The Venn diagram shows the sets $\mathbb{N}$, $\mathbb{Q}$, $\mathbb{R}$ and $\mathbb{Z}$.
(a) Add labels to show which region corresponds to each set.
(b) Put the following numbers into the correct set:
3, –3, 3.3 and π.



3. A set contains the elements x such that $-4 \le x < 6$, $x \in \mathbb{Z}$. Each element is equally likely to occur. One element is drawn at random.
Find the probability that it is in: (a) $\mathbb{N}$ (b) $\mathbb{Q}$.

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1.2 STANDARD FORM AND SI UNITS

WORKED EXAMPLE 1.2

If $x = 2.3 \times 10^{12}$ cm and $y = 4.8 \times 10^{-11}$ km:

(a) write x and y in metres

(b) find xy, giving your answer in metres squared in the form $a \times 10^k$ where $1 \le a < 10$ and $k \in \mathbb{Z}$.

> 1 km = 1000 m, 1 cm = 0.01 m, 1 mm = 0.001 m
> 1 kg = 1000 g, 1 mg = 0.001 g
> $1\text{ m}^2 = 100\text{ cm} \times 100\text{ cm} = 10\,000\text{ cm}^2$
> $1\text{ m}^3 = 100\text{ cm} \times 100\text{ cm} \times 100\text{ cm} = 1\,000\,000\text{ cm}^3$.

(a) $x = \dfrac{2.3 \times 10^{12}\text{ cm}}{100\text{ cm/m}} = 2.3 \times 10^{10}\text{ m}$

$y = 4.8 \times 10^{-11}\text{ km} \times 1000\text{ m/km} = 4.8 \times 10^{-8}\text{ m}$

(b) $xy = 2.3 \times 10^{10}\text{ m} \times 4.8 \times 10^{-8}\text{ m}$
$= 1104\text{ m}^2 \approx 1.10 \times 10^3\text{ m}^2$

> Converting to a larger unit means fewer of them, so divide. Converting to a smaller unit means more of them, so multiply.

> Make sure that you can convert numbers to standard form on your calculator. However, you must not write calculator notation, such as 10E3, in your answer.

Practice questions 1.2

4. Given that $a = 5.6 \times 10^{10}$ and $b = 1.6 \times 10^{-4}$, calculate the following, giving your answer in the form $c \times 10^k$ where $1 \le c < 10$ and $k \in \mathbb{Z}$:

(a) ab (b) $\dfrac{a}{b}$

5. Given that $x = 4.3 \times 10^8$ g and $y = 0.98$ hours, calculate $\dfrac{x}{y}$, giving your answer in kg per second in the form $a \times 10^k$ where $1 \le a < 10$ and $k \in \mathbb{Z}$.

6. A room measures 3.1 m by 4.4 m. Find the area of the room in:

(a) m^2 (b) cm^2

(c) Give your answer to (b) in the form $a \times 10^k$ where $1 \le a < 10$ and $k \in \mathbb{Z}$.

> If numbers are very big or very small, the GDC gives the answers in standard form automatically.

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1.3 APPROXIMATION AND ESTIMATION

WORKED EXAMPLE 1.3

(a) Write down $\sqrt{2}$ correct to two decimal places.
(b) Write down $\sqrt{2}$ correct to the nearest ten.
(c) Write down $\sqrt{2}$ correct to two significant figures.
(d) Calculate the percentage error if $\sqrt{2}$ is given correct to two significant figures.

In an exam question, if a specific degree of accuracy is not asked for, give your answer correct to three significant figures.

(a) 1.41

$\sqrt{2} = 1.41421\ldots$ To round to 2 decimal places, look at the digit in the third decimal place, which is 4. It is less than 5, so round down.

(b) 0

Remember that 0 is a multiple of ten.

(c) 1.4

Find the second significant figure and look at the digit after it. As 1 < 5, round down.

