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Excerpt
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Chapter 1

Mathematics for the 21st century

On a visit to a classroom, Nicola (N), an early childhood educator, had the following conversation with Tessa (T), a Year 3 child:[1]

N: What are you doing, Tessa?

T: Another maths sheet.

N: Oh great! I love doing maths. What is this one about?

T: Adding sums [said glumly].

N: Do you like doing maths, Tessa?

T: No. It's boring and anyway why should I when my teacher hates it too?

N: Does she really hate it? How do you know that?

T: She never smiles when we do maths.

This anecdote indicates the strength of young children's perceptions of the learning environment. It also highlights the fact that some adults lack confidence in and enthusiasm for mathematics. This book is designed to support teachers, parents and caregivers in providing a rich mathematical environment for children in their early years (birth to eight years of age). Young children who have worthwhile and interesting mathematical experiences have the opportunity to acquire mathematical understandings and positive attitudes towards mathematics. This enables them to become numerate and use mathematics effectively in their daily lives.

Early childhood educators use observations and interpretations of children's abilities, needs and interests as the starting point for designing activities that will support each individual's learning. Learning occurs when children:

- *explore* ideas through play, problem solving and problem posing
- are *actively* involved in what they are doing
- *engage* in a rich variety of multimodal learning experiences
- *pursue self-directed activities* and cooperate with others
- *complete activities* or pursue ideas to a logical conclusion
- *share* their ideas with others
- *make connections* between ideas and experiences in their everyday world.

These principles resonate with a social constructivist view of learning (see Vygotsky 1978) and align with viewing pedagogies as knowledge processes, as advocated under the Learning by Design model (e.g. Kalantzis & Cope 2012; Yelland, Cope & Kalantzis 2008). They provide a guide for designing effective learning experiences that enable young children to actively construct knowledge. Parents, teachers and

[1] The sex of a child has been designated as female or male in alternating chapters throughout this book to avoid the use of the terms 'he or she' and 'his or her'.

caregivers need to find ways to promote these principles in meaningful experiences to enable mathematical understandings to flourish, so that each child can become numerate.

Mathematics and society

Mathematics is an integral part of everyday life. Therefore, to function effectively people need to develop mathematical proficiency – this is what we mean by being *numerate*. For example, going shopping requires expertise with the concepts of number and money. When children accompany adults on shopping trips and observe money being exchanged for goods and change being given, some understanding about number and money is developed. Children's understanding of number and money can extend further when they play shops and take on the role of customers and shopkeepers, who 'buy and sell' goods. However, children need a wide variety of experiences so that they are able to broaden their understandings about number and money in this context. The use of number in our society is complex. Number is used to identify a quantity (e.g. three cars), to signify order (e.g. the third car) and in labels (e.g. a car registration plate, EAB303).

The mathematics that is taught in schools is influenced by the mathematics required for everyday life. In the past, school mathematics focused on learning tables and 'doing sums'. This was understandable in an era when speed and accuracy in computation were important for jobs, such as those of a shop assistant or a clerk. However, shopkeeper arithmetic is inadequate for the 'digital era' of the 21st century in which access to information has become a valuable commodity, just like material goods. Rapid technological innovations in recent decades have had an enormous impact on everyday lives. For example, items are electronically scanned at the supermarket and shopping via the internet is an ubiquitous part of everyday lives.

With the advent of information and communication technologies (ICT), there were high hopes that it would improve children's outcomes in mathematics and justify the substantial expenditure by schools on hardware, software and internet connections. However, this did not occur because although teachers changed the mode of delivery, they did not capitalise on the technologies and use innovative approaches to learning. For example, teachers replaced number fact worksheets with number fact software that looked like worksheets. Over time, there have been 'pockets' of innovation where teachers have begun to use ICT differently and software designers have created interactive learning products to engage young learners. These include 'apps' – the games, communication media, editing features and organisational applications that are available for use on smartphones and tablets; see Chapters 3 and 5–9 for examples. These innovations by teachers and software designers have influenced thinking about the teaching and learning of mathematics in contemporary times and led to new ways of thinking about how we become numerate in the 21st century (see Yelland 2005, 2007).

A further way mathematics has changed in the technologically oriented world is in the enormous collection and generation of data. Hence, it has become important for children to learn how to represent, organise and interpret data, and to create and interpret tables and graphs. Technological advancements have also impacted on the types of problems that students are required to solve in mathematics. In a rapidly changing world, students need to acquire skills that empower them to deal with novelty. Computational skills alone are insufficient for the solution of novel (non-routine) problems. To solve novel problems, students need to have a broad mathematical knowledge base, to employ thinking skills and to have a positive attitude towards mathematics. As a consequence of new thinking about learning using ICTs, the demand for skills in data collection, analysis and representation, and the need to deal with novel mathematics situations, aspects of today's mathematics program should be vastly different to that of even a decade ago.

