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A Double Hall Algebra Approach to Affine Quantum Schur–Weyl Theory

Bangming Deng, Jie Du and Qiang Fu

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