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Operator Methods for Boundary Value Problems

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Preface

The theory of unbounded operators, which dates back to the early 1930s, was developed by J. von Neumann and M.H. Stone. Of course the earlier work of H. Weyl on boundary eigenvalue problems and of T. Carleman on singular integral operators should be mentioned as stepping stones for the general abstract treatment. One of the underlying ideas was to put quantum mechanics on a rigorous mathematical foundation. J. von Neumann used a distinction between symmetric (Hermitian) and selfadjoint (hypermaximal) operators and referred to E. Schmidt for this. Once this distinction was made, it was natural to determine all selfadjoint extensions of a symmetric operator (necessarily with equal deficiency indices). J. von Neumann gave such a description for densely defined symmetric operators by means of his well-known formulas. This description requires the knowledge of the deficiency spaces of the symmetric operator. Another approach involving abstract boundary conditions was developed by J.W. Calkin in his 1937 Harvard doctoral dissertation, which was written under the direction of Stone, who suggested the topic. Unfortunately Calkin's work on boundary value problems did not receive the attention it deserved; probably because he never returned to it after his mathematical work related to World War II.

A revival of interest in applications of this approach to boundary value problems is due to M.G. Kreĭn, M.I. Vishik, M.S. Birman, and R. S. Phillips in the 1950s and, later, to G. Grubb, F.S. Rofe-Beketov, and M.L. Gorbachuk. The notion of boundary triplet was proposed in the 1970s by A.N. Kochubei and V.M. Bruk. The boundary triplet idea was used to introduce the associated Weyl function: an abstract analog of the classical Titchmarsh-Weyl m-function. This function formed a connection with the classical work of M.A. Naimark and M.G. Kreĭn from the early 1940s about exit space extensions. A densely defined symmetric operator with possibly unequal deficiency indices always has selfadjoint extensions once exit space extensions are allowed. In fact, Kreĭn had discovered the formula bearing his name describing the resolvents of the selfadjoint extensions, possibly in exit spaces; this procedure involves an operator-valued analytic function, the so-called Q-function, which equals the above Weyl function up to an additive constant. In the same period, during the 1970s, the theory was facilitated by the systematic introduction of the notion of linear relation (multivalued linear operator), so that it was not necessary to restrict attention to densely defined operators any longer. The background of this extension employed an idea of von Neumann, which was to use the graph to analyze an unbounded operator.

In Kreĭn's formula the Weyl function and a possibly multivalued parameter family play similar roles. A geometric interpretation of the Kreĭn formula gave rise to the notion of boundary relation by treating both the Weyl function and the parameter family on an even basis. The notion of boundary relation is based on an abstract form of Green's or Lagrange's identity, and this is where isometric and unitary relations in Kreĭn spaces come into play. Simultaneously, Kreĭn space methods received increasing attention in system theory. Recently, system theoretic equivalents for boundary triplets and boundary relations have been developed. The theory of boundary relations and triplets offers another approach to extension theory and operator theoretic methods to concrete boundary value problems. Various forms of boundary relations have appeared recently in conjunction with boundary value problems for elliptic operators (Dirichlet-to-Neumann mappings) and with local point interactions.

The idea of the present volume arose during a workshop at the Lorentz Center in Leiden, the Netherlands (December 14–December 18, 2009), entitled *Boundary relations, extension theory, and applications* and organized by the present editors. Several of the talks were devoted to recent developments in boundary triplet methods, system theory, and partial differential equations. This volume presents more or less a coherent presentation of these new developments with a historical review, beginning with Calkin's work from the late 1930s. Different aspects of boundary value problems are treated in contributions written by specialists in the field from their own point of view. We thank the Lorentz Center for their support in organizing the workshop and their hospitality during the conference. Moreover, we thank Sam Harrison of Cambridge University Press for his pleasant and constructive cooperation.

Seppo Hassi, Henk de Snoo and Franek Szafraniec