

COLLOIDAL SUSPENSION RHEOLOGY

Colloidal suspensions are encountered in a multitude of natural, biological, and industrially relevant products and processes. Understanding what affects the flow behavior, or rheology, of colloid suspensions, and how this flow behavior can be manipulated, is important for successful formulation of products such as paint, polymers, foods, and pharmaceuticals. This book is the first devoted to the study of colloidal rheology in all its aspects. With material presented in an introductory manner, and complex mathematical derivations kept to a minimum, the reader will gain a strong grasp of the basic principles of colloid science and rheology. Beginning with purely hydrodynamic effects, the contributions of Brownian motion and interparticle forces are covered, before the reader is guided through specific problem areas such as thixotropy and shear thickening; special classes of colloid suspensions are also treated. The techniques necessary for measuring colloidal suspension rheology are presented along with methods to correlate and interpret the results. An essential guide for academic and industrial researchers, this book is also ideal for graduate course use.

JAN MEWIS is Emeritus Professor of the Chemical Engineering Department at the Katholieke Universiteit Leuven. He is involved in industrial and academic research in complex fluids such as suspensions and polymer blends. Professor Mewis has lectured all over the world and has written over 200 publications on colloid science and rheology. He was Chairman of the International Committee on Rheology and is a recipient of the Gold Medal of the British Society of Rheology and the Bingham Medal of The Society of Rheology (USA).

NORMAN J. WAGNER received his Doctorate from Princeton University and is a named Professor and Chair of the Department of Chemical Engineering at the University of Delaware. He has extensive international teaching and research experience, and leads an active research group covering fields such as rheology, complex fluids, polymers, nanotechnology, and particle technology. Professor Wagner has received several awards for his research developments, has co-authored over 150 scientific publications and patents, and is on the editorial boards of five international journals. He currently serves on the executive boards of the Society of Rheology and the Neutron Scattering Society of America.

CAMBRIDGE SERIES IN CHEMICAL ENGINEERING

Series Editor

Arvind Varma, *Purdue University*

Editorial Board

Christopher Bowman, *University of Colorado*

Edward Cussler, *University of Minnesota*

Chaitan Khosla, *Stanford University*

Athanassios Z. Panagiotopoulos, *Princeton University*

Gregory Stephanopolous, *Massachusetts Institute of Technology*

Jackie Ying, *Institute of Bioengineering and Nanotechnology, Singapore*

Books in Series

Chau, *Process Control: A First Course with MATLAB*

Cussler, *Diffusion: Mass Transfer in Fluid Systems, Third Edition*

Cussler and Moggridge, *Chemical Product Design, Second Edition*

Denn, *Chemical Engineering: An Introduction*

Denn, *Polymer Melt Processing: Foundations in Fluid Mechanics and Heat Exchangers*

Duncan and Reimer, *Chemical Engineering Design and Analysis: An Introduction*

Fan and Zhu, *Principles of Gas-Solid Flows*

Fox, *Computational Models for Turbulent Reacting Flows*

Leal, *Advanced Transport Phenomena: Fluid Mechanics and Convective Transport*

Mewis and Wagner, *Colloidal Suspension Rheology*

Morbideilli, Gavrilidis, and Varma, *Catalyst Design: Optimal Distribution of Catalyst in Pellets, Reactors, and Membranes*

Noble and Terry, *Principles of Chemical Separations with Environmental Applications*

Orbey and Sandler, *Modeling Vapor–Liquid Equilibria: Cubic Equations of State and their Mixing Rules*

Petyluk, *Distillation Theory and its Applications to Optimal Design of Separation Units*

Rao and Nott, *An Introduction to Granular Flow*

Russell, Robinson, and Wagner, *Mass and Heat Transfer: Analysis of Mass Contactors and Heat Exchangers*

Slattery, *Advanced Transport Phenomena*

Varma, Morbidelli, and Wu, *Parametric Sensitivity in Chemical Systems*

Colloidal Suspension Rheology

JAN MEWIS

Katholieke Universiteit Leuven

NORMAN J. WAGNER

University of Delaware



CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom
 One Liberty Plaza, 20th Floor, New York, NY 10006, USA
 477 Williamstown Road, Port Melbourne, VIC 3207, Australia
 314-321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi - 110025, India
 79 Anson Road, #06-04/06, Singapore 079906

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning and research at the highest international levels of excellence.

www.cambridge.org

Information on this title: www.cambridge.org/9781107622807

© J. Mewis and N. Wagner 2012

This publication is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press.

