INTRODUCTORY SUMMARY

MY object in the following brief essay is to propound a certain analysis of Categorical Propositions of the forms *S is P*, *S is not P*, to show that this is the only general analysis which it is possible to accept, and to indicate its bearing upon logical science. According to the analysis in question, *S is P* asserts Identity of Denotation in Diversity of Intension, and *S is not P* denies this. The example given by Professor Frege (whose analysis of *S is P* I understand to agree roughly with mine) is

" The morning star is the evening star" $\left(\begin{smallmatrix} \text{M. S.} \\ \text{E. S.} \end{smallmatrix}\right)$: the

terms " morning star " and " evening star " apply to *one thing*, but the meaning, intension, or qualitative implication of " morning star " is not the same as that of " evening star." " The largest city in the world is the Metropolis of England " is another illustration, where again it is clear that the two names or terms, the Subject and Predicate of the Assertion, apply to *one place* but have different meanings or definitions. *S is not P* asserts Difference of Denotation (Otherness) in Difference of Intension (Diversity)—e.g. " Cambridge is not Oxford,"

$\begin{smallmatrix} \text{C} \end{smallmatrix}$ $\begin{smallmatrix} \text{O} \end{smallmatrix}$. *A is related to B* implies *A is not B*, $\begin{smallmatrix} \text{A} \end{smallmatrix}$ $\begin{smallmatrix} \text{B} \end{smallmatrix}$.

J. 1

We need propositions of the form *S is P*, *S is not P*, for significant assertion, and without them no satisfactory statement can be given of the "three fundamental Laws of Thought," which are put forward as the basis of logical science. The first two of these Laws are commonly formulated as: (1) *A is A*, (2) *A is not non-A*, and the third sometimes as **A** *is either A or non-A* (3). Desperate efforts have been made by logicians to give a valuable meaning to *A is A*; but if *A is A*, interpreted as *A is A*, is retained as the first fundamental Law, there is no possible passage from it to *A is B*, and *A is A* or *A is B* (*S is P*) must be given up. This is fully recognised by Lotze, who gives up (theoretically) *S is P*. *A is A* tells us no more than *A is A*, and if we begin with it, we must also end with it, if we are to be consistent. I maintain that we must not begin with it, but must begin instead with a Law of significant assertion—assertion of the forms *S is P*, *S is not P*, forms which provide the only straightforward and effective statement of the second and third Laws of Thought, thus:

S is P {cannot both be true (L. of Contradiction)
S is not P {cannot both be false (L. of Excluded Middle).

It follows from these two Laws that of any Subject of Predication (*S*) either *P* or *not-P* can be affirmed. Thus from them, and *S is P*, *S is not P*, analysed as above, we obtain the principle that:

Every Subject of Predication is an Identity-in-Diversity.

It follows further that every Predicate (*P*) is necessarily incompatible with *not-P* (absence of intension *P*) and necessarily compatible with *not-not-P*. (This suggests a formal principle of necessary connection of attributes.)

I contend that if we start, not with *A is A*, but with the principle that *Every Subject of Predication is an identity* (*of denotation*) *in diversity* (*of intension*), this Law (1), and the Laws of (2) Contradiction and (3) Excluded Middle (of which (1) for the first time makes logically possible the formulation given above) do furnish a real and adequate and obvious basis and starting-point of "Formal" Logic. Granted propositions of the form *S is P*, with the identity-in-diversity analysis and the corresponding analysis of *S is not P*, together with the traditional Laws of Contradiction and Excluded Middle, the whole scheme of Immediate and Mediate Inference can be built up systematically and explicitly, as I hope to show. The possibility of Conversion, e.g. implies that the Predicate, as well as the Subject, of any Proposition has Denotation, and a Denotation that is implicitly quantified; the one indispensable condition of Mediate Inference is identity of Denotation of the Middle Term in both premisses. Without propositions of the forms *S is P*, *S is not P*, thought cannot live or move; but the disastrous acceptance of *A is A*, with its baffling ambiguities, has stood in the way of their being rightly analysed by logicians and explicitly recognised by them as fundamental forms of significant assertion, without which not even the Laws of Contradiction and Excluded Middle can receive satisfactory expression[1].

---

[1] In the following pages I have occasionally borrowed from writings of my own in cases where I have not felt able to improve upon the statement already printed.

A NEW "LAW OF THOUGHT" AND ITS LOGICAL BEARINGS

> "I am the pillars of the house,
>   The keystone of the arch am I;
>   Take me away, and roof and wall
>   Would fall to ruin utterly."
>
> K. TYNAN.

STATEMENT OF THE CASE.