(d) $\varepsilon = \left|\frac{v_A - v_E}{v_E}\right| \times 100\% = \left|\frac{1.4 - \sqrt{2}}{\sqrt{2}}\right| \times 100\%$

$= 1.01\%$ (3 SF)

Substitute the rounded value, $v_A = 1.4$, and the exact value, $v_E = \sqrt{2}$, into the formula for percentage error. The modulus sign simply means that we remove any negative sign which occurs.

Practice questions 1.3

7. (a) Write π correct to three decimal places.
(b) Find the percentage error when π is given correct to three decimal places.

8. (a) Write down $\sqrt{46}$ and π correct to three significant figures.
(b) Write down the value of $\pi^{\sqrt{46}}$, giving all the digits shown on your calculator.
(c) Write down the value of $\pi^{\sqrt{46}}$ using the approximate values found in part (a), giving all the digits shown on your calculator.
(d) To how many significant figures is your result in part (c) correct?
(e) What is the percentage error in your answer to part (c)?

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1.4 CURRENCY CONVERSIONS

WORKED EXAMPLE 1.4

The table shows the exchange rates for US dollars and euros:

	? USD	? EUR
1 USD	1	0.78
1 EUR	p	1

(a) Find the value of p to two decimal places.
(b) What is the value of \$150 in euros?
(c) Jamie changes €300 into dollars. She is charged 6% commission. How many dollars does she receive?

(a) 1 USD = 0.78 EUR
$1 \div 0.78$ USD = 1 EUR
so $p = 1.28205\ldots = 1.28$ (2 DP)

We treat this as an equation and divide both sides by 0.78.

(b) \$150 = 150×0.78 EUR = 117 EUR

Use the exchange rate 1 USD = 0.78 EUR.

(c) $0.06 \times 300 = 18$ EUR
$300 - 18 = 282$ EUR
$282 \times 1.28205\ldots = 361.54$ USD

First work out the amount of commission paid. The exchange rate is then applied to the remaining amount.

⚠ Use the full accuracy of the conversion rate, not a rounded result.

Practice questions 1.4

9. The table gives the conversion rates between British pounds (GBP) and Australian dollars (AUD). Find x, and hence find the number of pounds which can be bought with 1000 AUD if there is a 4% commission.

	? GBP	? AUD
1 GBP	1	1.58
1 AUD	x	1

10. A currency office buys one South African rand (SAR) for 10.95 Japanese yen and sells one SAR for 11.05 yen.

(a) Blaise converts 1000 SAR to yen for a holiday. While on holiday he spends half of this money. On his return he converts the remainder back to SAR. How many SAR will he get back?

(b) Robbie also converts 1000 SAR to yen for a holiday. He then cancels his holiday and changes all the yen back to SAR. How much has Robbie lost after the two transactions? Express your answer as a percentage of Robbie's original 1000 SAR.

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Mixed practice 1

1. If $x = 1.23 \times 10^5$ and $y = 1.46 \times 10^{-3}$:

(a) write y to four decimal places

(b) calculate xy, giving your answer in decimal form

'Decimal form' means as an ordinary number.

(c) calculate $\frac{y}{x}$, giving your answer in the form $a \times 10^k$ where $1 \le a < 10$ and $k \in \mathbb{Z}$.

2. Look at this list of numbers:

3.14×10^2, 100π, $\frac{2200}{7}$, $\sqrt{100\,000}$, 310

(a) Which of these numbers is largest?

(b) List all the numbers which are members of: (i) $\mathbb{Z}$ (ii) $\mathbb{Q}$.

(c) What is the largest number of significant figures to which all five numbers are equal?

3. A farmer wants to plant a new forest in a field covering a rectangular area of 3 km by 5 km.

(a) Find the area of the forest in m^2, giving your answer in the form $a \times 10^k$ where $1 \le a < 10$ and $k \in \mathbb{Z}$.