First published 2012

A catalogue record for this publication is available from the British Library

Library of Congress Cataloging in Publication data

Mewis, J.

Colloidal suspension rheology / Jan Mewis, Norman J. Wagner.

p. cm. – (Cambridge series in chemical engineering)

Includes bibliographical references and index.

ISBN 978-0-521-51599-3

1. Rheology. 2. Suspensions (Chemistry) 3. Colloids. I. Wagner, Norman Joseph, 1962– II. Title.

TP156.R45M49 2012

531'.1134 – dc23 2011029383

ISBN 978-0-521-51599-3 Hardback

ISBN 978-1-107-62280-7 Paperback

Cambridge University Press has no responsibility for the persistence or accuracy of URLs for external or third-party internet websites referred to in this publication, and does not guarantee that any content on such websites is, or will remain, accurate or appropriate.

Cambridge University Press
978-1-107-62280-7 — Colloidal Suspension Rheology
Jan Mewis , Norman J. Wagner
Frontmatter
[More Information](#)

To Ria and Sabine

This landmark book thoroughly details the basic principles of colloid science and uniquely covers all aspects of the rheology of colloidal suspensions, including difficult and often controversial topics such as yield stress, thixotropy, shape effects and shear thickening, as well as latest developments in microrheology and interfacial rheology. The elegant presentation style, focusing on the fundamental concepts, bridging engineering and physics, experiment and theory, and paying attention to the interplay between microstructure and rheology, reflects the vast teaching and research experience of the authors, and makes the book a much needed reference for practitioners, researchers and graduate students.

Dimitris Vlassopoulos
IESL-FORTH, Greece

Appropriately, the first book to span the subject of suspension rheology is authored by Jan Mewis, a pioneer in the field, and Norm Wagner, whose research has advanced many of the modern frontiers. Their text emerges from a long-standing collaboration in short courses that have introduced graduate students, young faculty, and industrial researchers to the fundamentals and the practicalities of rheological phenomena and their underlying principles. After a brief introduction to colloid science and rheology the book teaches the consequences of the relevant forces, i.e., hydrodynamic, Brownian, electrostatic, polymeric, and van der Waals, through data from model systems and results from fundamental theory. Then time-dependent phenomena, shear thickening, and the effects of viscoelastic media, in which the two have paved the way, receive special attention. The treatment closes with brief accounts of microrheology, electro- and magnetorheology, and two-dimensional suspensions. There is much to learn from this tome!

William B Russel
Princeton University

Ever since I learned that Mewis and Wagner were preparing *Colloidal Suspension Rheology* I have been eagerly awaiting its arrival. I was not disappointed! The book is very logically laid out. The reader is told what is coming and key ideas are summarized at the end of every section. I especially like the “landmark observations” that focus each chapter. Every chapter has a table of notation and is extensively referenced including titles of articles. The concise review of colloidal phenomena in chapter 1 is outstanding and the Advanced Topics in the final chapter (microrheology, electro and magneto-rheology and 2 dimensional rheology) are a special treat.

Colloid Suspension Rheology is the first text in this field and will be much appreciated. Suspensions are growing rapidly in academic importance and are the key to so many new industrial products. Rheology is a rapid and sensitive tool to characterize both their microstructure and performance. This text will be of great excellent supplement to courses in colloids and rheology.

Chris Macosko
University of Minnesota & IPRIME

Contents

Preface	<i>page</i> xiii
General list of symbols	xvi
Useful physical constants and values	xx
1 Introduction to colloid science and rheology	1
1.1 Colloidal phenomena	1
1.1.1 Forces acting on individual colloids	2
1.1.2 Colloidal interactions	6
1.1.3 Phase behavior and colloidal stability	15
1.2 Principles of rheology	18
1.2.1 Basic concepts	18
1.2.2 Generalized Newtonian fluids	24
1.2.3 Viscoelasticity	25
1.2.4 Application to colloidal dispersions	30
A look ahead	32
Appendix: Second virial coefficients	32
Chapter notation	33
References	34
2 Hydrodynamic effects: Non-colloidal particles	36
2.1 Introduction	36
2.2 Landmark observations	37
2.2.1 Summary	40
2.3 Dilute systems	41
2.3.1 Flow around and motion of single particles	41
2.3.2 Viscosity of dilute suspensions	43
2.3.3 Summary	46
2.4 Semi-dilute suspensions	46
2.4.1 Summary	51
2.5 Concentrated suspensions	51
2.5.1 Microstructure	52
2.5.2 Viscosity	54
2.5.3 Other stress components	60
	vii

