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To the memory of my parents
(MIRCEA SOFONEA)

To my family
(ANDALUZIA MATEI)

Contents

Preface

Contact processes between deformable bodies abound in industry and everyday life and, for this reason, considerable efforts have been made in their modelling and analysis. Owing to their inherent complexity, contact phenomena lead to new and interesting mathematical models. Here and everywhere in this book by a mathematical model we mean a system of partial differential equations, associated with boundary conditions and initial conditions, eventually, which describes a specific contact process.

The purpose of this book is to introduce the reader to some representative mathematical models which arise in Contact Mechanics. Our aim is twofold: first, to present a sound and rigorous description of the way in which the mathematical models are constructed; second, to present the mathematical analysis of such models which includes the variational formulation, existence, uniqueness and convergence results. To this end, we use results on various classes of variational inequalities in Hilbert spaces, that we present in an abstract functional framework. Also, we use various functional methods, including monotonicity, compactness, penalization, regularization and duality methods. Moreover, we pay particular attention to the mechanical interpretation of our results and, in this way, we illustrate the cross fertilization between modelling and applications on the one hand, and nonlinear analysis on the other hand.

This book is intended as a unified and readily accessible source for graduate students, as well as mathematicians, engineers and scientists. Its reading requires only basic knowledge of linear algebra, general topology, functional analysis and mechanics of continua.

The book is divided into two parts and seven chapters.

Part I contains Chapters 1–3 and represents a brief introduction to the study of variational inequalities. The material presented here was selected with emphasis on useful mathematical tools needed in the study of contact problems.

Part II contains Chapters 4–7 and represents the main part of the book. It is devoted to the modelling and analysis of representative frictionless or frictional contact problems and includes original results.

A brief description of the chapters of the book follows.

Chapter 1 is devoted to preliminaries on functional analysis which are fundamental to the developments later in this book. The material presented in this chapter is standard and can be found in many textbooks and monographs. For this reason we provide proofs only for the results which are frequently used in the rest of the book, i.e. the Banach fixed point theorem, the projection lemma, the Riesz representation theorem and the Weierstrass theorem, among others.

Chapter 2 is devoted to the study of several classes of elliptic variational inequalities with strongly monotone Lipschitz continuous operators. For each class of inequalities, we establish existence and uniqueness results for the solution by using arguments of monotonicity, fixed point or compactness. We also prove existence results by using penalization and regularization arguments. And, sometimes, we state and prove convergence results.

Chapter 3 deals with the study of a special class of variational inequalities, the so-called history-dependent variational inequalities, and is based on our original research. We present existence, uniqueness and convergence results, then we particularize them in the study of various evolutionary variational inequalities. Our aim in this chapter is to introduce a general framework in which a large number of quasistatic contact problems can be cast, and to construct mathematical tools which are useful in the study of these problems.

Chapter 4 introduces preliminary material needed in the study of contact problems. Here we introduce the spaces of functions used in Contact Mechanics, together with their main properties. We also present basic notions on modelling of contact problems, including the constitutive laws, the contact conditions and the friction laws used in the rest of the book. Finally, we pay particular attention to the modelling of contact processes with piezoelectric materials.

Chapter 5 is devoted to the study of static frictionless or frictional contact problems with nonlinearly elastic materials. The contact is either bilateral or is modelled with normal compliance or with the Signorini condition. Friction is modelled with versions of Coulomb's law, in which the friction bound is either given or depends on the normal stress. For each of the problems studied in this chapter we provide a variational formulation, which is in the form of an elliptic variational or quasivariational inequality for the displacement field. Then we use the abstract results

of Chapter 2 in order to establish existence, uniqueness and convergence results.

Chapter 6 deals with quasistatic frictionless and frictional contact problems. We model the material's behavior with viscoelastic or viscoplastic constitutive laws and, in the case of viscoelastic constitutive laws, we consider the cases of both short and long memory. The contact is either bilateral or is modelled with normal compliance with or without unilateral constraint, or with the so-called normal damped response condition. Friction is modelled with versions of Coulomb's law, in which the friction bound is either given or depends on the normal stress. For each of the problems studied in this chapter we provide a variational formulation, which is in the form of a history-dependent or evolutionary variational inequality for the displacement or the velocity field. Then we use the abstract results of Chapter 3 in order to establish existence, uniqueness and convergence results.

Chapter 7 deals with static and quasistatic frictionless and frictional contact problems for piezoelectric materials. We model the material's behavior with electro-elastic, electro-viscoelastic or electro-viscoplastic constitutive laws. The contact is either bilateral or is modelled with normal compliance. The foundation is assumed to be either electrically conductive or an insulator. For each of the problems studied in this chapter we provide a variational formulation, which is in the form of a system of variational inequalities for the displacement and the electric potential field. Then we use the abstract results of Chapters 2 and 3 in order to establish existence and uniqueness results.

Most of the results presented in this book can be extended to more general cases; however, since our aim is to provide an accessible presentation of the mathematical models which arise in the study of contact problems, we restrict ourselves to some introductory problems and we avoid giving the results in the most general abstract form, so that it is easier for the reader to understand more clearly the essential ideas involved. For instance, in the study of the variational inequalities presented in Part I of the book we restrict ourselves to the Hilbertian case, convex functions and strongly monotone Lipschitz continuous operators; also, in the study of history-dependent inequalities we avoid using the Bochner–Sobolev spaces $W^{k,p}(0,T;X)$ and we restrict ourselves to using spaces of continuous and continuously differentiable functions defined on the time interval $[0,T]$ with values in X. Nevertheless, we refer the reader to the section of bibliographical notes at the end of the book, where we provide references in which more information on the topics related to the body of the text can be found.

The present book is a result of cooperation between the authors during the past several years and was supported by the CNRS (France) and the Romanian Academy, under the *LEA Math-Mode* program, as well as the Romanian National Authority for Scientific Research (CNCS – UEFiSCDI) within the project PN-II-RU-TE-2011-3-0223. Its contents are based on

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