

## Index

- $A(\mathbf{j}, r)$ , BLM basis element, 77
- $A[\mathbf{j}, r]$ , BLM basis element for affine Schur algebra, 185
- $A_\lambda$ , matrix defining the semisimple representation  $S_\lambda$ , 13
- $C = C_{\Delta(n)}$ , Cartan matrix of a cyclic quiver, 19
- $I$ , index set  $\mathbb{Z}/n\mathbb{Z} = \{1, 2, \dots, n\}$  of simple generators, 12
- $M(A)$ , representation of  $\Delta(n)$  associated with  $A$ , 13
- $M * N$ , generic extension, 14
- $R_j^\lambda$ ,  $j \in \mathbb{Z}$ , partition of  $\mathbb{Z}$ , 70
- $S_\lambda$ , semisimple representation, 13
- $S_i[l]$ , indecomposable representation of  $\Delta(n)$ , 12
- $\mathfrak{H}_\Delta(n)$ , Ringel–Hall algebra of  $\Delta(n)$ , 15
- $\Lambda(n, r)$ , set of compositions of  $r$  into  $n$  parts, 11
- $\Lambda^+(n, r)$ , set of partitions in  $\Lambda(n, r)$ , 135
- $\Lambda^+(r)$ , set of partitions of  $r$ , 133
- $\Omega$ , free  $\mathcal{Z}$ -module with basis  $\{\omega_i \mid i \in \mathbb{Z}\}$ , 75
- $\mathcal{F}_\Delta = \mathcal{F}_{\Delta, n}$ , set of cyclic flags of period  $n$ , 63
- $\mathcal{F}_{\Delta, \lambda}$ ,  $G$ -orbit in  $\mathcal{F}_\Delta$ , 63
- $\mathcal{H}_\Delta(r)$ , affine Hecke algebra, 64
- $\Lambda_\Delta(n, r)$ , set of sequences  $\lambda \in \mathbb{Z}_\Delta^n$  with  $\sigma(\lambda) = r$ , 11
- $\Theta_\Delta(n)$ , set of  $\mathbb{Z} \times \mathbb{Z}$ -matrices over  $\mathbb{N}$  with  $n$  periodicity, 11
- $\Theta_\Delta^-(n)$ , set of strictly lower triangular matrices in  $\Theta_\Delta(n)$ , 96
- $\Theta_\Delta^+(n)$ , set of strictly upper triangular matrices in  $\Theta_\Delta(n)$ , 13
- $\Theta_\Delta^+(n)^*$ , subset of non-zero matrices in  $\Theta_\Delta^+(n)$ , 27
- $\Theta_\Delta^\pm(n)$ , set of 0-diagonal matrices in  $\Theta_\Delta(n)$ , 77
- $\Theta_\Delta(n, r)$ , set of matrices  $A \in \Theta_\Delta(n)$  with  $\sigma(A) = r$ , 11
- $\widehat{\mathcal{K}}_\Delta(n)_\mathbb{Q}$ , completion of  $\mathcal{K}_\Delta(n)_\mathbb{Q}$ , 171
- $\mathbb{Z}_\Delta^n$ , set of infinite sequences of integers with  $n$  periodicity, 11
- $\mathcal{K}_\Delta(n)$ , algebra isomorphic to  $\bigoplus_{r \geq 0} \mathcal{S}_\Delta(n, r)$ , 169
- $\mathfrak{S}_{\Delta, r}$ , affine symmetric group, 68
- $\mathbf{U}_\Delta(n)$ , extended quantum affine  $\mathfrak{sl}_n$ , 43
- $\mathbf{U}_\Delta(n, r)$ , homomorphic image of  $\mathbf{U}_\Delta(n)$ , 154
- $\mathfrak{C}_\Delta(n)^\pm$ , subalgebras of  $\mathfrak{D}_\Delta(n)$ ,  $\cong \mathfrak{C}_\Delta(n)$ , 40
- $\mathbf{d}(A)$ , dimension vector of  $M(A)$ , 13
- $\iota_\lambda$ , idempotent  $[\text{diag}(\lambda)]$ , 99, 151, 154
- $\sigma(A)$ , sequence of hook sums, 101
- $\text{co}(A)$ , column sum vector, 10, 169