Teaching and learning mathematics

Figure 1.1: *Thinking and problem solving are central to mathematical development*

The need for high-quality mathematics education has been endorsed worldwide. Landmark reports in Australia (Chubb 2012), the United Kingdom (ACME 2011) and the United States (Ball 2003; National Academies 2006) have all highlighted the importance of supporting students to become mathematically proficient to ensure they can function effectively in contemporary times. However, Steen (1997) argues that mathematical proficiency is essential in everyday life because 'an innumerate citizen of today is as vulnerable as the illiterate peasant in Gutenberg's time' (p. xv). The achievement of the essential goal of mathematical proficiency requires attention to *concepts*, *processes* and *attitudes* in the teaching of mathematics.

Concepts are the 'big ideas' of mathematics. The key ones are *number, measurement, space* (i.e. geometry), and *chance* and *data*. Concepts are discussed further throughout the book.

Processes are the thinking skills that children use to work mathematically and to develop an understanding of the concepts. Problem solving is the core process in mathematics. An important goal in mathematics education is for children to become problem solvers. Reasoning, communicating, making connections and representing ideas are also important processes for the 21st century. These processes are complex and involve the use of other processes, such as describing attributes, matching, sorting or classifying, comparing, ordering, patterning, counting, calculating, estimating, measuring, explaining, validating, analysing, inferring and organising.

Attitude affects a child's learning in mathematics. If a child is motivated and confident, she is likely to persevere when faced with a challenge. Experiences need to be interesting, enjoyable and have personal relevance for the child. Adult support should be carefully matched to the child's needs. If insufficient support is provided, the child may become frustrated and give up. If too much support is provided, she may feel helpless and unable to proceed independently. A child's attitude is also influenced by the attitudes of others. If someone believes in a child's capabilities, she usually fulfils that expectation.

Beliefs about how children learn are evident from the ways in which adults interact with learners. For example, when an adult claps and smiles as a child attempts to walk, there is recognition of the importance of encouragement in the learning process. The role of an adult in supporting children's learning is underpinned by beliefs about learning. Previously, there was a widespread belief that children learnt by absorbing knowledge. From this perspective, the role of the teacher at school was to 'transmit knowledge' to children and there was a focus on drill and practice activities in the classroom. However, it is now widely accepted that children learn by actively constructing knowledge and making connections between new knowledge and prior knowledge. For example, based on her prior experiences, a young child might initially think that all dogs are brown. But when she sees a black dog in a picture book and an adult identifies this animal as a dog, her understanding of the term 'dogs' is expanded. An improved understanding of how children learn has implications for the role of adults in facilitating children's learning. The following section focuses on the ways in which young children's mathematical understandings develop.

Figure 1.2: *There are many opportunities for mathematical explorations*

Early mathematical experiences

Parents, teachers and caregivers all play a significant role in establishing a child's confidence and competence in mathematics. Mathematical understandings are enhanced when a child observes and explores an object; thinks about and tests an idea; and communicates findings or asks questions. Early mathematical experiences occur in a variety of settings that might include the home, the early childhood centre, the school, the park, the zoo, the beach, or a social occasion such as a birthday party. Planned experiences, such as weighing objects, are structured to provide opportunities promoting mathematical understandings. However, spontaneous experiences can also provide valuable opportunities for exploring mathematical ideas and enriching meaning making. For example, a child learns about *number* when an adult counts the stairs that she climbs or about *space* when she is asked to look for a grocery item in the supermarket. As adults play and work with young children, there are many opportunities to enrich mathematical understanding through explorations and interactions. Throughout this book, there are specific examples of ways in which adults can support young children's mathematical understandings. To become a successful problem solver, a child needs to understand conceptual knowledge, to establish processes for investigating a problem and to acquire a positive attitude. Therefore, a young child needs a range of early mathematical experiences to develop the necessary concepts, processes and attitudes for problem solving. During problem-solving and problem-posing experiences, a child may encounter or need to use a variety of representations. Important representations in mathematics are described in the following section.

Mathematics and representations

Representations are the ways that children experience, think about and record the ideas that they have. In mathematics, a variety of representations are used. These include real-world situations, oral and written communication, manipulatives (e.g. counters), pictures, new technologies and symbols. Table 1.1 shows the role of these representations in mathematics.

Table 1.1: *Representations in mathematics*

REPRESENTATION	DESCRIPTION	EXAMPLE/S
Real-world situations	Provide a context for understanding and interpreting mathematical concepts.	The concept of *chance* is explored in a game involving dice.
Oral and written communication	Is a fundamental aspect of early mathematical learning. The child needs to be encouraged to talk about her mathematical experiences with others. Through conversation, mathematical terms become an integral part of the child's vocabulary. The child also needs to read and write about mathematical situations.	Literature can provide a context and a rich base for mathematical explorations. A child's personal writing provides a record of an exploration. Reading a child's work provides the teacher with an insight into the child's level of understanding.
Manipulatives, such as real objects (e.g. counters) or models of real objects	Enable ideas to be physically represented and the relationship between the objects acted out.	Plastic animals or counters could be used to represent the following situation: *There are 4 dogs and 2 ran away.*
Pictures	Facilitate the link between aspects of the real world and an abstract idea. They also provide the basis for ideas to be internalised as mental images.	Through concrete experiences, a child may form an image of 5 as a row of 3 counters and a row of 2 counters. This image may be of assistance in an addition (e.g. 3 + 2) or subtraction situation (e.g. 5 – 3).
New technologies (e.g. software, learning objects, apps)	Have extended the ways people think and have the potential to enhance mathematical understandings.	Children can take digital photos or movies of what they experience and view. They can play with a large range of apps to move things around in two-dimensional space to create scenarios and images.

