
Contents

<i>Preface</i>		<i>page xi</i>
1	Brownian Motion	1
1.1	Preliminaries and Notation	1
1.2	Definition and Basic Properties	1
1.3	Wiener Integral	7
1.4	Wiener Space	9
1.5	Brownian Filtration	9
1.6	Markov Property	10
1.7	Martingales Associated with Brownian Motion	11
1.8	Strong Markov Property	14
	Exercises	16
2	Stochastic Calculus	18
2.1	Stochastic Integrals	18
2.2	Indefinite Stochastic Integrals	23
2.3	Integral of General Processes	28
2.4	Itô's Formula	30
2.5	Tanaka's Formula	35
2.6	Multidimensional Version of Itô's Formula	38
2.7	Stratonovich Integral	40
2.8	Backward Stochastic Integral	41
2.9	Integral Representation Theorem	42
2.10	Girsanov's Theorem	44
	Exercises	47
3	Derivative and Divergence Operators	50
3.1	Finite-Dimensional Case	50
3.2	Malliavin Derivative	51
3.3	Sobolev Spaces	53
3.4	The Divergence as a Stochastic Integral	56

viii	<i>Contents</i>	
3.5	Isonormal Gaussian Processes	57
	Exercises	61
4	Wiener Chaos	63
4.1	Multiple Stochastic Integrals	63
4.2	Derivative Operator on the Wiener Chaos	65
4.3	Divergence on the Wiener Chaos	68
4.4	Directional Derivative	69
	Exercises	72
5	Ornstein–Uhlenbeck Semigroup	74
5.1	Mehler’s Formula	74
5.2	Generator of the Ornstein–Uhlenbeck Semigroup	78
5.3	Meyer’s Inequality	80
5.4	Integration-by-Parts Formula	83
5.5	Nourdin–Viens Density Formula	84
	Exercises	86
6	Stochastic Integral Representations	87
6.1	Clark–Ocone formula	87
6.2	Modulus of Continuity of the Local Time	90
6.3	Derivative of the Self-Intersection Local Time	96
6.4	Application of the Clark–Ocone Formula in Finance	97
6.5	Second Integral Representation	99
6.6	Proving Tightness Using Malliavin Calculus	100
	Exercises	103
7	Study of Densities	105
7.1	Analysis of Densities in the One-Dimensional Case	105
7.2	Existence and Smoothness of Densities for Random Vectors	108
7.3	Density Formula using the Riesz Transform	111
7.4	Log-Likelihood Density Formula	113
7.5	Malliavin Differentiability of Diffusion Processes	118
7.6	Absolute Continuity under Ellipticity Conditions	122
7.7	Regularity of the Density under Hörmander’s Conditions	123
	Exercises	129
8	Normal Approximations	131
8.1	Stein’s Method	131
8.2	Stein Meets Malliavin	136
8.3	Normal Approximation on a Fixed Wiener Chaos	138
8.4	Chaotic Central Limit Theorem	143
8.5	Applications to Fractional Brownian Motion	146

<i>Contents</i>		ix
8.6	Convergence of Densities	150
8.7	Noncentral Limit Theorems	153
	Exercises	156
9	Jump Processes	158
9.1	Lévy Processes	158
9.2	Poisson Random Measures	160
9.3	Integral with respect to a Poisson Random Measure	163
9.4	Stochastic Integrals with respect to the Jump Measure of a Lévy Process	164
9.5	Itô's Formula	168
9.6	Integral Representation Theorem	172
9.7	Girsanov's Theorem	174
9.8	Multiple Stochastic Integrals	175
9.9	Wiener Chaos for Poisson Random Measures	177
	Exercises	180
10	Malliavin Calculus for Jump Processes I	182
10.1	Derivative Operator	182
10.2	Divergence Operator	187
10.3	Ornstein–Uhlenbeck Semigroup	191
10.4	Clark–Ocone Formula	192
10.5	Stein's Method for Poisson Functionals	193
10.6	Normal Approximation on a Fixed Chaos	194
	Exercises	199
11	Malliavin Calculus for Jump Processes II	201
11.1	Derivative Operator	201
11.2	Sobolev Spaces	205
11.3	Directional Derivative	208
11.4	Application to Diffusions with Jumps	212
	Exercises	220
Appendix A	Basics of Stochastic Processes	221
A.1	Stochastic Processes	221
A.2	Gaussian Processes	222
A.3	Equivalent Processes	223
A.4	Regularity of Trajectories	223
A.5	Markov Processes	223
A.6	Stopping Times	224
A.7	Martingales	225
	<i>References</i>	228
	<i>Index</i>	235