IT will be admitted that up to the present time no adequate and unquestionable basis of the Science of Logic has been found—that the Method of Logic, itself the Science of Method, is not wholly satisfactory. Logic is often defined as the Science of the Laws of Thought— the Laws, that is, of Identity, Contradiction, and Excluded Middle; but on the one hand the statement of these Laws is not uniform, and the interpretation of at least the first of them, the Law of Identity (A is A, whatever is is, Everything is what it is), is matter of perpetual dispute; on the other hand no one of these Laws alone, nor all of them together, can or do take account of, or can explain and justify, the common indispensable form of Categorical Assertion S is P—e.g. Trees are green, All Men are mortal, George V is the present King of England, Perseverance is admirable, Honesty is the best policy, The quality of Mercy is twice bless'd. On the contrary, A is A appears to exclude it, and there is no passage from A is A

to *A is B.* And if anyone who accepts *A is A*, and the corresponding expression of the Law of Contradiction, *A is not not-A*, is driven into giving *A is B or not-B* as the Law of Excluded Middle, it is for him to show what logical connexion there is between the last "Law" and the two previous ones. Logic undoubtedly, like all other Sciences, like literature, like common thought and common speech, uses the forms *S is P, S is not P*—uses them at every step. It must use them, of course; it has no choice; without them, it would be impossible to affirm or deny; but it adopts them in the same fashion as Bentham adopted the *Greatest Happiness of the Greatest number* as his ultimate ethical principle—that is to say, without any reasoned justification. No "plain man" certainly, would be expected to give any reason why he should use propositions of the form *A is B* rather than of the form *A is A*; but a logician who declares that *A is A* is the first Law of Thought, and (if he is consistent), that *A is not not-A* and *A is either A or not A*[1] are the other two, may fairly be called upon to explain the fact that he habitually says that Roses are red and Violets are blue, rather than Roses are roses, Red is red, Violets are violets, and so on. For logicians to find fault with a so-called "Law" which is a pure tautology, which is expressed in a form which may indeed have important uses, and may be employed epigrammatically or rhetorically, but in which no ordinary sensible person would think of trying to convey straightforward information, or matter of fact— much less a fundamental principle—is no new thing.

[1] This, however, is generally stated *A is either B or not B* and sometimes the *A is B* form is slipped into even in stating the Law of Contradiction, by upholders of the *A is A* Law of Identity.

To lay it down (1) that we can never legitimately affirm of any subject a predicate different from itself, while at the same time (2) it has to be allowed that this rule cannot be even stated without being broken, without using assertions of the form S is P, was we know, a state of mind possible in the time of Plato; it was possible because those who asserted (1) thought it self-evident that the Predicate ought always to be the same as its Subject, " that to apply many Predicates to one and the same Subject is to make one thing into many things." And as for (2), they could not deny it; while to give up (1) seemed to be a denial of self-evident truth, to give up (2) was sheerly impossible. The situation is rather intolerable.

That there is a difficulty about S is P we need not question, that logicians who accept A is A are imperatively called upon to show how this " Law " can be adapted to propositions of form A is B (S is P) is too obvious to need pointing out. Some writers have tried to give a meaning to A is A which does not seem to prohibit diversity of Predicate from Subject—a meaning which is itself expressed in the A is B, not in the tautological A is A, form ; Mr Bradley e.g. interprets the Law of Identity to mean that " if what I say is really true, it stands for ever." A is A thus expounded into A is B does not of course exclude propositions of A is B form. Dr Bosanquet frankly admits that, while he would not accept either A is B or A is A as a schematic expression of the Law of Identity, he would prefer A is B to A is A[1].

[1] " If I were asked " he says, " how I should represent a true Identity, such as a judgment must express, in a schematic form with

The only logician, as far as I know, who, while re-
taining *A is A* in its purity has made a determined effort
to reconcile it with propositions of the *A is B* (*S is P*)
form, is Lotze.   He holds (*Logic*, Bk I. ch. II.) that "our
thought is subject to a limitation, has to conform to a
law...in the categorical judgment each constituent can
only be conceived as self-same [= ?].   This primary law of
thought, the *principle of identity*, we express positively
in the formula $A = A$."   He states the conclusion to

symbolic letters, I should say the problem was insoluble.   Every A is B
would be much better than Every A is A ; but as the letters are not parts
in any whole of meaning, they are 'things cut asunder with an axe'."
(Dr Bosanquet in *Mind*, 1888, p. 357.)   (The objection that in *A is B*
"the letters are not parts in any whole of meaning" seems either

(1) inaccurate, for there is a symbolic whole, viz. (A, B,) which has

a meaning and an important one,—or (2) irrelevant, if what is meant is
a concrete special "whole of meaning.")