(b) Each tree needs an area of $3.4\,m^2$. Find the maximum number of trees which could be planted. Give your answer to the nearest thousand trees.

4. A travel agent converts dollars and yen at an exchange rate of \$1 = 103 yen. They charge 5% commission on all transactions.

(a) If Dima converts 10 000 yen to dollars, how much will he receive?

(b) If he converts \$500 to yen, how much will he receive?

5. (a) Write 125.987 in the form $a \times 10^k$ where $1 \le a < 10$ and $k \in \mathbb{Z}$.

(b) Write 125.987 to two significant figures.

(c) What is the percentage error when rounding 125.987 to two significant figures?

6. Nicole wants to convert pounds to euros. She can choose between two different offers.

- Offer 1: an exchange rate of 1 pound to 1.26 euros with no commission.
- Offer 2: an exchange rate of 1 pound to 1.30 euros with 5% commission.

Which offer provides Nicole with the better deal?

7. Mark each cell to indicate which number set(s) the number belongs to. The first row has been completed for you.

Number	$\mathbb{N}$	$\mathbb{Z}$	$\mathbb{Q}$	$\mathbb{R}$
-5	✗	✓	✓	✓
0				
$\tan 45°$				
$\tan 60°$				
9.9×10^{24}				
1×10^{-2}				
π				

8. The Earth can be modelled by a perfect sphere with a radius of 6700 km.

(a) Find the volume of the Earth in km^3.

(b) Find the volume of the Earth in cm^3, giving your answer in the form $a \times 10^k$ where $1 \le a < 10$ and $k \in \mathbb{Z}$.

(c) If the average density of the earth is $6.7\ \text{g/cm}^3$, find the mass of the Earth.

The formula for the volume of a sphere is given in the Formula booklet.

Going for the top 1

1. The value of x is quoted as 20 to the nearest 10. The value of y is quoted as 1.6 to two significant figures.

(a) Write an inequality in the form $a \le x < b$ showing the range that x can lie in.

(b) What is the smallest value $3x - 7$ could take?

(c) Find the largest possible percentage error if $\frac{x}{y}$ is quoted as being 12.5.

2. The table below shows the values of different currencies compared to one US dollar:

	GBP	CHF	EUR	JPY	AUD
1 USD	0.66	0.97	0.78	102.48	1.02

(a) How many US dollars can you get with one British pound (GBP)?

(b) What is the exchange rate for euros (EUR) to Japanese yen (JPY)?

(c) Camille converts 1000 US dollars to British pounds, then Swiss francs (CHF), then euros, then Japanese yen, then Australian dollars, and then back to US dollars. For each transaction, she pays 5% commission. How much does she have left at the end, to the nearest US dollar?

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2 SEQUENCES AND SERIES

WHAT YOU NEED TO KNOW

- The notation for sequences and series:
  - u_n represents the nth term of the sequence u.
  - S_n denotes the sum of the first n terms of the sequence.
- An arithmetic sequence has a constant difference, d, between terms. If the first term is u_1, then:
  - $u_n = u_1 + (n-1)d$
  - $S_n = \frac{n}{2}[2u_1 + (n-1)d] = \frac{n}{2}(u_1 + u_n)$
- A geometric sequence has a constant ratio, r, between terms:
  - $u_n = u_1 r^{n-1}$
  - $S_n = \frac{u_1(r^n - 1)}{r-1} = \frac{u_1(1-r^n)}{1-r}$, $r \neq 1$
- Practical problems involving growth or decay, such as questions about interest or depreciation, can be solved using geometric sequences.
  - The formula for compound interest is $FV = PV \times \left(1 + \frac{r}{100k}\right)^{kn}$

    where FV = future value, PV = present value, n = number of years, k = number of compounding periods per year, $r\%$ = nominal annual rate of interest.