viii **Contents**

2.5.4	Summary	62
2.6	Other flow phenomena	63
2.6.1	Diffusion or migration	63
2.6.2	Inertial effects	67
2.6.3	Sedimentation	68
2.6.4	Summary	69
	Appendix A: Derivation of Einstein expression for intrinsic viscosity	70
	Appendix B: Derivation of phenomenological equations for suspension viscosity	71
	Chapter notation	72
	References	73
3	Brownian hard spheres	80
3.1	Introduction	80
3.2	Landmark observations	81
3.3	Structure and thermodynamic properties of the hard sphere fluid	85
3.3.1	Pressure in a hard sphere fluid	86
3.3.2	Brownian forces in concentrated dispersions	87
3.4	Rheology of dilute and semi-dilute dispersions	90
3.5	Concentrated dispersions	95
3.5.1	Zero shear viscosity	95
3.5.2	Linear viscoelasticity	101
3.5.3	Steady shear rheology	105
3.5.4	Hard sphere colloidal glass transition and mode-coupling theory	109
	Summary	112
	Appendix A: Principles of the Smoluchowski equation for dispersion micromechanics	113
	Appendix B: The role of hydrodynamic interactions	114
	Appendix C: Osmotic pressure for a hard sphere solid	115
	Chapter notation	116
	References	116
4	Stable systems	122
4.1	Introduction	122
4.2	Landmark observations	123
4.3	Electrostatically stabilized systems	127
4.3.1	Dilute and semi-dilute suspensions	127
4.3.2	Concentrated suspensions	131
4.4	Sterically stabilized systems	137
4.4.1	Mechanism	137
4.4.2	Dilute systems	138
4.4.3	Non-dilute systems	139
4.5	Electrosterically stabilized systems	147

Contents	ix
Summary	149
Chapter notation	149
References	150
5 Non-spherical particles	155
5.1 Introduction	155
5.2 Landmark observations	155
5.3 Particle motion	160
5.4 Rheology of dilute suspensions of non-spherical particles	163
5.5 Semi-dilute suspensions of non-spherical particles	167
5.6 Concentrated suspensions of non-spherical particles	168
5.7 Charged non-spherical particles	172
Summary	173
Appendix: Structural description of the stresses in fiber suspensions	173
Chapter notation	175
References	176
6 Colloidal attractions and flocculated dispersions	180
6.1 Introduction	180
6.1.1 Methods to induce interparticle attraction and flocculation	181
6.2 Landmark observations	183
6.3 Phase behavior, microstructure, and state diagrams	187
6.3.1 Equilibrium phase behavior	187
6.3.2 Flocs and fractals	188
6.3.3 Effect of flow on floc structure	190
6.3.4 Stable clusters	193
6.3.5 Percolation, gelation, jamming, and vitrification	193
6.4 Rheology at low volume fractions	201
6.5 Concentrated dispersions	203
6.6 Rheology of gelled systems	206
6.6.1 Moduli and yield stress of gels	207
6.7 Kinetics of aggregating systems	214
6.8 Polymer bridge flocculation	214
Summary	215
Appendix: Influence of weak attractions on near hard sphere dispersion rheology	216
Chapter notation	217
References	218
7 Thixotropy	228
7.1 Introduction	228
7.2 The concept of thixotropy	228
7.2.1 Definition	228
7.2.2 Thixotropy versus viscoelasticity	230
7.3 Landmark observations	231

