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Wavelets: A Student Guide

This text offers an excellent introduction to the mathematical theory of wavelets for senior undergraduate students. Despite the fact that this theory is intrinsically advanced, the author's elementary approach makes it accessible at the undergraduate level.

Beginning with thorough accounts of inner product spaces and Hilbert spaces, the book then shifts its focus to wavelets specifically, starting with the Haar wavelet, broadening to wavelets in general, and culminating in the construction of the Daubechies wavelets. All of this is done using only elementary methods, bypassing the use of the Fourier integral transform. Arguments using the Fourier transform are introduced in the final chapter, and this less elementary approach is used to outline a second and quite different construction of the Daubechies wavelets. The main text of the book is supplemented by more than 200 exercises ranging in difficulty and complexity.

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PETER NICKOLAS
University of Wollongong, New South Wales

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Preface

Overview

The overall aim of this book is to provide an introduction to the theory of wavelets for students with a mathematical background at senior undergraduate level. The text grew from a set of lecture notes that I developed while teaching a course on wavelets at that level over a number of years at the University of Wollongong.

Although the topic of wavelets is somewhat specialised and is certainly not a standard one in the typical undergraduate syllabus, it is nevertheless an attractive one for introduction to students at that level. This is for several reasons, including its topicality and the intrinsic interest of its fundamental ideas. Moreover, although a comprehensive study of the theory of wavelets makes the use of advanced mathematics unavoidable, it remains true that substantial parts of the theory can, with care, be made accessible at the undergraduate level.

The book assumes familiarity with finite-dimensional vector spaces and the elements of real analysis, but it does not assume exposure to analysis at an advanced level, to functional analysis, to the theory of Lebesgue integration and measure or to the theory of the Fourier integral transform. Knowledge of all these topics and more is assumed routinely in all full accounts of wavelet theory, which make heavy use of the Lebesgue and Fourier theories in particular.

The approach adopted here is therefore what is often referred to as 'elementary'. Broadly, full proofs of results are given precisely to the extent that they can be constructed in a form that is consistent with the relatively modest assumptions made about background knowledge. A number of central results in the theory of wavelets are by their nature deep and are not amenable in any straightforward way to an elementary approach, and a consequence is that while most results in the earlier parts of the book are supplied with complete proofs, a few of those in the later parts are given only partial proofs or are proved only in special cases or are stated without proof. While a degree of intellectual danger is inherent in giving an exposition that is incomplete in this way, I am careful to acknowledge gaps where they occur, and I think that any minor disadvantages are outweighed by the advantages of being able to introduce such an attractive topic at the undergraduate level.

Structure and Contents

If a unifying thread runs through the book, it is that of exploring how the fundamental ideas of an orthonormal basis in a finite-dimensional real inner product space, and the associated expression of a vector in terms of its projections onto the elements of such a basis, generalise naturally and elegantly to suitable infinite-dimensional spaces. Thus the work starts in the familiar and concrete territory of Euclidean space and moves towards the less familiar and more abstract domain of sequence and function spaces.

The structure and contents of the book are shown in some detail by the Contents, but brief comments on the first and last chapters specifically may be useful.

Chapter 1 is essentially a miniaturised version of the rest of the text. Its inclusion is intended to allow the reader to gain as early as possible some sense of what wavelets are, of how and why they are used and of the beauty and unity of the ideas involved, without having to wait for the more systematic development of wavelet theory that starts in Chapter 5.

As noted above, the Fourier transform is an indispensable technical tool for the rigorous and systematic study of wavelets, but its use has been bypassed in this text in favour of an elementary approach in order to make the material as accessible as possible. Chapter 8, however, is a partial exception to this principle, since we give there an overview of wavelet theory using the powerful extra insight provided by the use of Fourier analysis. For students with a deeper background in analysis than that assumed earlier in the text, this chapter brings the work of the book more into line with standard approaches to wavelet theory. In keeping with this, we work in this chapter with complex-valued rather than real-valued functions.

Pathways

The book contains considerably more material than could be covered in a typical one-semester course, but lends itself to use in a number of ways for such a course. It is likely that Chapters 3 and 4 on inner product spaces and Hilbert spaces would need to be included however the text is used, since this material is required for the work on wavelets that follows but is not usually covered in

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such depth in the undergraduate curriculum. Beyond this, coverage will depend on the knowledge that can be assumed. The following three pathways through the material are proposed, depending on the assumed level of mathematical preparation.

At the most elementary level, the book could be used as an introduction to Hilbert spaces with applications to wavelets, by covering just Chapters 1–5, perhaps with excursions, which could be quite brief, into Chapters 6 and 7. At a somewhat higher level of sophistication, the coverage could largely or completely bypass Chapter 1, survey the examples of Chapter 2 briefly and then proceed through to the end of Chapter 7. A third pathway through the material is possible for students with a more substantial background in analysis, including significant experience with the Fourier transform and, preferably, Lebesgue measure and integration. Here, coverage could begin comfortably with Chapter 3 and continue to the end of Chapter 8.

Exercises

The book contains about 230 exercises, and these should be regarded as an integral part of the text. Although they are referred to uniformly as 'exercises', they range substantially in difficulty and complexity: from short and simple problems to long and difficult ones, some of which extend the theory or provide proofs that are omitted from the body of the text. In the longer and more difficult cases, I have generally provided hints or outlined suggested approaches or broken possible arguments into steps that are individually more approachable.

Sources

Because the book is an introductory one, I decided against including references to the literature in the main text. At the same time, however, it is certainly desirable to provide such references for readers who want to consult source material or look more deeply into issues arising from the text, and the Appendix provides these.

Acknowledgements

The initial suggestion of converting my lecture notes on wavelets into a book came from Jacqui Ramagge. Adam Sierakowski read and commented on the notes in detail at an early stage and Rodney Nillsen likewise read and commented on substantial parts of the book when it was in something much closer to its final form. I am indebted to these colleagues, as well as to two anonymous referees.