EXAM TIPS AND COMMON ERRORS

- Questions often contain parts asking you to find a term of a sequence and a sum of a sequence.
- Many questions on sequences and series involve forming and then solving simultaneous equations.
- You may need to use the list feature on your calculator to solve problems involving sequences.
- You only ever need to use the first sum formula for geometric sequences.
- Sometimes it is useful to remember that $d = u_2 - u_1 = u_3 - u_2 = \ldots$ and $r = \frac{u_2}{u_1} = \frac{u_3}{u_2} = \ldots$
- Always check whether a question is asking for the interest or the total amount.
- Your calculator may have a finance application which can be helpful.

2.1 ARITHMETIC SEQUENCES AND SERIES

WORKED EXAMPLE 2.1

The third term of an arithmetic sequence is 15 and the sixth term is 27.

(a) Find the tenth term.
(b) Find the sum of the first ten terms.
(c) The sum of the first n terms is 5250. Find the value of n.

(a) $u_3 = u_1 + 2d = 15 \quad \cdots(1)$
$u_6 = u_1 + 5d = 27 \quad \cdots(2)$
$(2) - (1) \Rightarrow 3d = 12$, so $d = 4$
$\therefore u_1 = 15 - 2 \times 4 = 7$
Hence $u_{10} = u_1 + 9d = 7 + 9 \times 4 = 43$

We need to find the first term and the common difference. Write down the given information in the form of simultaneous equations and then solve.

(b) $S_{10} = \frac{10}{2}(u_1 + u_{10})$
$= 5(7 + 43)$
$= 250$

There are two formulae for the sum of an arithmetic sequence. Since we know the first and last terms, we use $S_n = \frac{n}{2}(u_1 + u_n)$.

(c) $5250 = \frac{n}{2}(2u_1 + (n-1)d)$
$= \frac{n}{2}(14 + 4(n-1))$

From GDC, the solutions are $n = 50$ or $n = -52.5$; but n must be a positive integer, so $n = 50$.

We need to find n, which will be the only unknown in the other sum formula. Form an equation and solve it using a GDC.

Practice questions 2.1

1. The fifth term of an arithmetic sequence is 8 and the eighth term is 17.
(a) Find the twentieth term.
(b) Find the sum of the first twenty terms.

2. The first four terms of an arithmetic sequence are 8, 7.5, 7, 6.5.
(a) Find the tenth term.
(b) Which term is equal to zero?
(c) The sum of the first n terms is equal to 65. Find the value of n.

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2.2 GEOMETRIC SEQUENCES AND SERIES

WORKED EXAMPLE 2.2

Consider the geometric sequence 2, 6, 18, …

(a) Write down the common ratio.
(b) Which is the first term whose value exceeds 1000?
(c) Find the sum of the first 10 terms.

(a) $r = 3$

'Write down' means that little or no calculation should be necessary. You do not need to show your working.

(b) The sequence continues as …, 54, 162, 486, 1458, …
So the 7th term is the first to exceed 1000.

Use the rule $u_n = u_1 \times r^{n-1}$ with $u_1 = 2$ and $r = 3$, i.e. $u_n = 2 \times 3^{n-1}$, and the list function on your GDC to extend the sequence. This could also be done using a graph or table.

(c) $S_{10} = \dfrac{2(3^{10} - 1)}{3 - 1} = 59\,048$

We can use the formula for the sum of a geometric sequence: $S_n = \dfrac{u_1(r^n - 1)}{r - 1}$.

Practice questions 2.2

3. Consider the geometric sequence 5, 10, 20, 40, ...
   (a) Find the tenth term.
   (b) What is the value of the first term to exceed 3000?
   (c) Find the sum of the first twelve terms.

4. The fifth term of a geometric sequence is 128 and the sixth term is 512.
   (a) Find the common ratio and the first term.
   (b) Which term has a value of 32 768?
   (c) How many terms are needed before the sum of all the terms in the sequence exceeds 100 000?

5. The sum of the first six terms of a geometric sequence is 1365 times its first term. Find the common ratio.