x Contents

7.4 Rheological phenomena	232
7.4.1 Start-up and intermittent flows	232
7.4.2 Hysteresis loops	234
7.4.3 Stepwise changes in shear	235
7.4.4 Creep tests	236
7.4.5 Oscillatory flow	237
7.5 Constitutive equations	239
7.5.1 Structure kinetics models	239
7.5.2 Integral models	244
Summary	245
Appendix: Parameter estimation and model assessment	245
Chapter notation	248
References	248
8 Shear thickening	252
8.1 Introduction	252
8.2 Landmark observations	253
8.3 Shear thickening colloidal dispersions	261
8.3.1 Dilute dispersions	261
8.3.2 Concentrated dispersions	265
8.3.3 Non-spherical particle dispersions	274
8.3.4 Extensional thickening, confinement, and field effects	275
8.3.5 Elasto-hydrodynamic limit of shear thickening	277
8.3.6 Models for predicting the onset of shear thickening	278
8.4 Dilatancy and shear thickening in suspensions	281
Summary	283
Chapter notation	284
References	284
9 Rheometry of suspensions	291
9.1 Introduction	291
9.2 Basic measurement geometries	291
9.2.1 Cone and plate	292
9.2.2 Parallel disks	292
9.2.3 Coaxial cylinders	293
9.2.4 Capillary flow	294
9.3 Measurement problems and basic procedures	295
9.3.1 Measurement problems	295
9.3.2 Selection of measurement geometry	297
9.3.3 General measurement procedures	299
9.4 Specific measurement procedures	300
9.4.1 Wall slip	300
9.4.2 Yield stress (shear)	305
9.4.3 Compressive yield stress	313
9.4.4 Thixotropy	315
9.4.5 Large amplitude oscillatory shear flow (LAOS)	316

Contents	xi
Appendix: Characterization of wall slip	318
Chapter notation	319
References	320
10 Suspensions in viscoelastic media	325
10.1 Introduction	325
10.2 Landmark observations	326
10.3 Particle motion	329
10.4 Rheological behaviour of dilute suspensions	335
10.5 Rheological behavior of concentrated suspensions	336
10.5.1 Steady state shear flow	337
10.5.2 Dynamic moduli	340
10.5.3 Relaxation function	341
10.5.4 Uniaxial extensional flow	341
10.5.5 Fiber suspensions	341
10.5.6 Filled polymer melts and nanocomposites	343
Summary	346
Chapter notation	347
References	347
11 Advanced topics	354
11.1 Special methods for bulk rheometry	354
11.1.1 Stress jumps	354
11.1.2 Superposition moduli	355
11.2 Microrheology	357
11.2.1 Passive microrheology	359
11.2.2 Active microrheology	360
11.2.3 Nonlinear microrheology	361
11.2.4 Concluding remarks	364
11.3 Field-response systems: Electrorheological and magnetorheological suspensions	365
11.3.1 Electrorheological fluids	365
11.3.2 Magnetorheological fluids	371
11.4 Two-dimensional colloidal suspensions	373
11.4.1 Interactions and structure in 2D suspensions	374
11.4.2 Interfacial rheometry	376
11.4.3 Rheological properties of 2D suspensions	378
11.4.4 Flow visualization using 2D suspensions	380
Chapter notation	381
References	382
Index	388

Preface

Colloidal dispersions played an important role in the early history of rheology as it evolved into a defined branch of science and engineering. Bingham's model for yield stress fluids was based on experiments on dispersions, namely oil paints. About the same time, systematic measurements on colloidal systems were performed in Europe, especially in Freundlich's laboratory in Berlin. This work culminated in one of the first books on rheology: *Thixotropie* (Paris, 1935). In the subsequent decades the interest in rheology gradually shifted to polymers and the theory of viscoelasticity.

Understanding Brownian motion and its consequences motivated Einstein's work on intrinsic viscosity and von Smoluchowski's study of colloidal aggregation nearly a century ago. However, it was not until the theoretical work of G. K. Batchelor in the early 1970s that a full micromechanical framework for colloidal suspension rheology combining statistical mechanics and hydrodynamics existed. This stimulated much important work and since then the number of researchers and the progress of our understanding of the subject has increased dramatically. The result is a rapidly growing body of scientific and technical papers – experimental and theoretical work as well as simulations – contributed by chemists, physicists, biologists, and engineers alike.

Whereas there is a vast and expanding literature, there are no sources that provide a systematic introduction to the field. The growing number of newcomers to the field have available to them numerous textbooks on rheology and many more on colloid science, as well as specialized overviews in the research literature, yet no book with the sole focus on colloidal suspension rheology. Interest in the rheology of colloidal dispersions is not restricted to academia. Increasingly, the available knowledge is being applied effectively to solve formulation and processing problems, e.g., for coatings, inks, filled polymers and nanocomposites, metal and ceramic slurries, cement and concrete, mine tailings, drilling muds, pharmaceuticals, and consumer products. The lack of a basic text dedicated to this subject stimulated the writing of the present book, which is intended as a general introduction to colloidal suspension rheology for a beginner in the field. Its purpose is to provide a systematic overview of the established, central elements of the field. Practical examples are presented and discussed within a framework for understanding the underlying structure-property relationships. Emphasis is on understanding the various phenomena that contribute to the rheological properties of colloidal suspensions, such as available relations and

xiv Preface

scaling laws, as well as on the underlying micromechanical explanations. It is our intention that the micromechanical understanding of the model systems presented herein can assist in the formulation and investigation of systems of specific or practical interest to the reader. To that end, basic theoretical results are presented, but without mathematical derivation. Extensive references guide the reader to more detailed and advanced information.

