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**Foundations of
Computational Mathematics,
Budapest 2011**

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Preface

The Society for the Foundations of Computational Mathematics supports and promotes fundamental research in computational mathematics and its applications, interpreted in the broadest sense. It fosters interaction among mathematics, computer science and other areas of computational science through its conferences, workshops and publications. As part of this endeavour to promote research across a wide spectrum of subjects concerned with computation, the Society brings together leading researchers working in diverse fields. Major conferences of the Society have been held in Park City (1995), Rio de Janeiro (1997), Oxford (1999), Minneapolis (2002), Santander (2005), Hong Kong (2008), and Budapest (2011). The next conference is expected to be held in 2014. More information about FoCM is available at its website <http://focm-society.org>.

The conference in Budapest on July 4 – 14, 2011, was attended by some 450 scientists. FoCM conferences follow a set pattern: mornings are devoted to plenary talks, while in the afternoon the conference divides into a number of workshops, each devoted to a different theme within the broad theme of foundations of computational mathematics. This structure allows for a very high standard of presentation, while affording endless opportunities for cross-fertilization and communication across subject boundaries. Workshops at the Budapest conference were held in the following nineteen fields:

- Approximation theory
- Asymptotic analysis and high oscillation
- Computational algebraic geometry
- Computational dynamics
- Computational harmonic analysis, image and signal processing
- Computational number theory
- Continuous optimization
- Flocking, swarming, and control of distributed systems
- Foundations of numerical PDEs
- Geometric integration and computational mechanics
- Information-based complexity

- Learning theory
- Multiresolution and adaptivity in numerical PDEs
- Numerical linear algebra
- Random matrix theory, computations & applications
- Real-number complexity
- Special functions and orthogonal polynomials
- Stochastic computation
- Symbolic analysis

In addition to the workshops, eighteen plenary lectures, covering a broad spectrum of topics connected to computational mathematics, were delivered by some of the world's foremost researchers. One of these plenary lectures was presented by Snorre H. Christiansen, as the first recipient of the Stephen Smale Prize awarded by the Society for the Foundations of Computational Mathematics.

This volume is a collection of articles based on the plenary talks presented at FoCM 2011. The topics covered in the lectures and in this volume reflect the breadth of research within computational mathematics as well as the richness and fertility of interactions between seemingly unrelated branches of pure and applied mathematics. The Budapest gathering proved itself to be a stimulating meeting place of researchers in computational mathematics and of theoreticians in mathematics and computer science, with emphasis on multidisciplinary interaction across subjects and disciplines in an informal and friendly atmosphere.

We hope that this volume will be of interest to researchers in the field of computational mathematics and also to non-experts who wish to gain some insight into the state of the art in this active and significant field.

We wish to express our gratitude to the organizing company Scope Meetings Ltd, the Budapest University of Technology and Economics, the host institute, for their technical support, and the local organizing committee for making FoCM 2011 such an outstanding success. We also thank the National Science Foundation (award no. 1068800), and the Commission for Developing Countries of the International Mathematical Union for their financial assistance. We would like to thank the authors of the articles in this volume for producing in short order such excellent contributions. Above all, however, we wish to express our gratitude to all the participants of FoCM 2011 for attending the meeting and making it such an exciting, productive and scientifically stimulating event.

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