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978-1-107-63012-3 - Theory of Differential Equations: Part II: Ordinary Equations,
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Andrew Russell Forsyth

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