The book starts with a brief introduction to basic concepts of colloid science and rheology as the bare necessities for those without prior knowledge of these disciplines and as a review and establishment of nomenclature useful for those already familiar with the subjects. The systematic study of colloid rheology begins with hydrodynamic effects. These are always present in dispersions and are the dominant contribution to the rheology of suspensions with large, non-colloidal particles (Chapter 2). The rheology of colloidal suspensions with increasing levels of complexity is treated systematically in the following chapters. Chapter 3 explores hard sphere dispersions, where Brownian motion is included and its effects analyzed. Next, repulsive interparticle forces are added to give colloidally stable systems (Chapter 4). Special features arising because of non-spherical particle shapes are discussed in Chapter 5. Chapter 6 examines the effects of attractive interparticle forces, leading to more complex microstructures, phase behavior, and thus rheology. Important time-dependent effects, such as thixotropy, are treated explicitly in Chapter 7. Chapter 8 is dedicated to the important phenomenon of shear thickening. Discussion of the rheological properties of colloidal dispersions is not complete without also covering specific problems related to accurate and precise rheological measurement, as well as the design of effective rheological experiments; this is the subject of Chapter 9. Whereas in all these chapters the suspending medium is assumed to be a Newtonian fluid, Chapter 10 considers the effects of suspending particles in viscoelastic media, covering the important cases of filled polymer solutions and melts and nanocomposites. The final chapter (11) provides a brief introduction to some advanced topics in suspension rheology, including sections on some special colloidal systems, more specifically electro- and magneto-rheological systems and colloids at interfaces: the so-called 2D dispersions. The latter section includes contributions by Professor J. Vermant (Katholieke Universiteit Leuven). In addition, some special rheological techniques are discussed, such as large amplitude oscillatory shear, superposition measurements, and microrheology, the latter section contributed by Professor E. Furst (University of Delaware). We thank these two colleagues for their valuable contributions to this text.

This book owes much to the scholarship of Professors W. B. Russel, W. R. Schowalter and the late D. A. Saville, all of Princeton University (and authors of *Colloidal Dispersions*), as well as the late Professor A. B. Metzner of the University of Delaware. It is through the schools of colloid science and rheology established at Princeton and Delaware that we became acquainted and started our research collaborations – their scholarship and mentoring motivated and influenced much of the science presented herein. Our many colleagues and mentors in the rheology community are also gratefully acknowledged. We thank Professors D. T. Leighton (University of Notre Dame), J. Morris (The City University of New York), D. Klingenberg (University of Wisconsin), J. Vermant (Katholieke Universiteit Leuven),

Preface**xv**

and W. B. Russel for commenting on early drafts of some chapters. Many colleagues, co-authors, and especially former and current students provided us with very valuable suggestions and information for the book – although all of their names cannot be mentioned here, one can readily find their work presented and cited throughout the text. For help in preparing the figures we thank J. Coffman, D. Kalman, A. Eberle, A. Golematis, E. Hermans, N. Reddy, and A. Schott, as well as the many students at Delaware who commented on and helped proofread versions of the text.

Many funding agencies helped support our research in this area during the time we wrote the manuscript, including the US National Science Foundation, the International Fine Particle Research Institute, the US Army Research Office, and corporations including Kodak, DuPont, Unilever, and Proctor & Gamble. The presentation of materials has benefitted from the short courses we developed and taught for the US Society of Rheology, as well as other institutions around the world. We especially thank the University of Delaware and Katholieke Universiteit Leuven for supporting us and our collaborations over the years, which made this book possible.

We sincerely hope you enjoy reading this book as much as we have enjoyed writing it. Our experience continues to be that the growing fields of colloid science and rheology are not only intellectually stimulating but of significant practical importance. We find these fields to be particularly collegial, and the participants have been very helpful as we have selected and assembled materials. Space limitations necessitated omitting many fine examples of colloidal suspension rheology and associated phenomena, but we hope the extensive referencing will aid the reader in exploration beyond what we could present here.