- $\mathfrak{C}_\Delta(n)$ , composition algebra of  $\Delta(n)$ , 17
- $\mathfrak{D}_\Delta(n)$ , double Ringel–Hall algebra, 37
- $\mathfrak{D}_\Delta(n)$ , integral form of  $\mathfrak{D}_\Delta(n)$ , 48
- $\mathfrak{D}_\Delta(n)^0$ , 0-part of  $\mathfrak{D}_\Delta(n)$ , 48
- $\mathfrak{D}_\Delta(n)^\pm$ ,  $\pm$ -part of  $\mathfrak{D}_\Delta(n)$ , 48
- $\mathfrak{S}_\lambda$ , Young subgroup associated with  $\lambda$ , 70
- $\alpha_A$ , order of  $\text{Aut}(M(A))$ , 14
- $\mathfrak{d}(A)$ , dimension of  $M(A)$ , 13
- $J_\Delta$ , bijection between double cosets and  $\Theta_\Delta(n, r)$ , 70
- $\langle \cdot, \cdot \rangle$ , Euler form, 15
- $\leq$ , order on  $\mathbb{Z}_\Delta^n$  or  $\mathbb{Z}^n$ , 11
- $\leq_{\text{dg}}$ , degeneration order, 14
- $\mathbb{C}^*$ , set of non-zero complex numbers, 11
- $v'$ , dual partition of  $v$ , 133

- $\preceq$ , order on  $M_{\Delta,n}(\mathbb{Z})$  or  $\Theta_{\Delta}(n)$ , 96
- ro( $A$ ), row sum vector, 10, 169
- $\mathcal{E}_{H,\mathbb{C}}$ , isomorphism  $\mathfrak{D}_{\Delta,\mathbb{C}}(n) \xrightarrow{\sim} U_{\mathbb{C}}(\widehat{\mathfrak{gl}}_n)$ , 54, 136
- $\mathcal{E}_H$ , isomorphism  $\mathfrak{D}_{\Delta}(n) \xrightarrow{\sim} U(\widehat{\mathfrak{gl}}_n)$ , 54
- $\mathcal{U}(\widehat{\mathfrak{gl}}_n)$ , universal enveloping algebra of  $\widehat{\mathfrak{gl}}_n$ , 179
- $\mathcal{Z} = \mathbb{Z}[v, v^{-1}]$ , Laurent polynomial ring in indeterminate  $v$ , 11, 14, 64, 122
- $\mathcal{X}$ , set of complete cyclic flags  $\mathcal{B}_{\Delta,r}(q)$ , 65
- $\mathcal{Y}$ , set of cyclic flags  $\mathcal{F}_{\Delta,n}(q)$ , 65
- $\sigma(A)$ , sum of  $n$  consecutive rows of  $A$ , 11
- $\sigma(\lambda)$ , sum of  $n$  consecutive components of  $\lambda$ , 11
- ${}^tA$ , transpose of  $A$ , 63
- $\tilde{C}_m, \tilde{C}_{\infty}$ , Borcherds–Cartan matrices, 42
- $\tilde{K}_{\alpha}, \tilde{K}_i$ , 27, 35, 42
- $\tilde{\mathfrak{D}}_{\Delta}(n)$ , candidate of Lusztig form, 102
  - $\mathfrak{H}_{\Delta}(n)^+$ , +-part of  $\tilde{\mathfrak{D}}_{\Delta}(n)$ , 49, 102
  - $\mathfrak{H}_{\Delta}(n)^-$ , --part of  $\tilde{\mathfrak{D}}_{\Delta}(n)$ , 102
- $\Delta(n)$ , cyclic quiver with  $n$  vertices, 12
- $\mathfrak{D}_A^A$ , PBW type basis element for  $\mathcal{S}_{\Delta}(n, r)$ , 101
- $\varphi_{B_1, \dots, B_m}^A$ , Hall polynomial, 14
- $\xi_r$ , epimorphism  $\mathfrak{D}_{\Delta}(n) \rightarrow \mathcal{S}_{\Delta}(n, r)$ , 86, 102
- $\xi_r^{\vee}$ , homomorphism  $\mathcal{H}_{\Delta}(r) \rightarrow \text{End}_{\mathfrak{D}_{\Delta}(n)}(\Omega^{\otimes r})$ , 86
- $\xi_{r,\mathbb{C}}$ , epimorphism  $\mathfrak{D}_{\Delta,\mathbb{C}}(n) \rightarrow \mathcal{S}_{\Delta}(n, r)_{\mathbb{C}}$ , 87, 137
