

Cambridge University Press  
978-1-107-63065-9 - Calculating Instruments and Machines  
Douglas R. Hartree  
Frontmatter  
[More information](#)

---

**CALCULATING INSTRUMENTS AND MACHINES**

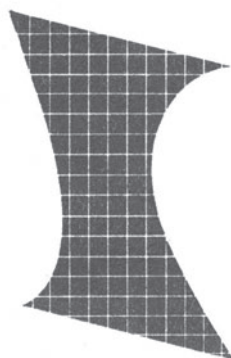
Cambridge University Press  
978-1-107-63065-9 - Calculating Instruments and Machines  
Douglas R. Hartree  
Frontmatter  
[More information](#)

---

Cambridge University Press  
978-1-107-63065-9 - Calculating Instruments and Machines  
Douglas R. Hartree  
Frontmatter  
[More information](#)

# CALCULATING INSTRUMENTS AND MACHINES

DOUGLAS R. HARTREE  
Plummer Professor of Mathematical Physics  
University of Cambridge



CAMBRIDGE  
AT THE UNIVERSITY PRESS  
1950

Cambridge University Press  
978-1-107-63065-9 - Calculating Instruments and Machines  
Douglas R. Hartree  
Frontmatter  
[More information](#)

CAMBRIDGE UNIVERSITY PRESS  
Cambridge, New York, Melbourne, Madrid, Cape Town,  
Singapore, São Paulo, Delhi, Mexico City

Cambridge University Press  
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

[www.cambridge.org](http://www.cambridge.org)  
Information on this title: [www.cambridge.org/9781107630659](http://www.cambridge.org/9781107630659)

© Cambridge University Press 1950

This publication is in copyright. Subject to statutory exception  
and to the provisions of relevant collective licensing agreements,  
no reproduction of any part may take place without the written  
permission of Cambridge University Press.

First published 1950  
First paperback edition 2012

*A catalogue record for this publication is available from the British Library*

ISBN 978-1-107-63065-9 Paperback

Cambridge University Press has no responsibility for the persistence or  
accuracy of URLs for external or third-party internet websites referred to in  
this publication, and does not guarantee that any content on such websites is,  
or will remain, accurate or appropriate.

## PREFACE

IN THE early fall of 1948 I visited the University of Illinois at the invitation of Dean Louis N. Ridenour, Dean of the Graduate School, to give a short series of lectures on calculating instruments and machines. These lectures, with only minor modifications, form the content of this book.

These lectures were to be devoted primarily to recent developments in the subject. But Dean Ridenour invited me to give some attention to its historical side, and I have been glad to do so if only to pay tribute to the remarkable vision and foresight, as it now seems, of two pioneers of thought in this field, Charles Babbage and Lord Kelvin.

Like the lectures, this book is intended as a general introduction to those who have no specialised knowledge in the subject, not as a detailed account for those already expert in it. To anyone who is, or has been, engaged in development work on any of the equipment mentioned, or on similar projects, this account will probably appear sketchy and inadequate; and probably no one group will consider that its own contribution to the subject is adequately represented. All I can hope is that this survey will be a useful introduction to the subject for those to whom it is primarily addressed.

In a series of lectures such as this, it seems appropriate for the lecturer to draw on his own first-hand experience, and to follow his own bias of interest, to a greater degree than would be suitable in a formal text-book, and this I have done, particularly in Chapter 3 and parts of Chapters 7, 8, and 9. If I am thought to have given too much prominence in Chapter 7 to two particular machines, the Harvard “Mark I Calculator” and the Eniac, I must explain that this prominence is deliberate; not because I happen to be better acquainted with these machines than with some others, but because I regard them as being outstanding steps in the development of automatic general-purpose machines, the one as the first practical realisation of such a machine and the other as the first electronic digital machine.

The subjects of calculating instruments (analogue machines) and calculating machines (digital machines) are here treated almost entirely separately, and the reader who is mainly interested in the latter can omit Chapters 2, 3, and 4 without missing anything important to the later argument. I have regarded desk machines and standard punched-card equipment as outside the scope of these lectures; the only digital machines with which I have been concerned are the automatic general-purpose machines.

In this field particularly, the subject is a live one, in which vigorous development is taking place. Since these lectures were given, a simple form of machine using Professor Williams’ form of electrostatic storage (p. 96) has been put into operation at the University of Manchester (ref. 117).

Cambridge University Press  
978-1-107-63065-9 - Calculating Instruments and Machines  
Douglas R. Hartree  
Frontmatter  
[More information](#)

---

Also quite recently the EDSAC at the Mathematical Laboratory of Cambridge University (p. 97) has been completed and proved to work satisfactorily, confirming that for this purpose a storage based on the use of mercury delay lines (p. 95) is a practicable project, and reports of satisfactory tests of a machine using the same form of storage have come from the United States. There is little doubt that the next few years will see further substantial developments in this field.

I wish to express my thanks to the Institute of Electrical Engineers for permission to draw on some of the material of my Kelvin Lecture to the Institution (ref. 90) for parts of Chapters 3 and 4; to the Metropolitan-Vickers Electrical Co. Ltd., to the Director of the Mathematical Laboratory, Cambridge University, and to Professor S. H. Caldwell for the photographs of differential analysers; to Professor H. H. Aiken for the photographs of the Harvard Mark I and Mark II Calculators; and to the U. S. War Department for the photographs of the Eniac.

