

Many processes in materials science and engineering, such as the load deformation behavior of certain solids and structures, exhibit nonlinear characteristics. The computer simulation of such processes therefore requires a deep understanding of both the theoretical aspects of nonlinearity and the associated computational techniques.

This book provides a complete set of exercises and solutions in the field of theoretical and computational nonlinear continuum mechanics, and is the perfect companion to *Nonlinear Continuum Mechanics for Finite Element Analysis*, where the authors set out the theoretical foundations of the subject. It employs notation consistent with the theory book and serves as a great resource to students, researchers, and those in industry interested in gaining confidence by practicing through examples. Instructors of the subject will also find the book indispensable in aiding student learning.

JAVIER BONET is a Professor of Engineering and the Head of the College of Engineering at Swansea University, and a visiting Professor at the Universitat Politècnica de Catalunya in Spain. He has extensive experience teaching topics in structural mechanics, including large strain nonlinear solid mechanics, to undergraduate and graduate engineering students. He has been active in research in the area of computational mechanics for more than 25 years, with contributions in modeling superplastic forming, large strain dynamic analysis, membrane modeling, finite element technology including error estimation, and meshless methods (smooth particle hydrodynamics). He has given invited, keynote, and plenary lectures at numerous international conferences.

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Cambridge University Press & Assessment

978-1-107-60361-5 — Worked Examples in Nonlinear Continuum Mechanics for Finite Element Analysis  
Javier Bonet , Antonio J. Gil , Richard D. Wood

Frontmatter

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To Catherine, Clare and Doreen

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WORKED EXAMPLES IN NONLINEAR  
CONTINUUM MECHANICS FOR  
FINITE ELEMENT ANALYSIS

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# WORKED EXAMPLES IN NONLINEAR CONTINUUM MECHANICS FOR FINITE ELEMENT ANALYSIS

**Javier Bonet**

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**Antonio J. Gil**

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**CAMBRIDGE**  
UNIVERSITY PRESS

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## CAMBRIDGE UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314-321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi - 110025, India

103 Penang Road, #05-06/07, Visioncrest Commercial, Singapore 238467

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Information on this title: [www.cambridge.org/9781107603615](http://www.cambridge.org/9781107603615)

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First published 2012

*A catalogue record for this publication is available from the British Library*

ISBN 978-1-107-60361-5 Paperback

Additional resources for this publication at [www.flagshyp.com](http://www.flagshyp.com)

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## PREFACE

This worked examples text is intended primarily as a companion to the second edition of the textbook *Nonlinear Continuum Mechanics for Finite Element Analysis* by Javier Bonet and Richard D. Wood. However, to be reasonably self-contained, where necessary key equations from the textbook are replicated in each chapter.

Textbook equation numbers given at the beginning of each chapter are indicated in square brackets.

Exercises are presented in a mix of direct (tensor), matrix, or indicial notation, whichever provides the greater clarity. Indicial notation is used only when strictly necessary and with summations clearly indicated.

The textbook is augmented by a website, [www.flagshyp.com](http://www.flagshyp.com), which contains corrections, software, and sample input data. Updates to this worked examples text will also be included on the website as necessary.