- $\xi'_{r,\mathbb{C}}$ , epimorphism  $U_{\mathbb{C}}(\widehat{\mathfrak{gl}}_n) \rightarrow \mathcal{S}_{\Delta}(n, r)_{\mathbb{C}}$ , 137
- ${}^l\mathfrak{D}_{\Delta}(n)$ ,  $\mathfrak{D}_{\Delta}(n)$  with reduced 0-part, 45
- $d_A$ , as in  $[A] := v^{-d_A}e_A$ , 66
- $d'_A$ , as in  $\tilde{u}_A := v^{d'_A}u_A$ , 15
- $u_A = u_{[M(A)]}$ , basis elements of  $\mathfrak{H}_{\Delta}(n)$ , 15
- $u_{\mathbf{a}}$ , semisimple generator, 22
- $u_i$ , simple generators for  $\mathfrak{H}_{\Delta}(n)$ , 17
- $u_i^{\pm}$ , simple generators for  $\mathfrak{D}_{\Delta}(n)$ , 40
- $w_{0,\lambda}$ , longest element of  $\mathfrak{S}_{\lambda}$ , 71
- $\mathbf{Z}_{\Delta}(n)$ , central subalgebra, 45, 158
- affine Branching Rule, 136
- affine Hecke algebra
  - $\sim$  at a prime power  $q$ ,  $\mathcal{H}_{\Delta,q}$ , 64
  - $\sim$  at a prime power  $q$ ,  $\mathbb{C}_G(\mathcal{X} \times \mathcal{X}')$ , 65
  - $\sim$  of  $\mathfrak{S}_{\Delta,r}$ ,  $\mathcal{H}(\mathfrak{S}_{\Delta,r})$ , 72
  - $\sim$  over  $\mathcal{Z}$ ,  $\mathcal{H}_{\Delta}(r)$ , 64
  - $\sim$  over  $\mathbb{Q}(v)$ ,  $\mathcal{H}_{\Delta}(r)$ , 72
- affine quantum Schur algebra
  - $\sim$  at a prime power  $q$ ,  $\mathcal{S}_{\Delta,q}$ , 64
  - $\sim$  at a prime power  $q$ ,  $\mathbb{C}_G(\mathcal{Y} \times \mathcal{Y}')$ , 65
  - $\sim$  over  $\mathcal{Z}$ ,  $\mathcal{S}_{\Delta}(n, r)$ , 64
- $\sim$  over  $\mathbb{Q}(v)$ ,  $\mathcal{S}_{\Delta}(n, r)$ , 77
- the  $\pm$ , 0-part,  $\mathcal{S}_{\Delta}(n, r)^{\pm}$ ,  $\mathcal{S}_{\Delta}(n, r)^0$ , 98
- the Hecke algebra definition,  $\mathcal{S}_{\Delta}^{\mathcal{H}}(n, r)$ , 72
- the tensor space definition,  $\mathcal{S}_{\Delta}^{\mathcal{t}}(n, r)$ , 76
- affine symmetric group, 68
- affine Weyl group of type  $A$ ,  $W$ , 68
- aperiodic matrix, 13
- Bernstein presentation, 74
- BLM basis, 78
- BLM spanning set, 78
- Cartan datum, 25
- Cartan matrix, 19
  - $\sim$  of a cyclic quiver, 19
  - Borcherds– $\sim$ ,  $\tilde{C}_m, \tilde{C}_{\infty}$ , 42
- central elements  $z_m^+$  of  $\mathfrak{D}_{\Delta}(n)$ , 40, 99, 183
- central elements  $c_m$  of  $\mathfrak{H}_{\Delta}(n)$ , 21
- central subalgebra of  $\mathfrak{D}_{\Delta}(n)$ , 45, 153, 158
- commutator formula, 61
- commutator relation, 59
- composition algebra, 17
- convolution product, 65
- cyclic flag, 63
- cyclic quiver, 12
- defining relations
  - (QGL1)–(QGL8), 42
  - (QLA1)–(QLA7), 50
  - (QS1')–(QS6'), (QS0'), 165
  - (QS1)–(QS6), 156
  - (QSL0)–(QSL7), (QSL6'), (QSL7'), 20
  - (R1)–(R8), 43
- degeneration order, 14, 96
- dimension vector, 12
- double Ringel–Hall algebra
  - ${}^l\mathfrak{D}_{\Delta}(n)$ ,  $\sim$  with reduced 0-part, 45
  - $\mathfrak{D}_{\Delta}(n)$ ,  $\sim$ , 37
- Drinfeld double, 32, 36
  - reduced  $\sim$ , 36