General list of symbols

a	particle radius [m]
a_i	particle radius of species/size i [m]
A	Hamaker constant [J]
c	mass concentration [kg m^{-3}]
$\mathbf{D}$	rate-of-strain tensor [s^{-1}]
D_f	fractal dimension [-]
D_{ij}	components of the rate-of-strain tensor [s^{-1}]
D	diffusivity tensor [$\text{m}^2 \text{s}^{-1}$]
D_0	Stokes-Einstein-Sutherland diffusivity, Eq. (1.5) [$\text{m}^2 \text{s}^{-1}$]
D_{ij}	components of the diffusivity tensor [$\text{m}^2 \text{s}^{-1}$]
$\mathcal{D}_r$	rotational diffusivity [s^{-1}]
$\mathcal{D}_{r,0}$	limiting rotational diffusivity for zero volume fraction [s^{-1}]
$\mathcal{D}^s$	self-diffusivity tensor [$\text{m}^2 \text{s}^{-1}$]
$\mathcal{D}_{ij}^s$	components of the self-diffusivity tensor [$\text{m}^2 \text{s}^{-1}$]
$\mathcal{D}^{ss}$	short-time self-diffusion coefficient [$\text{m}^2 \text{s}^{-1}$]
E	elasticity modulus [Pa]
e	electronic charge [C]
F	force [N]
g	gravity or acceleration constant [m s^{-2}]
$g(r)$	radial distribution function [-]
G	modulus [N m^{-2}]
G'	storage modulus [N m^{-2}]
G''	loss modulus [N m^{-2}]
G_{pl}	plateau modulus [N m^{-2}]
h	surface-to-surface distance between particles [m]
$\mathbf{I}$	unit tensor [-]
k	coefficient in the power law model [$\text{N s}^n \text{m}^{-1}$]
k'	coefficient in the Cross equation, Eq. (1.35) [s]
k_B	Boltzmann's constant [J K^{-1}]
k_H	Huggins coefficient [-]
L	length [m]
m	power law index in Cross model [-]
n	number density [m^{-3}]
N	number of particles [-]

General list of symbols

xvii

N_A	Avogadro's number [mol^{-1}]
N_i	i th normal stress difference [Pa]
P	pressure [Pa]
P_y	compressive yield stress [Pa]
q	scattering vector [nm^{-1}]
R	radius [m]
R_g	radius of gyration [m]
r	distance from center of particle [m]
S	entropy [J K^{-1}]
t	time [s]
T	temperature [K]
U	relative velocity between particles [m s^{-1}]
v	local speed [m s^{-1}]
V	volume [m^3]
$\mathbf{v}$	velocity vector [m]
v_i	velocity component in the i direction, $i = x, y$, or z [m s^{-1}]
W	stability ratio [-]
W^{shear}	stability ratio for shear-induced cluster formation [-]
x	Cartesian coordinate, in simple shear flow the flow direction [m]
y	Cartesian coordinate, in simple shear flow the velocity gradient direction [m]
z	Cartesian coordinate, in simple shear flow the vorticity direction [m]
II_i	second invariant of tensor i

Greek symbols

γ	strain [-]
γ_0	peak strain [-]
$\dot{\gamma}$	shear rate [s^{-1}]
δ	phase angle [-]
Δ	half width of a square-well potential [m]
ϵ	dielectric constant [-]
ϵ	depth of a square-well potential [J]
ϵ_0	permittivity of vacuum [$8.85 \times 10^{-12} \text{ F m}^{-1}$]
η	(suspension) viscosity [Pa s]
η'	dynamic viscosity [Pa s]
$[\eta]$	intrinsic viscosity [$\text{cm}^3 \text{ g}^{-1}$]
$[\eta]'$	dimensionless intrinsic viscosity [-]
κ	Debye-Hückel constant [m]
ν	number of particles or molecules per volume [m^{-3}]
Π	osmotic pressure [Pa]
θ	polar coordinate [-]
ρ	density [kg m^{-3}]
σ	shear stress tensor [Pa]

xviii **General list of symbols**

σ	shear stress in simple shear flow [Pa]
σ_y	yield stress [Pa]
σ_y^B	Bingham yield stress [Pa]
σ_y^d	dynamic yield stress [Pa]
τ	relaxation time [s]
τ_B	Baxter stickiness parameter [-]
ϕ	particle volume fraction [-]
Φ	particle interaction potential [J]
Ψ_i	<i>i</i> th normal stress coefficient [Pa s ²]
ψ	electrostatic potential [V]
Ψ	dimensionless electrostatic potential [-]
ψ_s	surface potential [V]
Ψ_s	dimensionless surface potential [-]
ζ	zeta potential [V]
ω	frequency [rad s ⁻¹]
Ω	rotational speed [s ⁻¹]

