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The references are to pages. The following is a list of the abbreviations used.

alg. = algebraic,	equiv. = equivalent,	parat. = paratomy,
arb. = arbitrary,	extrav. = extravagant,	rat. = rational,
compl. = complementary,	homog. = homogeneous,	recip. = reciprocal,
conj. = conjugate,	horiz. = horizontal,	reg. = regular,
corresp. = correspondent,	inc. = incident,	sim. = similar,
corresp. = corresponding,	int. = intersecting,	sp. = spacelet,
degen. = degenerate,	intersec. = intersection,	sq. = square,
detant = determinant,	inv. = inverse,	sym. = symmetric,
detoid = determinoid,	non-int. = non-intersecting,	transfn. = transformation,
diag. = diagonal,	mut. = mutually,	vert. = vertical,
el. = element,	orthog. = orthogonal,	uncon. = unconnected,
equigr. = equigradent,	orthot. = orthotomy,	undeg. = undegenerate.

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See sub-heads.
- Matrices or matrix (qualifying terms indicating relations between matrices):
 Co-joint (of minor detants); conjugate; conjugate reciprocal; connected (horizontally, vertically); equi-extravagant; equigradent; equisignant; equivalent (horizontally, vertically); incident (lying in, containing); inverse; normal or orthogonal (horizontally, vertically); reciprocal; skew-conjugate; unconnected.
See sub-heads.
- Matrices or matrix (properties of a single matrix):
 Cores; degeneracy; determinant (of a sq. matrix); determinoid; derangements; elements (diagonal, non-diagonal); extravagances or extravagance; invariants; minor determinants (diagonal, non-diagonal, regular, simple); minor determinoids; minor matrices (diagonal, simple, square); orders; Pfaffian (of a skew-symmetric matrix of even order); rank; rows (long, short, horizontal, vertical, extravagant, non-extravagant, orthogonal, connections between); signants and signature; standard forms.
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- Matrices (properties of two or more):
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- Matrices or matrix (operations on or with):
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