- Drinfeld polynomials, 130, 132
- Euler form, 15
  - symmetric  $\sim$ ,  $(, )$ , 25
- extended quantum affine  $\mathfrak{sl}_n$ , 43, 153
- extended Ringel–Hall algebra, 27, 28
- Gaussian polynomial, 12
- generators for  $\mathcal{S}_{\Delta}(n, r)$ , 104
  - indecomposable  $\sim$ , 179
  - semisimple  $\sim$ , 172, 175
- generators for  $\mathfrak{H}_{\Delta}(n)$ 
  - homogeneous indecomposable  $\sim$ , 23

- homogeneous semisimple  $\sim$ , 21
- Schiffmann–Hubery  $\sim$ , 21
- generators for  $\mathfrak{D}_\Delta(n)$ , 45
- semisimple  $\sim$ , 55, 61, 86
- indecomposable representation of  $\Delta(n)$ , 12
- integral form
  - $\sim$  of  $\mathfrak{D}_\Delta(n)$ ,  $\mathfrak{D}_\Delta(n)$ , 48
  - candidate of Lusztig form,  $\tilde{\mathfrak{D}}_\Delta(n)$ , 102
- length function, 69
- loop algebra of  $\mathfrak{gl}_n$ , 9, 49, 179
- Morita equivalence, 124
- nilpotent representation, 12
  - aperiodic  $\sim$ , 13
  - periodic  $\sim$ , 13
- opposite Hopf algebra, 27
- PBW type basis, 15, 37, 81
- PBW type basis for  $\mathcal{S}_\Delta(n, r)$ , 101
- periodic matrix, 13
- polynomial identity, 115
- polynomial representation, 131
- primitive element, 37
- pseudo-highest weight module, 130, 131
- pseudo-highest weight vector, 130, 131
- quantum affine  $\mathfrak{sl}_n$ , 20
  - extended  $\sim$ ,  $\mathbf{U}_\Delta(n)$ , 43
- quantum affine  $\mathfrak{gl}_n$ , 50
- quantum loop algebra
  - $\sim$  of  $\mathfrak{gl}_n$ ,  $\mathbf{U}(\mathfrak{gl}_n)$ , 50
  - $\sim$  of  $\mathfrak{sl}_n$ ,  $\mathbf{U}(\mathfrak{sl}_n)$ , 20, 51
- realization conjecture, 171
- Ringel–Hall algebra
  - double  $\sim$ ,  $\mathfrak{D}_\Delta(n)$ , 37
  - extended  $\sim$ ,  $\mathfrak{H}_\Delta(n)^{\leq 0}$ , 28
  - extended  $\sim$ ,  $\mathfrak{H}_\Delta(n)^{\geq 0}$ , 27
- root datum, 26
- semi-opposite Hopf algebra, 27
- semisimple representation of  $\Delta(n)$ ,  $S_{\mathbf{a}}$ , 12
- simple representation
  - $\sim$  of  $\mathcal{H}_\Delta(r)_{\mathbb{C}}$ ,  $V_{\mathbf{s}}$ , 133
  - $\sim$  of  $\mathcal{S}_\Delta(n, r)_{\mathbb{C}}$ , 134, 149
  - $\sim$  of  $\mathbf{U}_{\mathbb{C}}(\widehat{\mathfrak{gl}}_n)$ ,  $L(\mathbf{Q})$ , 132
  - $\sim$  of  $\mathbf{U}_{\mathbb{C}}(\mathfrak{sl}_n)$ ,  $\bar{L}(\mathbf{P})$ , 130
  - $\sim$  of  $\Delta(n)$ ,  $S_i$ , 12
  - $\sim$  of  $\mathbf{U}_\Delta(n)_{\mathbb{C}}$ ,  $\bar{L}(\mathbf{P}, \lambda)$ , 150
- sincere, 12
- skew-Hopf pairing, 31
- symmetrization, 25
- tensor space
  - $\mathcal{T}_\Delta(n, r)$ , 65
  - $\sim$  over  $\mathbb{Q}(v)$ ,  $\Omega^{\otimes r}$ , 83
  - $\sim$  over  $\mathcal{Z}$ ,  $\Omega^{\otimes r}$ , 75
- transfer map, 172
- triangular decomposition of  $\mathcal{S}_\Delta(n, r)$ , 101
- triangular relation, 97