Last, but not least, I am glad to take this opportunity to thank Dean Ridenour for the opportunity of giving these lectures at the University of Illinois, and to express my warm appreciation of the hospitality and friendly kindness I met there. I also wish to thank Dr. Wilbur Schramm and the staff of the University of Illinois Press for their cooperation in the production of these lectures in book form.

D. R. HARTREE

Cavendish Laboratory,  
University of Cambridge,  
England.

May, 1949

### **PREFACE TO THE ENGLISH EDITION**

I WISH to thank the Cambridge University Press for undertaking the publication of this book in England, and the University of Illinois Press for agreeing to this course. I have taken the opportunity of this re-publication to make some corrections in the text and some additions to the list of references.

D. R. H.

May, 1950

## CONTENTS

1. INTRODUCTION . . . . .	1
2. THE DIFFERENTIAL ANALYSER . . . . .	4
2.1. The Nature of the Problem of Instrumental Solution of Differential Equations . . . . .	4
2.2. Integrating Mechanisms . . . . .	5
2.3. The General Idea of the Differential Analyser . . . . .	6
2.4. General Structure of the Earlier Forms of Differential Analyser	8
2.5. A New Differential Analyser . . . . .	14
2.6. Using the Differential Analyser . . . . .	17
2.7. Regenerative Connections . . . . .	20
2.8. Automatic Generation of Any Function . . . . .	21
2.9. Other Forms of Differential Analyser . . . . .	23
2.10. Boundary Conditions in Numerical and Mechanical Integration of Differential Equations . . . . .	24
2.11. Applications . . . . .	25
3. THE DIFFERENTIAL ANALYSER AND PARTIAL DIFFERENTIAL EQUATIONS . . . . .	26
3.1. Introduction . . . . .	26
3.2. Replacement of the $t$ -Derivative by a Finite Difference . . . . .	27
3.3. Replacement of the $x$ -Derivative by Finite Differences . . . . .	28
3.4. Discussion of the Two Methods . . . . .	30
3.5. Examples of Application to Equations of “Parabolic” Type . . . . .	34
3.6. Hyperbolic Equations . . . . .	37
3.7. Use of “Characteristics” . . . . .	38
3.8. Another Application of Characteristics . . . . .	41
4. SOME OTHER INSTRUMENTS . . . . .	44
4.1. Introduction . . . . .	44
4.2. Solution of Simultaneous Linear Algebraic Equations . . . . .	44
4.3. The Isograph . . . . .	46
4.4. Fourier Synthesisers . . . . .	47
4.5. Integrating Instruments . . . . .	48
4.6. Directors . . . . .	52

5.	INTRODUCTION TO LARGE AUTOMATIC DIGITAL MACHINES . . . . .	54
5.1.	Historical . . . . .	55
5.2.	Structure and Function in Calculating Machines . . . . .	56
5.3.	Functions to Be Provided in an Automatic Digital Machine . . . . .	56
5.4.	Representation of Numbers in the Machine . . . . .	59
5.5.	Arithmetical Operations . . . . .	61
5.6.	Serial (Successive) and Parallel (Simultaneous) Operations . . . . .	65
5.7.	Static and Dynamic Storage . . . . .	66
5.8.	Control and the Form of the Operating Instructions . . . . .	67
6.	CHARLES BABBAGE AND THE ANALYTICAL ENGINE . . . . .	69
6.1.	Babbage's Calculating Engines . . . . .	69
6.2.	Babbage's Analytical Engine . . . . .	70
6.3.	Control in the Analytical Engine . . . . .	71
6.4.	Organization of Calculations for the Analytical Engine . . . . .	72
7.	THE FIRST STAGE OF DEVELOPMENT . . . . .	74
7.1.	The Harvard Mark I Calculator . . . . .	74
7.2.	Control in the Harvard Mark I Machine . . . . .	79
7.3.	Relay Machines . . . . .	80
7.4.	The Eniac . . . . .	81
7.5.	Control in the Eniac . . . . .	83
7.6.	Centralised Control System of the Eniac . . . . .	87
7.7.	The I.B.M. Selective Sequence Electronic Calculator . . . . .	87
7.8.	An Application of the Eniac . . . . .	88
7.9.	The "Machine's-Eye View" in Programming a Calculation . . . . .	92
8.	PROJECTS AND PROSPECTS . . . . .	94
8.1.	The Main Directions of Development . . . . .	94
8.2.	Storage Systems . . . . .	94
8.3.	Serial Machines Using Delay-Line Storage . . . . .	97
8.4.	Functional Analysis of Serial Machines . . . . .	97
8.5.	Arithmetical Operation . . . . .	103
8.6.	Control . . . . .	106
8.7.	Parallel Machines . . . . .	109
8.8.	Other Types of Machine . . . . .	111
8.9.	Programming and Coding . . . . .	111



9. HIGH-SPEED AUTOMATIC DIGITAL MACHINES AND NUMERICAL ANALYSIS . . . . .	115
9.1. Introduction . . . . .	115
9.2. Iterative Methods . . . . .	116
9.3. Simultaneous Algebraic Equations . . . . .	119
9.4. Solution of Ordinary Differential Equations . . . . .	122
9.5. Solution of Ordinary Differential Equations with One-Point Boundary Conditions . . . . .	123
9.6. Solution of Ordinary Differential Equations with Two-Point Boundary Conditions . . . . .	125
9.7. Partial Differential Equations . . . . .	127
REFERENCES . . . . .	131
NAME INDEX . . . . .	135
SUBJECT INDEX . . . . .	136

Cambridge University Press  
978-1-107-63065-9 - Calculating Instruments and Machines  
Douglas R. Hartree  
Frontmatter  
[More information](#)

---