It is clear from other passages in the same article that for
Dr Bosanquet, individual identity is not distinguished from *qualitative
one-ness of two things*—e.g., he speaks of some "present impression"
as being "identical with a former impression" (p. 360), and says that
"the element of *identity* between two outlines can be accurately pointed
out and limited, but the moment they cease to be two, it ceases to be an
identity" (p. 359).   He objects to drawing "a sharp line between the
unity of the individual human being...and the unity of human beings
in identical sentiments, ideas, purposes or habits" (p. 362), and says
that a number of persons may have "a really *identical* purpose and
endeavour and *consciousness* of certain facts" (p. 364).   Again (p. 365),
he says "Any indiscernible resemblance [= ?] between two different
contents, in specified respects, will do whatever identity will do, because
it is identity under another name" (if so, what need is there of a
Distributed Middle in Syllogism?); and on p. 366 speaks of "indis-
cernible likeness [= ?] or identity."   With this meaning of identity it
certainly is not clear how "a true identity" could be satisfactorily
expressed as *A is B*.   Connotationally, qualitatively, *A is not B*.

which he is driven, thus: "This absolute connexion of
two concepts S and P, in which the one is unconditionally
the other and yet both stand over against each other as
different, is a relation quite impracticable in thought:
by means of *this* copula, the simple 'is' of the cate-
gorical judgment, two different contents cannot be con-
nected at all; they must either fall entirely within one
another, or they must remain entirely separate, and the
impossible judgment 'S *is* P' resolves itself into the
three others, 'S *is* S,' 'P *is* P,' 'S *is not* P'." (Engl.
transl. p. 59.)

Whether A *is* A is understood as A-*ness is* A-*ness*, or
in any other possible way in which A *is* A is honestly
interpreted as A *is* A (not as A *is* B), the acceptance of
it as a first and fundamental Law is absolutely suicidal
for Logic from a theoretical point of view. But it must
be confessed that its nominal acceptance does not appear
to have seriously affected the construction of the Science.
A *is* A cannot justify or support this, it even seems in-
consistent with it, but the restrictions logically imposed
by A *is* A have (almost universally) been not only not
respected, they have not even been borne in mind, and
A *is* A itself has received a variety of interpretations
(generally of the form A *is* B) which it was natural to
ignore as they mostly did not interfere with either theory
or practice, and it was thus easy for logicians to go on
systematising and constructing in complete independence
of the "First Law of Thought."

No doubt the speculative incompatibility between it
and ordinary assertion has been for the most part a
"contradiction that was not seen." When it has been
seen, common sense has had no hesitation in driving a coach-

and-six through the venerable but insubstantial obstacle.
Lotze, keenly aware of the contradiction and loyal to
tradition, but oblivious for the moment of the needs and
actualities of living thought, imagined that he must,
and could, give up S *is* P. The actual starting-point
of Logic has been not A *is* A, but the Law of Contra-
diction and the Law of Excluded Middle, and the effort
to analyse S *is* P (*not-P*); and in Conversion, Mediate
Inference etc., it is propositions of those forms that have
been dealt with. But those forms were accepted un-
critically, and together with A *is* A. Logic has lacked a
First Law which could furnish a legitimate and logical
starting-point and be capable of development and general
application, have a real and important difference from,
and connexion with, the Law of Contradiction and the
Law of Excluded Middle, be effective throughout the
Science of Logic, and justify, explain and support logical
procedure. Though A *is* A may be sometimes a con-
venient mode of expression, we cannot start from it as
the fundamental propositional form and we do not see
how to get from it to A *is* B. A *is* B is the inevitable
point of departure, and this has, as the limit on one side
(the side of tautology) A *is* A (which excludes diversity
of intension), and on the other (the side of Contra-
diction), A *is* *not-A* (which excludes identity of denota-
tion). A *is* A, $\left(\text{A, A}\right)$, is of course quite different from
$A = A$, $\left(\text{A}\right) = \left(\text{A.}\right)$.

I think that every name or term has two aspects:
   (1) the denotational, extensional, or applicational;
   (2) the intensional or connotational;

corresponding to the two aspects of the things of which they are names—i.e. the aspects of (1) Thatness and (2) Whatness, to use Mr Bradley's terms. Everything of which we can think or speak is (1) Something and (2) some definite sort of something. Everything must be thought as having (1) existence (in the widest sense— mere thing-hood) and (2) some fixed definite nature and constitution. For the sake of clearness, I propose in what follows to confine the term *identity* to denotational one-ness, as distinct from one-ness in the intensional sense, which makes possible general names, classing, and classi-fication. Without both (1) and (2) no assertion is possible, nothing can be Subject or Predicate of a proposition, The Law of Identity may have been an attempt to express the qualitative fixity of nature of anything in brief and self-evident form; if so, the expression *A is A* is unfortunately incapable of expressing what was in-tended. If it does express a meaning, that meaning is clearly not self-evident, for there is nothing about which there has been more dispute than the meaning of *A is A*. It seems to me that until we have *A is B (S is P)* there is nothing to accept or reject, nothing to doubt or dispute, and that the true significance of contradiction is to deny of something some predicate which has already been affirmed of it. It might seem that for conceptualists the problem of *A is A* was simplified, as their whole interest was in Quality, Intension, as distinct from Extension or Thatness; but it is demonstrable that no significant affirmation can be purely qualitative.

If we genuinely accept *A is A* as the expression of a fundamental and primary logical principle, the difficulty is, how theoretically to get beyond it. If we reject it,