Table 1.1: *cont.*

REPRESENTATION	DESCRIPTION	EXAMPLE/S
Symbols, such as numerals and operation signs	Are used as efficient representations of complex ideas in mathematics. Prior to the introduction of each symbol it is essential that the child has developed the understanding associated with the symbol.	In the equation 2 + 3, the symbol '+' will be meaningless if the child does not understand the concept of addition.

Each representation is important in the development of mathematical understandings. Observing how the child functions with a particular representation provides an insight into a child's understanding of a concept, such as *number*. Teachers, parents and caregivers can facilitate the process of mathematical explorations by providing opportunities for the child to become familiar with mathematical concepts through the use of a variety of representations. We refer to this as multimodal learning (e.g. Yelland 2007) – a defining characteristic of learning in the 21st century. Multimodal learning enables the child to engage with concepts in creative and interesting ways. In the early childhood years, the child is actively engaged in exploration through playing with concepts and ideas (see Yelland 2011).

Some representations are more difficult than others for particular children. For example, if a child is asked to find the total number of cats in a story in which two cats chase three cats, it may be easier for the child to represent the situation with manipulatives (e.g. 2 counters and 3 counters) than with symbols (e.g. 2 + 3). Knowing which representations are easier for particular children is one way of supporting the development of their mathematical understanding.

Although a child may initially use particular representations in isolation, it is important for a child to be able to move fluidly from one representation to another. For example, a child may be asked a question orally (e.g. 'How many blocks are needed to build a tower as tall as the cupboard?') and the child may use manipulatives to solve the problem. This process of changing from one representation to another is referred to as *translation*. Solving a problem rarely requires the use of just one representation. To be effective problem solvers, children need to be familiar with each representation and flexible in moving between representations.

About this book

The purpose of this book is to provide ideas for teachers, parents and caregivers of young children to encourage children from birth to eight years of age to become numerate and to create contexts for the children to enrich their mathematical understandings. The book provides background information about mathematics and processes and suggests ways to encourage mathematical explorations and interactions so that children become numerate in the 21st century.

Figure 1.3: *Mathematical moments can occur in a variety of contexts*

The book is organised so that information about and activities for very young children in the preschool years are considered first. Chapters 2 and 3 address the ways in which mathematical understandings can be supported in the years before formal schooling begins (birth to approximately five years of age). Chapter 4 provides an overview of mathematics for the young child beginning school. Chapters 5–9 are designed for the first three years of school and discuss concepts such as *number, measurement, patterns* and *algebra, space,* and *chance* and *data*. Background information about the concepts is provided at the start of these chapters, followed by research-based activities. Chapter 10 suggests the context and connections for mathematical understandings when conducting investigations and exploring topics. The examples address three varying focuses. These are planning a range of mathematical activities for a topic (e.g. *Ourselves*); using everyday events to encourage mathematical understandings (e.g. *Celebrations*); and finding ways to stimulate mathematical thinking in a variety of environments (e.g. *Out and About*). Finally, Chapter 11 provides an overview of ways in which teachers can design effective teaching and learning in a mathematics program.

References

Advisory Committee on Mathematics Education, 2011, *Mathematical Needs: The Mathematical Needs of Learners*, London: ACME

Ball, D. L. (Chair), 2003, *Mathematical Proficiency for All Students: Toward a Strategic Research and Development Program in Mathematics Education*, RAND Mathematics Study Panel, Arlington and Santa Monica: RAND Education

Chubb, I., 2012, *Mathematics, Engineering and Science in the National Interest*, Canberra: Commonwealth of Australia

Kalantzis, M. and Cope, B., 2012, *New Learning: Elements of a Science of Education*, 2nd edn, Melbourne: Cambridge University Press

National Academies, 2006, *Learning to Think Spatially*, Washington, D.C.: The National Academies Press

Steen, L. A., 1997, *Why Numbers Count: Quantitative Literacy for Tomorrow's America*, New York: The College Board

Vygotsky, L. S., 1978, *Mind in Society: The Development of Higher Psychological Processes*, Cambridge, MA: Harvard University Press

Yelland, N. J., 2011, 'Reconceptualising Play and Learning in the Lives of Children', *Australasian Journal of Early Childhood*, 36(2), 4–12

——2007, *Shift to the Future: Rethinking Learning with New Technologies in Education*, New York: Routledge

——2005, 'Curriculum Practice and Pedagogies with ICT in the Information Age', in N. Yelland (ed.), *Critical Issues in Early Childhood* (pp. 224–42), Buckingham, UK: Oxford University Press

Yelland, N., Cope, W. and Kalantzis, M., 2008, 'Learning By Design: Creating Pedagogical Frameworks for Knowledge Building in the 21st Century', *Asia Pacific Journal of Teacher Education*, 36(3), 197–213