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A Double Hall Algebra Approach to Affine Quantum Schur–Weyl Theory

Bangming Deng, Jie Du and Qiang Fu

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BANGMING DENG

Beijing Normal University

JIE DU

University of New South Wales, Sydney

QIANG FU

Tongji University, Shanghai



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We dedicate the book to our teachers:

Peter Gabriel

Shaoxue Liu

Leonard Scott

Jianpan Wang

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Abstract

Over its one-hundred year history, the theory of Schur–Weyl duality and its quantum analogue have had and continue to have profound influences in several areas of mathematics such as Lie theory, representation theory, invariant theory, combinatorial theory, and so on. Recent new developments include, e.g., walled Brauer algebras and rational Schur algebras, quantum Schur superalgebras, and the integral Schur–Weyl duality for types other than A . This book takes an algebraic approach to the affine quantum Schur–Weyl theory.

The book begins with a study of extended Ringel–Hall algebras associated with the cyclic quiver of n vertices and the Green–Xiao Hopf structure on their Drinfeld double—the double Ringel–Hall algebra. This algebra is presented in terms of Chevalley type and central generators and is proved to be isomorphic to the quantum loop algebra of the general linear Lie algebra. The rest of the book investigates the affine quantum Schur–Weyl duality on three levels. This includes

- the affine quantum Schur–Weyl reciprocity;
- the bridging role played by the affine quantum Schur algebra between the quantum loop algebra and the corresponding affine Hecke algebra;
- Morita equivalence of certain representation categories;
- the presentation of affine quantum Schur algebras; and
- the realization conjecture for the double Ringel–Hall algebra which is proved to be true in the classical case.

Connections with various existing works by Lusztig, Varagnolo–Vasserot, Schiffmann, Hubery, Chari–Pressley, Frenkel–Mukhin, and others are also discussed throughout the book.

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