Subscripts

<i>eff</i>	effective
<i>el</i>	elastic contribution
<i>ext</i>	extensional
<i>floc</i>	<i>floc</i>
<i>g</i>	glass
<i>gel</i>	gel
<i>lin</i>	linearity limit
<i>m</i>	suspending medium/mean value
<i>M</i>	Maxwell
<i>max</i>	maximum value
<i>p</i>	particle
<i>pl</i>	plastic
<i>r</i>	relative
<i>y</i>	yield condition
0	limiting value in the zero shear limit
∞	limiting value at high shear rate or frequency

Superscripts

<i>B</i>	Brownian, with yield stress Bingham
<i>C</i>	Casson
<i>d</i>	dispersion
<i>g</i>	gravity
<i>h</i>	hydrodynamic
<i>hcY</i>	hard core Yukawa (potential)

General list of symbols

xix

<i>h_s</i>	hard sphere
<i>H</i>	Herschel-Bulkley
<i>I</i>	interparticle contribution
<i>m</i>	power law index in Cross model
<i>n</i>	power law index for shear stress
<i>s</i>	surface
<i>*</i>	complex

Dimensionless numbers

<i>Bo</i>	Boussinesq number, Eq. (11.22)
<i>De</i>	Deborah number (ratio of characteristic material time to characteristic process time)
<i>Ha</i>	Hartmann number, Eq. (4.2)
<i>Mn</i>	Mason number, Eq. (11.13)
<i>Mn_{mag}</i>	magnetic Mason number, Eq. (11.17)
<i>Pe_D</i>	Péclet number for microrheology, Eq. (11.6)
<i>Pe_i</i>	Péclet number for the ions, Eq. (4.3)
<i>Pe_μ</i>	Péclet number for microrheology, Eq. (11.3)
<i>Re</i>	Reynolds number ($\rho V D/\eta$)
<i>Re_p</i>	particle Reynolds number ($\rho \dot{\gamma} a^2/\eta_m$), Eq. (2.11)
<i>St</i>	Stokes number ($m_p \dot{\gamma}/6\pi\eta_m a$)
<i>Wi</i>	Weissenberg number (N_1/σ)

Useful physical constants and values

Note that many CODATA internationally recommended values can be found at physics.nist.gov/cuu/Constants/.

Constant		Value
e	Elementary charge	$1.602\,176\,487 \times 10^{-19} \text{ C}$
g	Standard acceleration of gravity	$9.806\,65 \text{ m s}^{-2}$
k_B	Boltzmann’s constant	$1.380\,650\,4 \times 10^{-23} \text{ J K}^{-1}$
m_u	Atomic mass unit	$1.660\,538\,782 \times 10^{-27} \text{ kg}$
N_A	Avogadro’s number	$6.022\,214\,170 \times 10^{23} \text{ mol}^{-1}$
R	molar gas constant	$8.314\,472 \text{ J mol}^{-1} \text{ K}^{-1}$
ϵ_0	Electric permittivity of vacuum	$8.854\,187\,817 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2} [\text{F m}^{-1}]$
μ_0	Vacuum permeability	$4\pi \times 10^{-7} \text{ N A}^{-2}$

Characteristic values

$k_B T$	$4.1 \times 10^{-21} \text{ J}$ (at room temperature)
$k_B T/e$	25.7 mV (at room temperature)
κ^{-1}	3.08 nm for a 10 mM 1:1 electrolyte in water at room temperature
l_b	0.7 nm for water at room temperature
Q	typically of $\mathcal{O}(1) \mu\text{C cm}^{-2}$

Properties of water at 298 K

ϵ	relative dielectric constant	80
η	viscosity	$8.90 \times 10^{-4} \text{ Pa s}$
ρ	density	997 kg m^{-3}

Useful Hamaker constants in water (units of 10^{-20} J)

Decane	0.46
Fused silica	0.85
Gold	30
Polystyrene	1.3
Poly(methyl methacrylate)	1.05
Poly(tetrafluoroethylene)	0.33